I. Vypočtěte a upravte parciální derivace 2.řádu:

1. $z = x\sqrt{y} + \dfrac{y}{\sqrt[3]{x}}$ $\qquad z''_{xx} = \dfrac{4y}{9\sqrt[3]{x^7}}$, $z''_{yy} = \dfrac{-x}{4\sqrt{y^3}}$, $z''_{xy} = \dfrac{1}{2\sqrt{y}} - \dfrac{1}{3\sqrt[3]{x^4}}$

2. $z = y^{x+1}$ $\qquad z''_{xx} = y^{x+1}\ln^2 y$, $z''_{yy} = x(x+1)y^{x-1}$,
   $z''_{xy} = y^x\left[1 + (x+1)\ln y\right]$

3. $z = x\sin(x+y)$ $\qquad z''_{xx} = 2\cos(x+y) - x\sin(x+y)$, $z''_{yy} = -x\sin(x+y)$,
   $z''_{xy} = \cos(x+y) - x\sin(x+y)$

4. $z = xe^{\frac{y}{x}}$ $\qquad z''_{xx} = \dfrac{y^2}{x^3}e^{\frac{y}{x}}$, $z''_{yy} = \dfrac{1}{x}e^{\frac{y}{x}}$, $z''_{xy} = \dfrac{-y}{x^2}e^{\frac{y}{x}}$

5. $z = y\operatorname{arctg}\dfrac{y}{x}$ $\qquad z''_{xx} = \dfrac{2xy^2}{\left(x^2+y^2\right)^2}$, $z''_{yy} = \dfrac{2x^3}{\left(x^2+y^2\right)^2}$, $z''_{xy} = \dfrac{-2x^2y}{\left(x^2+y^2\right)^2}$

6. $z = \ln\dfrac{x-y}{x+y}$ $\qquad z''_{xx} = \dfrac{-4xy}{\left(x^2-y^2\right)^2} = z''_{yy}$, $z''_{xy} = \dfrac{2(x^2+y^2)}{\left(x^2-y^2\right)^2}$

7. $z = x^2ye^{xy}$ $\qquad z''_{xx} = e^{xy}\left(x^2y^3 + 4xy^2 + 2y\right)$, $z''_{yy} = e^{xy}x^3\left(y+2\right)$,
   $z''_{xy} = e^{xy}\left(x^3y^2 + 4x^2y + 2x\right)$

II. Pro danou funkci určete hodnotu uvedeného výrazu:

1. $z = \ln\left(e^x + e^y\right)$, $V = z''_{xx} \cdot z''_{xy} + \left(z''_{xy}\right)^2$ $\qquad V = 0$

2. $z = xy + xe^{\frac{y}{x}}$, $V = z''_{xx} + \dfrac{y}{x}z''_{yx}$ $\qquad V = \dfrac{y}{x}$

3. $z = xy\operatorname{arctg}\dfrac{x}{y}$, $V = \left(x^2+y^2\right)^2 \cdot \left(z''_{xx} - z''_{yy}\right)$ $\qquad V = 2\left(x^4 + y^4\right)$

4. $z = 2\cos^2\left(x - \dfrac{y}{2}\right)$, $V = 2z''_{yy} + z''_{xy}$ $\qquad V = 0$

5. $z = \operatorname{arctg}(2x-y)$, $V = z''_{xx} + 2z''_{xy} + x$ $\qquad V = x$

6. $z = \operatorname{arctg}\dfrac{x+y}{1-xy}$, $V = \dfrac{1+x^2}{2} z''_{xx} - 2z''_{xy}$ $\qquad V = \dfrac{-x}{1+x^2}$

7. $z = e^{x+y}\left(x^2 + y + 1\right)$, $V = x \cdot \left(z''_{xy} - z'_x\right) + y \cdot \left(z''_{yy} - z'_y\right)$ $\qquad V = e^{x+y}(x+y)$

III. Ověřte, že daná funkce vyhovuje uvedené diferenciální rovnici:

1. $z = xe^{-\frac{y}{x}}$ … $xz''_{xy} + 2\left(z'_x + z'_y\right) = y \cdot z''_{yy}$

2. $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$ … $(x^2 + y^2) \cdot z'_x \cdot z''_{xx} = z''_{xy} \cdot \dfrac{x}{y \cdot z'_x}$

3. $z = \ln\dfrac{xy}{x-y}$ … $z''_{xx} \cdot z'_y + z''_{yy} \cdot z'_x = z''_{xy} \cdot \left(-\dfrac{x+y}{xy}\right)$

4. $z = \arcsin\dfrac{x}{y}$ … $\sqrt{y^2 - x^2} \cdot z''_{xx} + y \cdot z'_x \cdot z'_y = 0$