Integrály, které se často vyskytují při řešení školních úloh z dif. rovnic 1. řádu:

1. $\int \frac{x}{x^2+1}\mathrm{d}x = \frac{1}{2}\int \frac{2x}{x^2+1}\mathrm{d}x = \frac{1}{2}\ln(x^2+1)+C$

2. $\int \frac{1}{\sqrt{y}}\mathrm{d}y = \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{y}+C$

3. $\int \frac{x^2}{x^2+1}\mathrm{d}x = \int (1-\frac{1}{x^2+1})\mathrm{d}x = x - \mathrm{arctg}\, x + C$

4. $\int \frac{1}{x^2}\mathrm{d}x = \frac{x^{-1}}{-1} = -\frac{1}{x}+C$

5. $\int \frac{x^2}{x+1}\mathrm{d}x = \int (x-1+\frac{1}{x+1})\mathrm{d}x = \frac{x^2}{2} - x + \ln|x+1| + C$

6. $\int \frac{y^2}{y-1}\mathrm{d}y = \int (y+1+\frac{1}{y-1})\mathrm{d}y = \frac{y^2}{2} + y + \ln|y-1| + C$

7. $\int \frac{1}{y^2}\mathrm{d}y = \frac{y^{-1}}{-1} = -\frac{1}{y}+C$

8. $\int \frac{y}{1-y^2}\mathrm{d}y = -\frac{1}{2}\ln|1-y^2|+C$

9. $\int \frac{1}{x^2-1}\mathrm{d}x = -\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+C$

10. $\int \frac{1}{\sqrt{1-y^2}}\mathrm{d}y = \arcsin y + C$

11. $\int \frac{1}{\sqrt{x}}\mathrm{d}x = 2\sqrt{x}+C$

12. $\int \frac{x^2+1}{x}\mathrm{d}x = \int (x+\frac{1}{x})\mathrm{d}x = \frac{x^2}{2}+\ln|x|+C$

13. $\int \frac{1}{y+1}\mathrm{d}y = \ln|y+1|+C$

14. $\int \frac{y}{2+y^2}\mathrm{d}y = \frac{1}{2}\ln(2+y^2)+C$

15. $\int \frac{1}{\mathrm{e}^y}\mathrm{d}y = -\mathrm{e}^{-y} = -\frac{1}{\mathrm{e}^y}+C$

16. $\int \frac{x+1}{x}\mathrm{d}x = \int (1+\frac{1}{x})\mathrm{d}x = x+\ln|x|+C$

17. $\int \frac{x}{x-1}\mathrm{d}x = \int (1+\frac{1}{x-1})\mathrm{d}x = x+\ln|x-1|+C$

18. $\int \frac{1}{y^2-1}\mathrm{d}y = -\int \frac{1}{1-y^2}\mathrm{d}y = -\frac{1}{2}\ln\left|\frac{1+y}{1-y}\right|+C$

19. $\int \frac{1}{1-y}\mathrm{d}y = -\int \frac{-1}{1-y}\mathrm{d}y = -\ln|1-y|+C$

20. $\int \frac{\mathrm{e}^x}{1+\mathrm{e}^x}\mathrm{d}x = \ln|1+\mathrm{e}^x|+C$

21. $\int \frac{y}{y^2-1}\mathrm{d}y = \frac{1}{2}\ln|y^2-1|+C$

22. $\int \frac{1}{y\ln y}\mathrm{d}y = \left|\begin{array}{c} \ln y = t \\ \frac{1}{y}\mathrm{d}y = \mathrm{d}t \end{array}\right| = \int \frac{1}{t}\mathrm{d}t = \ln|t| = \ln|\ln y|+C$

23. $\int \frac{x-1}{x^2}\mathrm{d}x = \int (\frac{1}{x}-\frac{1}{x^2})\mathrm{d}x = \ln|x| - \frac{x^{-1}}{-1} = \ln|x|+\frac{1}{x}+C$

24. $\int x\,\mathrm{e}^{x^2}\mathrm{d}x = \left|\begin{array}{c} x^2 = t \\ 2x\mathrm{d}x = \mathrm{d}t \end{array}\right| = \int \mathrm{e}^t \frac{1}{2}\mathrm{d}t = \frac{1}{2}\mathrm{e}^{x^2}+C$

A nakonec

$$\int x\,\mathrm{e}^{2x}\mathrm{d}x = \left|\begin{array}{cc} u' = \mathrm{e}^{2x} & u = \frac{1}{2}\mathrm{e}^{2x} \\ v = x & v' = 1 \end{array}\right| = \frac{1}{2}x\,\mathrm{e}^{2x} - \int \frac{1}{2}\mathrm{e}^{2x}\mathrm{d}x = \frac{1}{2}x\,\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x} = \mathrm{e}^{2x}(\frac{1}{2}x-\frac{1}{4})+C$$