DIFERENCIÁLNÍ ROVNICE 1. ŘÁDU

I. DR 1. řádu se separovatelnými proměnnými

1) $e^{-2x}y' - e^{-y} = 0$ $\quad y = \ln\left(\frac{1}{2}e^{2x} + c\right)$.

2) $e^{x} + y^{2}y' = 0\,,\ y \neq 0$ $\quad y^{3} = c - 3e^{x}$.

3) $x^{2}yy' = y^{2} + 2$ $\quad y^{2} = c \cdot e^{-\frac{2}{x}} - 2$.

4) $(x+1)y' + xy = 0\,,\ y(0) = 1$ $\quad y = (x+1)e^{-x}$. (Obecné řeš. $y = c(x+1)e^{-x}$)

5) $y\ln y + xy' = 0\ ,\ y(1) = 1$ $\quad y = 1$. (Obecné řešení $y = e^{\frac{c}{x}}, c \in R$)

6) $y\cos x - (\sin x)y' = 0\,,\ y\left(\frac{\pi}{2}\right) = 1$ $\quad y = \sin x$. (Obecné řešení $y = c \cdot \sin x, c \in R$)

7) $\frac{1}{2y+1}dy - \operatorname{cotg} x\,dx = 0\,,\ y\left(\frac{\pi}{4}\right) = \frac{1}{2}$ $\quad y = 2\sin^{2} x - \frac{1}{2}$.

8) $y'\operatorname{tg} x - y = 1, y\left(\frac{\pi}{2}\right) = 1$ $\quad y = 2\sin x - 1$.

9 $\frac{x}{1+y} - \frac{y}{1+x}y' = 0\,,\ y(0) = 1$ $\quad 2\left(y^{3} - x^{3}\right) + 3\left(y^{2} - x^{2}\right) - 5 = 0$.

10) $9(1+y) - \left(9 - x^{2}\right)yy' = 0\,,\ y(0) = 0$ $\quad y - \ln|1+y| = \frac{3}{2}\ln\left|\frac{3+x}{3-x}\right|$.

11) $\sin x\cos y + y' \cdot \operatorname{tg} y\cos x = 0$ $\quad \cos y = \frac{1}{\ln|\cos x| + c}$.

12) $yy' = xe^{x+y}$ $\quad e^{-y}(y+1) = e^{x}(1-x) + c$.

13) $xy' + y = y^{2}\ ,\ y(1) = \frac{1}{2}$ $\quad y = \frac{1}{x+1}$. (Obecné řešení $\frac{1-y}{y} = cx \Rightarrow y = \frac{1}{cx+1}$)

II. Lineární DR 1. řádu

1) $y' - \frac{2x}{1+x^2} y = 0$, $y(1) = 2$ — $y = 1 + x^2$. (Obecné řeš. $y = c(1+x^2)$)

$y' - \frac{2x}{1+x^2} y = 0$, $y(2) = 10$ — $y = 2(1+x^2)$.

2) $y' + \frac{1}{x} y = 3x$ — $y = x^2 + \frac{c}{x}$.

3) $(1+x^2)y' - 2xy = (1+x^2)^2$ — $y = (x+c)(1+x^2)$.

4) $y' - \frac{1}{\sqrt{x}} y = \mathrm{e}^{2\sqrt{x}}$ — $y = (x+1)\mathrm{e}^{2\sqrt{x}}$.

5) $x(y' - y) = (1+x^2)\mathrm{e}^x$, $y(1) = 0$ — $y = \left(\ln|x| + \frac{x^2}{2} - \frac{1}{2} \right) \mathrm{e}^x$.

6) $y' + 2y = 4x$ — $y = c\,\mathrm{e}^{-2x} + 2x - 1$.

7) $xy' = x + \frac{1}{2} y$ — $y = 2x + c\sqrt{x}$.

8) $(x+1)y' - 2y - (x+1)^4 = 0$ — $y = \left(\frac{x^2}{2} + x + c \right)(x+1)^2$.

9) $xy' - (1+x^2)y = 2x^3$ — $y = cx\,\mathrm{e}^{\frac{x^2}{2}} - 2x$.

10) $y' + y \operatorname{tg} x = \cos^2 x$ — $y = c \cdot \cos x + \sin x \cos x$.

11) $y' - \frac{x}{1+x^2} y = 1$, $y(0) = 3$ — $y = \sqrt{1+x^2} \left(\ln\left| x + \sqrt{1+x^2} \right| + 3 \right)$.

12) $y' - \frac{8x}{2x^2+1} y = 2x^2 + 1$ — $y = \frac{\sqrt{2}}{2} (2x^2+1)^2 \operatorname{arctg} x\sqrt{2} + c(2x^2+1)^2$.

13) $y' - y \sin x = \sin x \cos x$ — $y = c\,\mathrm{e}^{-\cos x} + 1 - \cos x$.

14) $y' = 2xy - x^3 + x$ — $y = c\,\mathrm{e}^{x^2} + \frac{1}{2} x^2$.

III. DR 1. řádu s homogenní funkcí

1) $y' = \frac{2y}{x}$ $\quad y = cx^2$.

2) $y' = \frac{y}{x}(1 + \ln y - \ln x)$ $\quad y = x \cdot e^{cx}$.

3) $xy' \cos\frac{y}{x} = y\cos\frac{y}{x} - x$ $\quad x e^{\sin\frac{y}{x}} = c$.

4) $xyy' - 2y^2 - x^2 = 0$ $\quad y^2 = cx^4 - x^2$.

5) $xy' - y = \sqrt{x^2 + y^2}$, $y(2) = 0$ $\quad y + \sqrt{x^2 + y^2} = \frac{1}{2}x^2$.

6) $(x + y)\mathrm{d}x + (x - y)\mathrm{d}y = 0$ $\quad y^2 - 2xy - x^2 = c$.

7) $xyy' = y^2 - 2xy - x^2$ $\quad \frac{y}{x} - \ln\left|\frac{y}{x} + 1\right| = -\ln|x| + c$ odtud po úpravě

$e^{\frac{y}{x}} \cdot \frac{x^2}{y + x} = c$.

8) $(y + x)\mathrm{d}y = (y - x)\mathrm{d}x$ $\quad \sqrt{x^2 + y^2} = c \cdot e^{-\mathrm{arctg}\frac{y}{x}}$.

9) $y^2 + x^2\frac{\mathrm{d}y}{\mathrm{d}x} = xy\frac{\mathrm{d}y}{\mathrm{d}x}$ $\quad y \cdot e^{-\frac{y}{x}} = c, y = 0$.

10) $y' = \frac{2xy}{x^2 - y^2}$ $\quad xy = cx(x^2 + y^2), y = 0$. (parc. zlomky)

11) $y' = \frac{2y}{x - y}$ $\quad \ln\left|\frac{y}{x}\right| - 2\ln\left|\frac{y}{x} + 1\right| = \ln|x| + c$ odtud po úpravě

$c = \frac{2(y + x)}{\sqrt{y}}$, $y = -x, y = 0$.