I. Několik řešených příkladů …

1) $y = \dfrac{x}{x^2+1}$

$$y' = \frac{x' \cdot \left(x^2+1\right) - x \cdot \left(x^2+1\right)'}{\left(x^2+1\right)^2} = \frac{1 \cdot \left(x^2+1\right) - x \cdot 2x}{\left(x^2+1\right)^2} = \frac{x^2+1-2x^2}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)^2}$$

2) $y = \dfrac{1-2x^3}{x^2}$

$$y' = \frac{\left(1-2x^3\right)' \cdot x^2 - \left(1-2x^3\right)\left(x^2\right)'}{\left(x^2\right)^2} = \frac{-6x^2 \cdot x^2 - \left(1-2x^3\right) \cdot 2x}{\left(x^2\right)^2} = \frac{-6x^4 - 2x + 4x^4}{x^4} =$$

$$= \frac{-2x^4 - 2x}{x^4} = -2\frac{x^3+1}{x^3}$$

3) $y = x^2 + \ln(x+1)$

$$y' = \left(x^2\right)' + \left[\ln(x+1)\right]' = 2x + \frac{1}{x+1}$$

4) $y = e^x\left(x^2-4x+4\right)$

$$y = (e^x)'\left(x^2-4x+4\right) + e^x\left(x^2-4x+4\right)' = e^x\left(x^2-4x+4\right) + e^x(2x-4) = e^x\left(x^2-2x\right)$$

5) $y = \left(x^2-1\right)\sin x + \left(x^2+2\right)\cos x$

$$y' = \left(x^2-1\right)' \sin x + \left(x^2-1\right)(\sin x)' + \left(x^2+2\right)' \cos x + \left(x^2+2\right)(\cos x)' =$$

$$= 2x\sin x + \left(x^2-1\right)\cos x + 2x\cos x + \left(x^2+2\right)(-\sin x) =$$

$$= \sin x\left(2x - x^2 - 2\right) + \cos x\left(x^2+2x-1\right)$$

6) $y = x\sqrt{x^2+3}$

$$y' = x'\sqrt{x^2+3} + x\left(\sqrt{x^2+3}\right)' = 1 \cdot \sqrt{x^2+3} + x \cdot \frac{1}{2}\left(x^2+3\right)^{-\frac{1}{2}} \cdot 2x =$$

$$= 1 \cdot \sqrt{x^2+3} + \frac{x \cdot 2x}{2\sqrt{x^2+3}} = \frac{x^2+3+x^2}{\sqrt{x^2+3}} = \frac{2x^2+3}{\sqrt{x^2+3}}$$

7) $y = x\ln\left(x^2-1\right)$

$$y' = x'\ln\left(x^2-1\right) + x\left[\ln\left(x^2-1\right)\right]' = 1 \cdot \ln\left(x^2-1\right) + x \cdot \frac{1}{x^2-1} \cdot 2x = \ln\left(x^2-1\right) + \frac{2x^2}{x^2-1}$$

8) $y = \left(\dfrac{x-2}{x+2}\right)^3$

$$y' = 3\left(\frac{x-2}{x+2}\right)^2 \cdot \left(\frac{x-2}{x+2}\right)' = 3\left(\frac{x-2}{x+2}\right)^2 \cdot \frac{(x-2)'(x+2) - (x-2)(x+2)'}{(x+2)^2} =$$

$$= 3\frac{(x-2)^2}{(x+2)^2}\cdot\frac{1\cdot(x+2)-(x-2)\cdot 1}{(x+2)^2} = 3\frac{(x-2)^2(x+2-x+2)}{(x+2)^4} = \frac{12(x-2)^2}{(x+2)^4}$$

9) $y = \ln\left(\frac{x^2+1}{x^2-1}\right)$

$$y' = \frac{1}{\frac{x^2+1}{x^2-1}}\cdot\left(\frac{x^2+1}{x^2-1}\right)' = \frac{x^2-1}{x^2+1}\cdot\frac{(x^2+1)'(x^2-1)-(x^2+1)(x^2-1)'}{(x^2-1)^2} =$$

$$= \frac{x^2-1}{x^2+1}\cdot\frac{2x(x^2-1)-(x^2+1)2x}{(x^2-1)^2} = \frac{2x^3-2x-2x^3-2x}{(x^2+1)(x^2-1)} = \frac{-4x}{x^4-1}$$

10) $y = \ln^4 tg(2x+1)$

$$y' = 4\ln^3 tg(2x+1)\cdot[\ln tg(2x+1)]' = 4\ln^3 tg(2x+1)\cdot\frac{1}{tg(2x+1)}[tg(2x+1)]' =$$

$$= 4\ln^3 tg(2x+1)\cdot\frac{1}{tg(2x+1)}\cdot\frac{1}{\cos^2(2x+1)}(2x+1)' =$$

$$= 4\ln^3 tg(2x+1)\cdot\frac{1}{tg(2x+1)}\cdot\frac{1}{\cos^2(2x+1)}\cdot 2$$

II. Vypočtěte derivaci a výsledek upravte

1) $y = x\ln x$ $\qquad y' = \ln x + 1$

2) $y = \frac{2+\sqrt{x}}{x}$ $\qquad y' = -\frac{4+\sqrt{x}}{2x^2}$

3) $y = \sqrt{\frac{x^2+1}{x^2-1}}$ $\qquad y' = \sqrt{\frac{x^2-1}{x^2+1}}\cdot\frac{-2x}{(x^2-1)^2}$

4) $y = \ln\frac{1}{x+\sqrt{x^2-1}}$ $\qquad y' = -\frac{1}{\sqrt{x^2-1}}$

5) $y = arctg\frac{x}{2} + \ln\sqrt{\frac{x-2}{x+2}}$ $\qquad y' = \frac{4x^2}{x^4-16}$

6) $y = \ln arctg\frac{1}{1+x}$ $\qquad y' = \frac{1}{arctg\frac{1}{1+x}}\cdot\frac{-1}{x^2+2x+2}$

7) $y = \arcsin\sqrt{\frac{1-x}{1+x}}$ $\qquad y' = -\frac{1}{(1+x)\sqrt{2x(1-x)}}$

8) $y=\frac{1}{\sqrt{3}}arctg\frac{x\sqrt{3}}{1-x^2}$ $\qquad y'=\frac{1+x^2}{1+x^2+x^4}$

9) $y=\sqrt{x+1}-\ln\left(1+\sqrt{x+1}\right)$ $\qquad y'=\frac{1}{2\left(1+\sqrt{x+1}\right)}$

10) $y=\frac{\sin x-x\cos x}{\cos x+x\sin x}$ $\qquad y'=\frac{x^2}{(\cos x+x\sin x)^2}$

III. Vypočtěte hodnotu 1.derivace funkce v daném bodě

1) $y=\frac{1+\cos x}{1-\cos x}$, $x_0=\frac{\pi}{2}$ $\qquad y'\left(\frac{\pi}{2}\right)=-2$

2) $y=xe^{1-\cos^2 x}$, $x_0=0$ $\qquad y'(0)=1$

3) $y=\ln\sqrt{\frac{1-\sin x}{1+\sin x}}$, $x_0=\pi$ $\qquad y'(\pi)=1$

4) $y=\frac{\sqrt{x}+1}{\sqrt{x}-1}$, $x_0=4$ $\qquad y'(4)=-\frac{1}{2}$

5) $y=\arcsin\frac{x}{\sqrt{4-x^2}}$, $x_0=1$ $\qquad y'(1)=\frac{4}{3\sqrt{2}}$

6) $y=2arctg\sqrt{\frac{2-x}{x}}$, $x_0=1$ $\qquad y'(1)=-1$

7) $y=\sqrt{e^{\left(x^2\right)}}$, $x_0=1$ $\qquad y'(1)=\sqrt{e}$

8) $y=\frac{8}{x}-\frac{3x^2}{\sqrt[3]{x}+1}$, $x_0=8$ $\qquad y'(8)=-\frac{1033}{72}$

9) $y=\frac{\cos^3 x+1}{\sqrt{x}}-\frac{2}{x^2}$, $x_0=\pi$ $\qquad y'(\pi)=\frac{4}{\pi^3}$

10) $y=\frac{1}{arctg\sqrt{x}}+x\frac{2-\sqrt[4]{x}}{x+1}$, $x_0=1$ $\qquad y'(1)=\frac{1}{8}-\frac{4}{\pi^2}$