

Vypočítejte následující limity pomocí l'Hospitalova pravidla:

$$1. \lim_{x \rightarrow 1} \frac{x - 1}{\ln x} \quad [1]$$

$$2. \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} \quad [0]$$

$$3. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} \quad \left[\frac{1}{3}\right]$$

$$4. \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3} \quad \left[-\frac{1}{3}\right]$$

$$5. \lim_{x \rightarrow 0} \frac{xe^x + x - 2e^x + 2}{x^3} \quad \left[\frac{1}{6}\right]$$

$$6. \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{2 \sin^2 x + 2x \sin 2x} \quad \left[-\frac{1}{3}\right]$$

$$7. \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} \quad [2]$$

$$8. \lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^2 \sin x^2} \quad \left[\frac{1}{2}\right]$$

$$9. \lim_{x \rightarrow 0} \frac{x \operatorname{cotg} x - 1}{x^2} \quad \left[-\frac{1}{3}\right]$$

Vypočtete následující limity pomocí l'Hospitalova pravidla:

Typ $\frac{0}{0}$ a $\frac{\pm\infty}{\pm\infty}$.

$$1. \lim_{x \rightarrow \infty} \frac{x \ln x}{x^2 + x + 1} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow \infty} \frac{\ln x + 1}{2x + 1} \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2} \stackrel{\text{dosaz.}}{=} 0$$

$$2. \lim_{x \rightarrow 0} \frac{\arctg 2x}{\arcsin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\frac{2}{1+4x^2}}{\frac{1}{\sqrt{1-x^2}}} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 0} \frac{2\sqrt{1-x^2}}{1+4x^2} \stackrel{\text{dosaz.}}{=} 2$$

$$3. \lim_{x \rightarrow 1} \frac{x-1}{\ln x} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{x}{\frac{1}{x}} \stackrel{\text{dosaz.}}{=} 1$$

$$4. \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \left\| \frac{0}{0} \right\| \stackrel{\text{úprava}}{=} \frac{1}{2} \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} = 0$$

$$5. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tg 3x}{\tg x} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{3}{\cos^2 3x}}{\frac{1}{\cos^2 x}} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{úprava}}{=} 3 \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 3x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} 3 \lim_{x \rightarrow \frac{\pi}{2}} \frac{-2 \sin x \cos x}{-6 \sin 3x \cos 3x} = \left\| \frac{0}{0} \right\|$$

$$= \stackrel{\text{úprava}}{=} 3 \cdot \frac{2}{6} \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{\sin 3x} \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cos 3x} \stackrel{\text{úprava}}{=} - \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cos 3x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} - \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{-3 \sin 3x} \stackrel{\text{dosaz.}}{=} \frac{1}{3}$$

$$5B. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tg 3x}{\tg x} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{úprava}}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin 3x \cos x}{\sin x \cos 3x} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin 3x}{\sin x} \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cos 3x} = - \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cos 3x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=}$$

$$= - \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{-3 \sin 3x} \stackrel{\text{dosaz.}}{=} \frac{1}{3}$$

$$6. \lim_{x \rightarrow 0} \frac{x - \arctg x}{x^3} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x^2}}{3x^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 0} \frac{1}{3(1+x^2)} \stackrel{\text{dosaz.}}{=} -\frac{1}{3}$$

$$7. \lim_{x \rightarrow 0} \frac{xe^x + x - 2e^x + 2}{x^3} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x + xe^x + 1 - 2e^x}{3x^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^x + xe^x - 2e^x}{6x} \stackrel{\text{úprava}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{xe^x}{6x} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 0} \frac{e^x}{6} \stackrel{\text{dosaz.}}{=} \frac{1}{6}$$

$$8. \lim_{x \rightarrow 0} \frac{xe^x + x - 2e^{-x} + 2}{x^3} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x + xe^x + 1 + 2e^x}{3x^2} \stackrel{\text{dosaz.}}{=} \frac{4}{+0} = \infty$$

$$9. \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{2 \sin^2 x + 2x \sin 2x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{-2 \sin 2x}{4 \sin x \cos x + 2 \sin 2x + 4x \cos 2x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{-4 \cos 2x}{4 \cos x \cos x - 4 \sin x \sin x + 4 \cos 2x + 4 \cos 2x - 8x \sin 2x} \stackrel{\text{dosaz.}}{=} -\frac{1}{3}$$

$$10. \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{\cos x} \stackrel{\text{dosaz.}}{=} 2$$

$$11. \lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^2 \sin x^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{2x \sin x^2}{2x \sin x^2 + 2xx^2 \cos x^2} \stackrel{\text{úprava}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x^2}{\sin x^2 + x^2 \cos x^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{2x \cos x^2}{2x \cos x^2 + 2x \cos x^2 - 2xx^2 \sin x^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{úprava}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x^2}{\cos x^2 + \cos x^2 - x^2 \sin x^2} \stackrel{\text{dosaz.}}{=} \frac{1}{2}$$

12. $\lim_{x \rightarrow 0} \frac{x \cotg x - 1}{x^2} = \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^2 \sin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\cos x - x \sin x - \cos x}{2x \sin x + x^2 \cos x} \stackrel{\text{úprava}}{=} \\ = - \lim_{x \rightarrow 0} \frac{\sin x}{2 \sin x + x \cos x} \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\cos x}{2 \cos x + \cos x - x \sin x} \stackrel{\text{dosaz.}}{=} -\frac{1}{3}$
13. $\lim_{x \rightarrow \infty} \frac{x^2}{e^{x^2}} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow \infty} \frac{2x}{2xe^{x^2}} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow \infty} \frac{1}{e^{x^2}} \stackrel{\text{dosaz.}}{=} 0$
14. $\lim_{x \rightarrow 1} \frac{1-x}{\cotg \frac{\pi x}{2}} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{-1}{-\frac{\pi}{2} \sin^{-2} \frac{\pi x}{2}} \stackrel{\text{dosaz.}}{=} \frac{2}{\pi}$
15. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x \sin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x + x \cos x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{-\sin x}{\cos x + \cos x - x \sin x} \stackrel{\text{dosaz.}}{=} 0$
16. $\lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x \sin x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\cos x - x \sin x - \cos x}{\sin x + x \cos x} \stackrel{\text{úprava}}{=} - \lim_{x \rightarrow 0} \frac{-x \sin x}{\sin x + x \cos x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \\ = - \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{\cos x + \cos x - x \sin x} \stackrel{\text{dosaz.}}{=} 0$
17. $\lim_{x \rightarrow 0^+} \frac{e^x - 1 - x}{(e^x - 1)x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0^+} \frac{e^x - 1}{e^x - 1 - xe^x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0^+} \frac{e^x}{e^x - xe^x - e^x} \stackrel{\text{dosaz.}}{=} \frac{1}{2}$
18. $\lim_{x \rightarrow 1} \frac{x \ln x - x + 1}{(x-1) \ln x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{\ln x + x \frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 1} \frac{\ln x}{\frac{x-1}{x} + \ln x} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 1} \frac{x \ln x}{(x-1) + x \ln x} = \\ = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{1 + \ln x}{2 + \ln x} \stackrel{\text{dosaz.}}{=} \frac{1}{2}$
19. $\lim_{x \rightarrow 0} \frac{\arcsin x}{1 - e^x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{-e^x} \stackrel{\text{dosaz.}}{=} -1$
20. $\lim_{x \rightarrow e} \frac{\ln x - 1}{\ln^2 x + \ln x - 2} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow e} \frac{\frac{1}{x}}{\frac{1}{x} \ln x + \frac{1}{x}} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow e} \frac{1}{\ln x + 1} \stackrel{\text{dosaz.}}{=} \frac{1}{2}$
21. $\lim_{x \rightarrow 1} \frac{\ln x}{\cotg \frac{\pi x}{2}} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 \frac{\pi x}{2}} \frac{\pi}{2}} \stackrel{\text{dosaz.}}{=} -\frac{2}{\pi}$
22. $\lim_{x \rightarrow 0^+} \frac{\ln(e^x - 1)}{\ln x} = \left\| \frac{\infty}{\infty} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{e^x - 1} e^x}{\frac{1}{x}} \stackrel{\text{úprava}}{=} \lim_{x \rightarrow 0^+} \frac{xe^x}{e^x - 1} \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0^+} \frac{e^x + xe^x}{e^x} \stackrel{\text{dosaz.}}{=} 1$
23. $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{e^x}{2 \cos 2x} \stackrel{\text{dosaz.}}{=} \frac{1}{2}$
24. $\lim_{x \rightarrow 0} \frac{x - \sin x}{\sin^3 x} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{3 \sin^2 x \cos x} \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 0} \frac{\sin x}{6 \sin x \cos^2 x - 3 \sin^3 x} \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \\ = \lim_{x \rightarrow 0} \frac{\cos x}{6 \cos^3 x - 6 \cdot 2 \cdot \sin^2 x \cos x - 9 \sin^2 x \cos x} \stackrel{\text{dosaz.}}{=} \frac{1}{6}$
25. $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 8x + 15} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 3} \frac{1}{2x - 8} \stackrel{\text{dosaz.}}{=} -\frac{1}{2}$
26. $\lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{(x-1)^2} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{-\pi \sin \pi x}{2(x-1)} = \left\| \frac{0}{0} \right\| \stackrel{\text{l'H.p.}}{=} \lim_{x \rightarrow 1} \frac{-\pi^2 \cos \pi x}{2} \stackrel{\text{dosaz.}}{=} \frac{\pi^2}{2}$