Vypočtěte integrály pomocí vhodné substituce:
Evaluate the following integrals through method of substitution:

1) $$\int \frac{\sin(\ln x)}{x}\,\mathrm{d}x = \int \sin(\ln x)\frac{1}{x}\,\mathrm{d}x = \left|\begin{matrix}\ln x = t\\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}t\end{matrix}\right| = \int \sin t\,\mathrm{d}t = -\cos t = -\cos(\ln x)$$

2) $$\int \frac{x\,\mathrm{d}x}{x^4+16} = \left|\begin{matrix}x^2=t\\ 2x\,\mathrm{d}x = \mathrm{d}t\\ x\,\mathrm{d}x = \frac{1}{2}\,\mathrm{d}t\\ x^4 = t^2\end{matrix}\right| = \frac{1}{2}\int \frac{1}{t^2+16}\,\mathrm{d}t = \frac{1}{8}\operatorname{arctg}\frac{t}{4} = \frac{1}{8}\operatorname{arctg}\frac{x^2}{4}$$

3) $$\int xe^{1+x^2}\,\mathrm{d}x = \left|\begin{matrix}1+x^2=t\\ 2x\,\mathrm{d}x = \mathrm{d}t\\ x\,\mathrm{d}x = \frac{1}{2}\,\mathrm{d}t\end{matrix}\right| = \int \frac{1}{2}e^t\,\mathrm{d}t = \frac{1}{2}e^t = \frac{1}{2}e^{1+x^2}$$

4) $$\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,\mathrm{d}x = \int e^{\sqrt{x+1}}\frac{1}{\sqrt{x+1}}\,\mathrm{d}x\left|\begin{matrix}\sqrt{x+1}=t\\ \frac{1}{2\sqrt{x+1}}\,\mathrm{d}x = \mathrm{d}t\\ \frac{1}{\sqrt{x+1}}\,\mathrm{d}x = 2\,\mathrm{d}t\end{matrix}\right| = 2\int e^t\,\mathrm{d}t = 2e^t = 2e^{\sqrt{x+1}}$$

5) $$\int \frac{\mathrm{d}x}{3^x+1} = \left|\begin{matrix}3^x=t\\ x=\log_3 t\\ \mathrm{d}x = \frac{1}{\ln 3}\frac{1}{t}\,\mathrm{d}t\end{matrix}\right| = \frac{1}{\ln 3}\int \frac{1}{t(t+1)}\,\mathrm{d}t = \frac{1}{\ln 3}\int (\frac{1}{t} - \frac{1}{t+1})\,\mathrm{d}t = \frac{1}{\ln 3}\Big(\ln t - \ln(t+1)\Big) =$$
$$= \log_3 t - \log_3(t+1) = x - \log_3(3^x+1)$$

Vypočtěte integrály z iracionálních funkcí pomocí vhodné substituce:
Evaluate the following integrals of the irrational functions through method of substitution:

1) $$\int \frac{\sqrt[3]{x}+1}{\sqrt{x}}\,\mathrm{d}x = \left|\begin{matrix}\sqrt[6]{x}=t\\ x=t^6\\ \mathrm{d}x = 6t^5\,\mathrm{d}t\end{matrix}\right| = \int \frac{t^2+1}{t^3}\,6t^5\,\mathrm{d}t = 6\int t^4+t^2\,\mathrm{d}t = 1\Big(\frac{t^5}{5}+\frac{t^3}{3}\Big) = \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x}$$

2) $$\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}\,\mathrm{d}x = \left|\begin{matrix}\sqrt{x+1}=t\\ x+1=t^2\\ \mathrm{d}x = 2t\,\mathrm{d}t\end{matrix}\right| = \int \frac{t+1}{t-1}\,2t\,\mathrm{d}t = 2\int \frac{t^2+t}{t-1}\,\mathrm{d}t = 2\int t+2+\frac{2}{t-1}\,\mathrm{d}t =$$
$$= 2\Big(\frac{t^2}{2} + 2t + 2\ln|t-1|\Big) = x + 1 + 4\sqrt{x+1} + 4\ln|\sqrt{x+1}-1|$$

3) $$\int \frac{x+1}{\sqrt[3]{x-1}}\,\mathrm{d}x = \left|\begin{matrix}x-1=t^3\\ \mathrm{d}x = 3t^2\,\mathrm{d}t\\ x=t^3+1\\ t=\sqrt[3]{x-1}\end{matrix}\right| = \int \frac{t^2+2}{t}3t^2\,\mathrm{d}t = 2\int (t^2+2)t\,\mathrm{d}t = 3\int t^3+2t\,\mathrm{d}t = 3\Big(\frac{t^5}{5}+t^2\Big) =$$
$$= \frac{3}{5}\sqrt[3]{(x-1)^5} + 3\sqrt[3]{(x-1)^2}$$

4) $$\int \frac{x^2}{\sqrt{4-x}}\,\mathrm{d}x = \left|\begin{matrix}4-x=t^2\\ \mathrm{d}x = -2t\,\mathrm{d}t\\ x=4-t^2\\ t=\sqrt{4-x}\end{matrix}\right| = -\int \frac{(t^2-4)^2}{t}2t\,\mathrm{d}t = -2\int t^4 - 8t^2 + 16\,\mathrm{d}t = -2\Big(\frac{t^5}{5} - 8\frac{t^3}{3} + 16t\Big) =$$
$$= -2\Big(\frac{1}{5}\sqrt{(4-x)^5} - \frac{8}{3}\sqrt{(4-x)^3} + 16\sqrt{4-x}\Big)$$

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5) $\displaystyle\int \frac{x+\sqrt{2x}}{x-\sqrt{2x}}\,\mathrm{d}x = \left| \begin{smallmatrix} 2x=t^2 \\ \mathrm{d}x=t\,\mathrm{d}t \\ x=\frac{1}{2}t^2 \\ t=\sqrt{2x} \end{smallmatrix} \right| = \int \frac{t^2+2t}{t^2-2t} t\,\mathrm{d}t = \int \frac{t^2+2t}{t-2}\,\mathrm{d}t = \int t+4+\frac{8}{t-2}\,\mathrm{d}t =$

$\displaystyle = \frac{1}{2}t^2+4t+8\ln|t-2| = x+4\sqrt{2x}+8\ln|\sqrt{2x}-2|$

6) $\displaystyle\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}}\,\mathrm{d}x = \left| \begin{smallmatrix} \sqrt[4]{x}=t \\ x=t^4 \\ \mathrm{d}x=4t^3\,\mathrm{d}t \end{smallmatrix} \right| = \int \frac{1+t}{t^4+t^2}\,4t^3\,\mathrm{d}t = 4\int \frac{t^2+t}{t^2+1}\,\mathrm{d}t = 4\int 1+\frac{t}{t^2+1}-\frac{1}{t^2+1}\,\mathrm{d}t =$

$\displaystyle = 4\Big(t+\frac{1}{2}\ln(t^2+1)-\operatorname{arctg} t\Big) = 4\sqrt[4]{x}+2\ln(\sqrt{x}+1)-4\operatorname{arctg}\sqrt[4]{x}$

7) $\displaystyle\int \frac{\sqrt{2x+1}}{x}\,\mathrm{d}x = \left| \begin{smallmatrix} \sqrt{2x+1}=t \\ 2x+1=t^2 \\ \mathrm{d}x=t\,\mathrm{d}t \\ x=\frac{t^2-1}{2} \end{smallmatrix} \right| = \int \frac{t}{\frac{t^2-1}{2}}\,t\,\mathrm{d}t = \int \frac{2t^2}{t^2-1}\,\mathrm{d}t = 2\int 1+\frac{1}{t^2-1}\,\mathrm{d}t =$

$\displaystyle = 2\Big(t+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Big) = 2\sqrt{2x+1}+\ln\left|\frac{\sqrt{2x+1}-1}{\sqrt{2x+1}+1}\right|$

8) $\displaystyle\int \frac{3}{1+\sqrt{2x+1}}\,\mathrm{d}x = \left| \begin{smallmatrix} \sqrt{2x+1}=t \\ 2x+1=t^2 \\ \mathrm{d}x=t\,\mathrm{d}t \end{smallmatrix} \right| = \int \frac{3t}{1+t}\,\mathrm{d}t = \int 3-\frac{3}{t+1}\,\mathrm{d}t = 3t-3\ln|t+1| =$

$= 3\sqrt{2x+1}-3\ln|1+\sqrt{2x+1}|$

9) $\displaystyle\int \frac{\sqrt{x-3}}{x-2}\,\mathrm{d}x = \left| \begin{smallmatrix} \sqrt{x-3}=t \\ x-3=t^2 \\ \mathrm{d}x=2t\,\mathrm{d}t \\ x=t^2+3 \end{smallmatrix} \right| = \int \frac{2t^2}{t^2+1}\,\mathrm{d}t = \int 2-\frac{2}{t^2+1}\,\mathrm{d}t = 2t-2\operatorname{arctg} t =$

$= 2\sqrt{x-3}-2\operatorname{arctg}\sqrt{x-3}$

10) $\displaystyle\int \frac{1}{x\sqrt{x-1}}\,\mathrm{d}x = \left| \begin{smallmatrix} x-1=t^2 \\ x=t^2+1 \\ \mathrm{d}x=2t\,\mathrm{d}t \end{smallmatrix} \right| = \int \frac{2t}{t(t^2+1)}\,\mathrm{d}t = 2\int \frac{1}{t^2+1}\,\mathrm{d}t = 2\operatorname{arctg} t = 2\operatorname{arctg}\sqrt{x-1}$

11) $\displaystyle\int \frac{x}{\sqrt{x+1}+\sqrt[3]{x+1}}\,\mathrm{d}x = \left| \begin{smallmatrix} \sqrt[6]{x+1}=t \\ x+1=t^6 \\ \mathrm{d}x=6t^5\,\mathrm{d}t \end{smallmatrix} \right| = \int \frac{t^6-1}{t^3+t^2}\,6t^5\,\mathrm{d}t = 6\int \frac{t^9-t^3}{t+1}\,\mathrm{d}t =$

$\displaystyle = 6\int t^8-t^7+t^6-t^5+t^4-t^3\,\mathrm{d}t = 6\Big(\frac{1}{9}t^9-\frac{1}{8}t^8+\frac{1}{7}t^7-\frac{1}{6}t^6+\frac{1}{5}t^5-\frac{1}{4}t^4\Big), \text{kde } t=\sqrt[6]{x+1}$

12) $\displaystyle\int \frac{\sqrt{x+1}}{x(1+\sqrt{x+1})}\,\mathrm{d}x = \left| \begin{smallmatrix} x+1=t^2 \\ \sqrt{x+1}=t \\ x=t^2-1 \\ \mathrm{d}x=2t\,\mathrm{d}t \end{smallmatrix} \right| = \int \frac{2t^2}{(t^2-1)(t+1)}\,\mathrm{d}t = \int \frac{2t^2}{(t-1)(t+1)^2}\,\mathrm{d}t =$

$\displaystyle = \int \frac{1}{2}\frac{1}{t-1}+\frac{3}{2}\frac{1}{t+1}-\frac{1}{(t+1)^2}\,\mathrm{d}t = \frac{1}{2}\ln|t-1|+\frac{3}{2}\ln|t+1|+\frac{1}{t+1} =$

$\displaystyle = \frac{1}{2}\ln|1-\sqrt{x+1}|+\frac{3}{2}\ln(1+\sqrt{x+1})+\frac{1}{1+\sqrt{x+1}}$

13) $\displaystyle\int \frac{x}{\sqrt{x+2}(x+1)(\sqrt{x+2}-2)}\,\mathrm{d}x = \left| \begin{smallmatrix} x+2=t^2 \\ x=t^2-2 \\ \mathrm{d}x=2t\,\mathrm{d}t \\ t=\sqrt{x+2} \end{smallmatrix} \right| = 2\int \frac{t^2-2}{(t^2-1)(t-2)}\,\mathrm{d}t =$

$$= 2\int \frac{\frac{1}{2}}{t-1} - \frac{\frac{1}{6}}{t+1} + \frac{\frac{2}{3}}{t-2}\,\mathrm{d}t = \ln|t-1| - \frac{1}{3}\ln|t+1| + \frac{4}{3}\ln(t-2) =$$
$$= \ln|\sqrt{x+2}-1| - \frac{1}{3}\ln(\sqrt{x+2}+1) + \frac{4}{3}\ln|\sqrt{x+2}-2|$$

14) $$\int \frac{\sqrt{3x+2}-1}{x}\,\mathrm{d}x = \left| \begin{matrix} 3x+2=t^2 \\ 3\,\mathrm{d}x = 2t\,\mathrm{d}t \\ \mathrm{d}x=\frac{2}{3}t\,\mathrm{d}t \\ x=\frac{1}{3}(t^2-2) \\ t=\sqrt{3x+2} \end{matrix} \right| = \int \frac{t-1}{\frac{1}{3}(t^2-2)}\,\frac{2}{3}t\,\mathrm{d}t = 2\int \frac{t^2-t}{t^2-2}\,\mathrm{d}t = 2\int 1 + \frac{-t+2}{t^2-2}\,\mathrm{d}t =$$
$$= 2\int 1 - \frac{t}{t^2-2} + \frac{2}{t^2-2}\,\mathrm{d}t = 2\Big(t - \frac{1}{2}\ln|t^2-2| - \frac{2}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|\Big) =$$
$$= 2\sqrt{3x+2} - \ln|3x| - \sqrt{2}\ln\left|\frac{\sqrt{2}+\sqrt{3x+2}}{\sqrt{2}-\sqrt{3x+2}}\right|$$

15) $$\int \frac{x}{1+\sqrt{3x+2}}\,\mathrm{d}x = \left| \begin{matrix} 3x+2=t^2 \\ 3\,\mathrm{d}x=2t\,\mathrm{d}t \\ \mathrm{d}x=\frac{2}{3}t\,\mathrm{d}t \\ x=\frac{1}{3}(t^2-2) \end{matrix} \right| = \frac{2}{9}\int \frac{t^2-2}{t+1}t\,\mathrm{d}t = \frac{2}{9}\int \frac{t^3-2t}{t+1}\,\mathrm{d}t = \frac{2}{9}\int t^2-t-1+\frac{1}{t+1}\,\mathrm{d}t =$$
$$= \frac{2}{9}\Big(\frac{t^3}{3} - \frac{t^2}{2} - t + \ln|t+1|\Big) = \frac{2}{9}\Big(\frac{\sqrt{(3x+2)^3}}{3} - \frac{3x+2}{2} - \sqrt{3x+2} + \ln(1+\sqrt{3x+2})\Big)$$

16) $$\int \frac{2}{x+2}\sqrt{\frac{x}{x+2}}\,\mathrm{d}x = \left| \begin{matrix} \sqrt{\frac{x}{x+2}}=t \\ \frac{x}{x+2}=t^2 \\ x=\frac{2t^2}{1-t^2} \\ \mathrm{d}x=\frac{4t\,\mathrm{d}t}{(1-t^2)^2} \end{matrix} \right| = \int \frac{4t^2}{1-t^2}\,\mathrm{d}t = \int -4 + \frac{4}{1-t^2}\,\mathrm{d}t =$$
$$= -4\sqrt{\frac{x}{x+2}} - 2\ln\frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}+\sqrt{x+2}}$$

Vypočtěte integrály z goniometrických funkcí pomocí vhodné substituce:
Evaluate the following integrals of goniometric functions through method of substitution:

1) $$\int \cos^3 x\,\mathrm{d}x = \int (1-\sin^2 x)\cos x\,\mathrm{d}x = \left| \begin{matrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{matrix} \right| = \int (1-t^2)\,\mathrm{d}t = t - \frac{t^3}{3} = \sin t - \frac{1}{3}\sin^3 t$$

2) $$\int \frac{\sin x}{1+2\cos^2 x}\,\mathrm{d}x = \int \frac{1}{1+2\cos^2 x}\sin x\,\mathrm{d}x = \left| \begin{matrix} \cos x = t \\ -\sin x\,\mathrm{d}x = \mathrm{d}t \\ \sin x\,\mathrm{d}x = -\mathrm{d}t \end{matrix} \right| = \int -\frac{1}{1+2t^2}\,\mathrm{d}t =$$
$$= -\int \frac{1}{1+(\sqrt{2}t)^2}\,\mathrm{d}t = -\frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}t) = -\frac{\operatorname{arctg}(\sqrt{2}\cos x)}{\sqrt{2}}$$

3) $$\int \frac{\sin^3 x}{\cos^2 x}\,\mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x}\sin x\,\mathrm{d}x = \left| \begin{matrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\mathrm{d}t \end{matrix} \right| = \int \frac{t^2-1}{t^2}\,\mathrm{d}t = \int 1 - \frac{1}{t^2}\,\mathrm{d}t = t + \frac{1}{t} =$$
$$= \cos x + \frac{1}{\cos x}$$

4) $$\int \operatorname{tg}^3 x\,\mathrm{d}x = \int \frac{\sin^3 x}{\cos^3 x}\,\mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^3 x}\sin x\,\mathrm{d}x = \left| \begin{matrix} !!\cos x = t\ !! \\ \sin x\,\mathrm{d}x = -\mathrm{d}t \end{matrix} \right| = \int \frac{t^2-1}{t^3}\,\mathrm{d}t = \int \frac{1}{t} - \frac{1}{t^3}\,\mathrm{d}t =$$
$$= \ln|t| + \frac{1}{2t^2} = \ln|\cos x| + \frac{1}{2\cos^2 x}$$

5) $\displaystyle\int \frac{\sin^3 x}{1+\cos^2 x}\,\mathrm{d}x = \int \frac{1-\cos^2 x}{1+\cos^2 x}\sin x\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = \int \frac{t^2-1}{t^2+1}\,\mathrm{d}t = \int 1 - \frac{2}{t^2+1}\,\mathrm{d}t =$

$= t - 2\operatorname{arctg} t = \cos x - 2\operatorname{arctg}(\cos x)$

6) $\displaystyle\int \frac{\sin x}{6-5\cos x+\cos^2 x}\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = -\int \frac{1}{t^2-5t+6}\,\mathrm{d}t = -\int \frac{1}{t-3} - \frac{1}{t-2}\,\mathrm{d}t$

7) $\displaystyle\int \frac{1}{\sin x}\,\mathrm{d}x = \int \frac{1}{1-\cos^2 x}\sin x\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = \int \frac{\mathrm{d}t}{t^2-1} = \frac{1}{2}\ln\left|\frac{1-t}{1+t}\right| = \frac{1}{2}\ln\frac{1-\cos x}{\cos x+1}$

8) $\displaystyle\int \frac{\mathrm{d}x}{(2+\cos x)\sin x} = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = \int \frac{1}{(2+t)(1+t)(t-1)}\,\mathrm{d}t = \int -\frac{1}{2}\frac{1}{1+t} + \frac{1}{6}\frac{1}{t-1} + \frac{1}{3}\frac{1}{2+t}\,\mathrm{d}t =$

$\displaystyle= -\frac{1}{2}\ln|1+t| + \frac{1}{6}\ln|t-1| + \frac{1}{3}\ln|2+t| = -\frac{1}{2}\ln(1+\cos x) + \frac{1}{6}\ln(1-\cos x) + \frac{1}{3}\ln(2+\cos x)$

9) $\displaystyle\int \frac{\cos x}{\sin^3 x}\,\mathrm{d}x = \int \frac{1}{\sin^3 x}\cos x\,\mathrm{d}x = \left|\begin{smallmatrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{smallmatrix}\right| = \int t^{-3}\,\mathrm{d}t = \frac{t^{-2}}{-2} = -\frac{1}{2\sin^2 x}$

10) $\displaystyle\int \frac{1+\cos x}{\sin x}\,\mathrm{d}x = \int \frac{1+\cos x}{1-\cos^2 x}\sin x\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = \int -\frac{1+t}{1-t^2}\,\mathrm{d}t = \int \frac{1}{t-1}\,\mathrm{d}t = \ln|t-1| =$

$= \ln(1-\cos x)$

11) $\displaystyle\int \frac{\sin x+\sin^3 x}{\cos x+\cos^2 x}\,\mathrm{d}x = \int \frac{2-\cos^2 x}{\cos^2 x+\cos x}\sin x\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = \int \frac{t^2-2}{t+t^2}\,\mathrm{d}t = \int 1 - \frac{t+2}{t^2+t}\,\mathrm{d}t =$

$\displaystyle= \int 1 - \frac{2}{t} + \frac{1}{t+1}\,\mathrm{d}t = t - \ln t^2 + \ln|t+1| = \cos x - \ln\cos^2 x + \ln(1+\cos x)$

12) $\displaystyle\int \frac{\cos^2 x}{\sin x}\,\mathrm{d}x = \int \frac{\cos^2 x}{1-\cos^2 x}\sin x\,\mathrm{d}x = \left|\begin{smallmatrix} \cos x = t \\ \sin x\,\mathrm{d}x = -\,\mathrm{d}t \end{smallmatrix}\right| = -\int \frac{t^2}{1-t^2}\,\mathrm{d}t = \int 1 - \frac{1}{1-t^2}\,\mathrm{d}t =$

$\displaystyle= t - \frac{1}{2}\ln\frac{1+t}{1-t} = \cos x - \frac{1}{2}\ln\frac{1+\cos x}{1-\cos x}$

13) $\displaystyle\int \frac{\cos x+\cos^3 x}{\sin x}\,\mathrm{d}x = \int \frac{2-\sin^2 x}{\sin x}\cos x\,\mathrm{d}x = \left|\begin{smallmatrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{smallmatrix}\right| = \int \frac{2-t^2}{t}\,\mathrm{d}t = \ln t^2 - \frac{1}{2}t^2 =$

$\displaystyle= \ln(\sin^2 x) - \frac{1}{2}\sin^2 x$

14) $\displaystyle\int \cos^5 x\,\mathrm{d}x = \int (1-\sin^2 x)^2\cos x\,\mathrm{d}x = \left|\begin{smallmatrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{smallmatrix}\right| = \int (1-t^2)^2\,\mathrm{d}t = t - \frac{2}{3}t^3 + \frac{1}{5}t^5 =$

$\displaystyle= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x$

15) $\displaystyle\int \frac{\cos x(2+\sin x)}{3-\cos^2 x}\,\mathrm{d}x = \int \frac{2+\sin x}{2+\sin^2 x}\cos x\,\mathrm{d}x = \left|\begin{smallmatrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{smallmatrix}\right| = \int \frac{t+2}{t^2+2}\,\mathrm{d}t =$

$\displaystyle= \frac{1}{2}\ln(t^2+2) + \sqrt{2}\operatorname{arctg}\frac{t}{\sqrt{2}} = \frac{1}{2}\ln(2+\sin^2 x) + \sqrt{2}\operatorname{arctg}\frac{\sin x}{\sqrt{2}}$

16) $\displaystyle\int \frac{\sin^2 x}{\cos^3 x}\,\mathrm{d}x = \int \frac{\sin^2 x}{(1-\sin x)^2(1+\sin x)^2}\cos x\,\mathrm{d}x \left|\begin{smallmatrix} \sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t \end{smallmatrix}\right| = \int \frac{t^2}{(1-t)^2(1+t)^2}\,\mathrm{d}t =$

$\displaystyle= \int \frac{\frac{1}{4}}{t-1} + \frac{\frac{1}{4}}{(t-1)^2} - \frac{\frac{1}{4}}{t+1} + \frac{\frac{1}{4}}{(t+1)^2}\,\mathrm{d}t = \frac{1}{4}\ln\frac{1-t}{1+t} - \frac{1}{4}\frac{1}{t-1} - \frac{1}{4}\frac{1}{t+1} =$

$\displaystyle= \frac{1}{4}\ln\frac{1-\sin x}{1+\sin x} - \frac{1}{4}\frac{1}{\sin x-1} - \frac{1}{4}\frac{1}{\sin x+1}$

17) $$\int \frac{1+\cos x}{(1+\cos^2 x)\sin x}\,\mathrm{d}x = \int \frac{1+\cos x}{(1+\cos^2 x)(1-\cos^2 x)}\sin x\,\mathrm{d}x\Big|\begin{smallmatrix}\cos x=t\\ \sin x\,\mathrm{d}x=-\,\mathrm{d}t\end{smallmatrix}\Big| = \int \frac{t+1}{(1+t^2)(t^2-1)}\,\mathrm{d}t =$$
$$= \int \frac{1}{(t^2+1)(t-1)}\,\mathrm{d}t = \int -\frac{1}{2}\frac{t+1}{t^2+1} + \frac{1}{2}\frac{1}{t-1}\,\mathrm{d}t = -\frac{1}{4}\ln(t^2+1) - \frac{1}{2}\operatorname{arctg} t + \frac{1}{2}\ln|t-1| =$$
$$= -\frac{1}{4}\ln(\cos x^2+1) - \frac{1}{2}\operatorname{arctg}\cos x + \frac{1}{2}\ln(1-\cos x)$$

18) $$\int \frac{2+\cos x}{\sin x\cos^2 x}\,\mathrm{d}x = \int \frac{2+\cos x}{\cos^2 x(1-\cos^2 x)}\sin x\,\mathrm{d}x\Big|\begin{smallmatrix}\cos x=t\\ \sin x\,\mathrm{d}x=-\,\mathrm{d}t\end{smallmatrix}\Big| = \int \frac{t+2}{t^2(t^2-1)}\,\mathrm{d}t =$$
$$= \int -\frac{2}{t^2} - \frac{1}{t} + \frac{3}{2}\frac{1}{t-1} - \frac{1}{2}\frac{1}{t+1}\,\mathrm{d}t = \frac{2}{t} - \ln|t| + \frac{3}{2}\ln(1-t) - \frac{1}{2}\ln(1+t) =$$
$$= \frac{2}{\cos x} - \ln|\cos x| + \frac{3}{2}\ln(1-\cos x) - \frac{1}{2}\ln(1+\cos x)$$

19) $$\int \frac{\cos x + 2\cos^3 x}{(1+\sin x)(1+\sin^2 x)}\,\mathrm{d}x = \int \frac{3-2\sin^2 x}{(1+\sin x)(1+\sin^2 x)}\cos x\,\mathrm{d}x\Big|\begin{smallmatrix}\sin x=t\\ \cos x\,\mathrm{d}x=\mathrm{d}t\end{smallmatrix}\Big| =$$
$$= \int \frac{3-2t^2}{(1+t)(1+t^2)}\,\mathrm{d}t = \int \frac{1}{2}\frac{1}{1+t} - \frac{5}{2}\frac{t}{t^2+1} + \frac{5}{2}\frac{1}{t^2+1}\,\mathrm{d}t =$$
$$= \frac{1}{2}\ln(1+t) - \frac{5}{4}\ln(1+t^2) - \frac{5}{2}\operatorname{arctg} t = \frac{1}{2}\ln(1+\sin x) - \frac{5}{4}\ln(1+\sin^2 x) + \frac{5}{2}\operatorname{arctg}\sin x$$

20) $$\int \frac{\sin^5 x\cos^2 x}{1+\cos x}\,\mathrm{d}x = \int \frac{(1-\cos^2 x)^2\cos^2 x}{1+\cos x}\sin x\,\mathrm{d}x = \Big|\begin{smallmatrix}\cos x=t\\ \sin x\,\mathrm{d}x=-\,\mathrm{d}t\end{smallmatrix}\Big| = -\int \frac{(1-t^2)^2 t^2}{1+t}\,\mathrm{d}t =$$
$$= -\int t^2(t-1)(t^2-1)\,\mathrm{d}t = -\int -t^3 + t^5 + t^2 - t^4\,\mathrm{d}t = -\frac{t^6}{6} + \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} =$$
$$= -\frac{\cos^6 x}{6} + \frac{\cos^5 x}{5} + \frac{\cos^4 x}{4} - \frac{\cos^3 x}{3}$$

21) $$\int \frac{1}{\sin x\cos x}\,\mathrm{d}x = \int \frac{1}{(1-\cos^2 x)\cos x}\sin x\,\mathrm{d}x = \Big|\begin{smallmatrix}\cos x=t\\ \sin x\,\mathrm{d}x=-\,\mathrm{d}t\end{smallmatrix}\Big| = \int \frac{1}{(t^2-1)t}\,\mathrm{d}t =$$
$$= \int \frac{1}{2}\frac{1}{t+1} + \frac{1}{2}\frac{1}{t-1} - \frac{1}{t}\,\mathrm{d}t = \frac{1}{2}\ln|t+1| + \frac{1}{2}\ln|t-1| - \ln|t| = \frac{1}{2}\ln\frac{1-\cos^2 x}{\cos^2 x} = \ln|\operatorname{tg} x|$$

22) $$\int \frac{1-\cos x}{\sin^3 x}\,\mathrm{d}x = \int \frac{1-\cos x}{(1-\cos^2 x)^2}\sin x\,\mathrm{d}x = \Big|\begin{smallmatrix}\cos x=t\\ \sin x\,\mathrm{d}x=-\,\mathrm{d}t\end{smallmatrix}\Big| = -\int \frac{1-t}{(1-t^2)^2}\,\mathrm{d}t =$$
$$= \int \frac{1}{(t-1)(1+t)^2}\,\mathrm{d}t = \int \frac{1}{4}\frac{1}{t-1} - \frac{1}{4}\frac{1}{t+1} - \frac{1}{2}\frac{1}{(1+t)^2}\,\mathrm{d}t =$$
$$= \frac{1}{4}\ln|t-1| - \frac{1}{4}\ln|t+1| + \frac{1}{2}\frac{1}{1+t} = \frac{1}{4}\ln\frac{1-\cos x}{1+\cos x} + \frac{1}{2(1+\cos x)}$$

23) $$\int \frac{1-\sin x}{1+\cos x}\,\mathrm{d}x = \left|\begin{smallmatrix}\operatorname{tg}\frac{x}{2}=t\\ \cos x=\frac{1-t^2}{1+t^2}\\ \sin x=\frac{2t}{1+t^2}\\ \mathrm{d}x=\frac{2}{1+t^2}\,\mathrm{d}t\end{smallmatrix}\right| = \int \frac{t^2-2t+1}{t^2+1}\,\mathrm{d}t = \int 1 - \frac{2t}{1+t^2}\,\mathrm{d}t = t - \ln(1+t^2) =$$
$$= \operatorname{tg}\frac{x}{2} - \ln(1+\operatorname{tg}^2\frac{x}{2})$$