Derivujte zadané funkce.
Differentiate the following functions.

Př. $y = \dfrac{x}{x^2+1}$

$$\begin{aligned} y' &= \frac{(x)' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2} \\ &= \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^2} \\ &= \frac{1-x^2}{(1+x^2)^2} \end{aligned}$$

Př. $y = e^{\operatorname{arctg} x^2}$

$$\begin{aligned} y' &= e^{\operatorname{arctg} x^2} \cdot (\operatorname{arctg} x^2)' \\ &= e^{\operatorname{arctg} x^2} \cdot \frac{1}{1+(x^2)^2} \cdot (x^2)' \\ &= \frac{2x e^{\operatorname{arctg} x^2}}{1+x^4} \end{aligned}$$

Př. $y = x \ln^2 x$

$$\begin{aligned} y' &= (x)' \cdot \ln^2 x + x \cdot (\ln^2 x)' \\ &= 1 \cdot \ln^2 x + x \cdot 2 \ln x \cdot (\ln x)' \\ &= \ln^2 x + x \cdot 2 \ln x \cdot \frac{1}{x} \\ &= (2 + \ln x) \ln x \end{aligned}$$

Př. $y = \dfrac{1-x^3}{x^2}$

$$\begin{aligned} y' &= \frac{(1-x^3)' x^2 - (1-x^3) \cdot (x^2)'}{(x^2)^2} \\ &= \frac{-3x^2 x^2 - (1-x^3) 2x}{(x^2)^2} \\ &= \frac{-3x^4 - 2x + 2x^4}{x^4} \\ &= -\frac{2+x^3}{x^3} \end{aligned}$$

Př. $y = \left(\dfrac{x-1}{x+1}\right)^2$

$$\begin{aligned} y' &= 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)' \\ &= 2\frac{x-1}{x+1} \cdot \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2} \\ &= 2\frac{x-1}{x+1} \cdot \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2} \\ &= 2\frac{x-1}{x+1} \cdot \frac{2}{(x+1)^2} \\ &= 4\frac{x-1}{(x+1)^3} \end{aligned}$$

Př. $y = e^x(x^2 - 2x + 2)$

$$\begin{aligned} y' &= (e^x)'(x^2-2x+2) + e^x(x^2-2x+2)' \\ &= e^x(x^2-2x+2) + e^x(2x-2) \\ &= e^x(x^2-2x+2+2x-2) \\ &= e^x x^2 \end{aligned}$$

Př. $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$

$$\begin{aligned} y' &= \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-2/3}\left(\frac{1+x^3}{1-x^3}\right)' \\ &= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{(1+x^3)'(1-x^3) - (1+x^3)(1-x^3)'}{(1-x^3)^2} \\ &= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{3x^2(1-x^3) - (1+x^3)(-3x^2)}{(1-x^3)^2} \\ &= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{6x^2}{(1-x^3)^2} \\ &= \sqrt[3]{\frac{1+x^3}{1-x^3}}\frac{1-x^3}{1+x^3} \cdot \frac{2x^2}{(1-x^3)^2} \\ &= \sqrt[3]{\frac{1+x^3}{1-x^3}}\frac{2x^2}{1-x^6} \end{aligned}$$

Př. $y = x \ln(x^2-1)$

$$\begin{aligned} y' &= x' \cdot \ln(x^2-1) + x \cdot (\ln(x^2-1))' \\ &= 1 \cdot \ln(x^2-1) + x \cdot \frac{1}{x^2-1} \cdot (x^2-1)' \\ &= \ln(x^2-1) + x \cdot \frac{1}{x^2-1} \cdot 2x \\ &= \ln(x^2-1) + \frac{2x^2}{x^2-1} \end{aligned}$$

Př. $y = \frac{1}{4}\ln\frac{x^2-1}{x^2+1}$

$$\begin{aligned} y' &= \frac{1}{4}\frac{x^2+1}{x^2-1}\frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2} \\ &= \frac{x^2+1}{x^2-1}\frac{4x}{(x^2+1)^2} \\ &= \frac{x}{(x^2-1)(x^2+1)} \end{aligned}$$

Př. $y = \sqrt{x+1} - \ln(1+\sqrt{x+1})$

$$\begin{aligned} y' &= \frac{1}{2\sqrt{x+1}} - \frac{1}{1+\sqrt{x+1}}\cdot\frac{1}{2\sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}}\left(1-\frac{1}{1+\sqrt{x+1}}\right) \\ &= \frac{1}{2\sqrt{x+1}}\frac{\sqrt{x+1}}{1+\sqrt{x+1}} \\ &= \frac{1}{2(1+\sqrt{x+1})} \end{aligned}$$

Př. $y = \frac{\sin x - x\cos x}{\cos x + x\sin x}$

$$\begin{aligned} y' &= \frac{[\cos x - (\cos x - x\sin x)](\cos x + x\sin x) - (\sin x - x\cos x)[-\sin x + \sin x + x\cos x]}{(\cos x + x\sin x)^2} \\ &= \frac{x\sin x\cos x + x^2\sin^2 x - [x\sin x\cos x - x^2\cos^2 x]}{(\cos x + x\sin x)^2} \\ &= \frac{x^2}{(\cos x + x\sin x)^2} \end{aligned}$$

Př. $y = \sqrt{1-x}\cdot\arcsin\sqrt{x}$

$$\begin{aligned} y' &= (\sqrt{1-x})'\cdot\arcsin\sqrt{x} + \sqrt{1-x}\cdot(\arcsin\sqrt{x})' \\ &= \frac{1}{2\sqrt{1-x}}\cdot(1-x)'\cdot\arcsin\sqrt{x} + \sqrt{1-x}\cdot\frac{1}{\sqrt{1-(\sqrt{x})^2}}(\sqrt{x})' \\ &= -\frac{1}{2\sqrt{1-x}}\arcsin\sqrt{x} + \sqrt{1-x}\frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} - \frac{\arcsin\sqrt{x}}{2\sqrt{1-x}} \end{aligned}$$

1. $y = -\frac{1}{9}x^4 + \frac{2}{3}x^2$
2. $y = 4x^3 - 3x^4$
3. $y = -2 + 12x - x^3$
4. $y = x + \frac{4}{x}$
5. $y = \frac{x}{(x+1)^2}$
6. $y = x^2 - 2\ln x$
7. $y = (3-x)\sqrt{x}$
8. $y = 2\sqrt{x} - x$
9. $y = \frac{x^2}{1-x}$
10. $y = 1 + x^2 - \frac{x^4}{4}$
11. $y = \frac{x-2}{\sqrt{x^2+1}}$
12. $y = \frac{x^2}{x^2+1}$
13. $y = \left(\frac{1+x}{1-x}\right)^2$
14. $y = \left(\frac{1+x}{1-x}\right)^4$
15. $y = \sqrt[3]{2x^2 - x^3}$
16. $y = \frac{x}{1+x^2}$
17. $y = \frac{1-x^3}{x^2}$
18. $y = \frac{1+x^2}{1-x^2} = -1 + \frac{2}{1-x^2}$

19. $y = \dfrac{\ln^2 x}{x}$
20. $y = \dfrac{\ln x}{\sqrt{x}}$
21. $y = \dfrac{e^x}{1+x}$
22. $y = x^{\frac{2}{3}} e^{-x}$
23. $y = x^2 e^{-x}$
24. $y = x e^{\frac{1}{x}}$
25. $y = x - \ln(1+x)$
26. $y = x - \ln(1+x^2)$
27. $y = e^{-x}\sin x$
28. $y = x + \dfrac{2x}{1+x^2}$
29. $y = \dfrac{x^2}{2} + \dfrac{8}{x^3}$
30. $y = \sqrt{2x - x^2}$
31. $y = (x+1)^{10} e^{-x}$
32. $y = \dfrac{x^2}{2^x}$
33. $y = \arcsin(\sqrt{x})$
34. $y = x \arcsin(\sqrt{x})$
35. $y = \dfrac{\arctan(x)}{x^2+1}$
36. $y = \sin(\ln(1+\cos(x)))$
37. $y = \ln(\dfrac{\cos(x)}{(\sin(x))^2})$
38. $y = \sqrt{\dfrac{x^2+1}{x^2-1}}$
39. $y = x^2 \ln(x^2+x+1)$
40. $y = \left(x^2+1\right) \arctan(\sqrt{1-x^2})$
41. $y = \left(x^2 + 3\,x + 6\right) e^{1-x}$
42. $y = \left(x^3 + 2\,x - 1\right) \sin(2\,x)$
43. $y = \dfrac{x e^{-x}}{x+1}$
44. $y = \dfrac{x^2+1}{\ln(x)}$
45. $y = \ln(\dfrac{1+\sin(x)}{1-\sin(x)})$
46. $y = \left(x^2-2\right)\sqrt{x^2+1}$
47. $y = \ln(1-\sqrt{x+1})$
48. $y = \arcsin(x\sqrt{1-x})$
49. $y = \dfrac{\arcsin(x)}{x^2+1}$
50. $y = \ln(\sqrt{e^x - x^2})$
51. $y = \ln(\sqrt{e^{-x} - 4\,x^2})$
52. $y = \arctan(\ln(1+\sin(x)))$
53. $y = \sin(\ln(x + \sin(2\,x)))$
54. $y = x^2 e^{-\sin(x)}$
55. $y = \arctan(\sqrt{\ln(x^2-1)})$
56. $y = x^2 \ln(\arctan(2\,x-1))$
57. $y = \dfrac{\ln(x^3+1)}{x}$
58. $y = \dfrac{x^2+1}{\sqrt{2\,x-1}}$
59. $y = \dfrac{\sqrt{x^2+x+1}}{x+1}$
60. $y = \sqrt{\arctan(x^{-1})}$
61. $y = \arctan(1/3\,\sqrt{6-3\,x})$
62. $y = \left(x^2+1\right)\arctan(x^{-1})$
63. $y = \ln(x\sqrt{x^2+1})$
64. $y = \left(x^2+x+1\right)\sin(x) + x\cos(x)$
65. $y = \arctan(\sqrt{2}\sqrt{x^{-1}})$
66. $y = \dfrac{\sin(x) - \cos(x)}{x}$
67. $y = \dfrac{1+\cos(x)}{\sin(2\,x)}$
68. $y = \left(1-x^2\right)\arccos(x)$

Výsledky (výsledky jsou u prvních příkladů navíc ještě většinou rozloženy na součin), *solutions*:

1: $-\dfrac{4}{9}x(x^2-3)$, **2**: $12x^2(1-x)$, **3**: $3(2-x)(2+x)$, **4**: $\dfrac{(x-2)(x+2)}{x^2}$, **5**: $\dfrac{1-x}{(x+1)^3}$, **6**: $2\dfrac{(x-1)(x+1)}{x}$,
7: $\dfrac{3}{2\sqrt{x}}(1-x)$, **8**: $\dfrac{1-\sqrt{x}}{\sqrt{x}}$, **9**: $\dfrac{x(2-x)}{(1-x)^2}$, **10**: $-2x(x-1)(x+1)$, **11**: $\dfrac{2x+1}{(x^2+1)^{\frac{3}{2}}}$, **12**: $\dfrac{2x}{(1+x^2)^2}$,

13: $-4\dfrac{x+1}{(x-1)^3}$, **14**: $-8\dfrac{(x+1)^3}{(x-1)^5}$, **15**: $\dfrac{1}{3}\dfrac{4-3x}{\sqrt[3]{x(2-x)^2}}$, **16**: $\frac{1-x^2}{(1+x^2)^2}$, **17**: $-\dfrac{x^3+2}{x^3}$, **18**: $\frac{4x}{(1-x^2)^2}$,

19: $\dfrac{\ln x(2-\ln x)}{x^2}$, **20**: $\dfrac{2-\ln x}{2x^{\frac{3}{2}}}$, **21**: $\frac{xe^x}{(x+1)^2}$, **22**: $e^{-x}\dfrac{2-3x}{3\sqrt[3]{x}}$, **23**: $e^{-x}x(2-x)$, **24**: $e^{\frac{1}{x}}\dfrac{x-1}{x}$,

25: $\dfrac{x^2+x-1}{x+1}$, **26**: $\dfrac{(x-1)^2}{x^2+1}$, **27**: $e^{-x}(\cos x-\sin x)$, **28**: $\dfrac{x^4+3}{(x^2+1)^2}$, **29**: $\frac{x^5-24}{x^4}$, **30**: $\dfrac{1-x}{\sqrt{x(2-x)}}$,

31: $e^{-x}(x+1)^9(9-x)$, **32**: $\dfrac{x(2-x\ln 2)}{2^x}$, **33**: $1/2\,\dfrac{1}{\sqrt{x}\sqrt{1-x}}$, **34**: $\arcsin(\sqrt{x})+1/2\,\dfrac{\sqrt{x}}{\sqrt{1-x}}$,

35: $-\dfrac{-1+2\,\arctan(x)x}{\left(x^2+1\right)^2}$, **36**: $-\dfrac{\cos(\ln(1+\cos(x)))\sin(x)}{1+\cos(x)}$, **37**: $-\dfrac{(\cos(x))^2+1}{\sin(x)\cos(x)}$,

38: $-2\,x\dfrac{1}{\sqrt{\frac{x^2+1}{x^2-1}}}\left(x^2-1\right)^{-2}$, **39**: $2\,x\ln(x^2+x+1)+\dfrac{x^2\left(1+2\,x\right)}{x^2+x+1}$,

40: $2\,x\arctan(\sqrt{1-x^2})-\dfrac{\left(x^2+1\right)x}{\sqrt{1-x^2}\left(2-x^2\right)}$, **41**: $-e^{1-x}\left(x+3+x^2\right)$,

42: $\left(3\,x^2+2\right)\sin(2\,x)+2\,\left(x^3+2\,x-1\right)\cos(2\,x)$, **43**: $-\dfrac{e^{-x}\left(-1+x^2+x\right)}{\left(x+1\right)^2}$, **44**: $2\,\dfrac{x}{\ln(x)}-\dfrac{x^2+1}{(\ln(x))^2\,x}$,

45: $2\,(\cos(x))^{-1}$, **46**: $3\,\dfrac{x^3}{\sqrt{x^2+1}}$, **47**: $1/2\,\dfrac{1}{\sqrt{x+1}\left(-1+\sqrt{x+1}\right)}$, **48**: $-1/2\,\dfrac{-2+3\,x}{\sqrt{1-x}\sqrt{1-x^2+x^3}}$,

49: $\dfrac{1}{\sqrt{1-x^2}\left(x^2+1\right)}-2\,\dfrac{\arcsin(x)x}{\left(x^2+1\right)^2}$, **50**: $1/2\,\dfrac{e^x-2\,x}{e^x-x^2}$, **51**: $1/2\,\dfrac{-e^{-x}-8\,x}{e^{-x}-4\,x^2}$,

52: $\dfrac{\cos(x)}{(1+\sin(x))\left(1+(\ln(1+\sin(x)))^2\right)}$, **53**: $\dfrac{\cos(\ln(x+\sin(2\,x)))\,(1+2\,\cos(2\,x))}{x+\sin(2\,x)}$,

54: $2\,xe^{-\sin(x)}-x^2\cos(x)e^{-\sin(x)}$, **55**: $\dfrac{x}{\sqrt{\ln(x^2-1)}\,(x^2-1)\,(1+\ln(x^2-1))}$,

56: $2\,x\ln(\arctan(2\,x-1))+2\,\dfrac{x^2}{\left(1+(2\,x-1)^2\right)\arctan(2\,x-1)}$, **57**: $3\,\dfrac{x}{x^3+1}-\dfrac{\ln(x^3+1)}{x^2}$,

58: $\dfrac{3\,x^2-2\,x-1}{(2\,x-1)^{3/2}}$, **59**: $1/2\,\dfrac{-1+x}{\sqrt{x^2+x+1}\,(x+1)^2}$, **60**: $-1/2\,\dfrac{1}{\sqrt{\arctan(x^{-1})}\,(x^2+1)}$,

61: $3/2\,\dfrac{1}{\sqrt{6-3\,x}\,(-5+x)}$, **62**: $2\,x\arctan(x^{-1})-1$, **63**: $\dfrac{2\,x^2+1}{(x^2+1)\,x}$,

64: $\sin(x)+x\sin(x)+\cos(x)x^2+x\cos(x)+2\,\cos(x)$, **65**: $-1/2\,\dfrac{\sqrt{2}}{\sqrt{x^{-1}}x\,(x+2)}$,

66: $\dfrac{x\cos(x)+x\sin(x)-\sin(x)+\cos(x)}{x^2}$, **67**: $-\dfrac{\sin(x)}{\sin(2\,x)}-2\,\dfrac{(1+\cos(x))\cos(2\,x)}{(\sin(2\,x))^2}$,

68: $-2\,x\arccos(x)-\sqrt{1-x^2}$ (bez záruky, no warranty)

Př. $y = \frac{x}{2}\sqrt{x^2+2x+2} + \ln\left(x+1+\sqrt{x^2+2x+2}\right)$

$$
\begin{aligned}
y' &= \left(\frac{1}{2}x\right)' \cdot \sqrt{x^2+2x+2} + \frac{1}{2}x \cdot \left(\sqrt{x^2+2x+2}\right)' + \frac{1}{x+1+\sqrt{x^2+2x+2}}\left(x+1+\sqrt{x^2+2x+2}\right)' \\
&= \frac{1}{2}\cdot\sqrt{x^2+2x+2} + \frac{1}{2}x\cdot\frac{1}{2}(x^2+2x+2)^{-\frac{1}{2}}(x^2+2x+2)' \\
&\quad + \frac{1}{x+1+\sqrt{x^2+2x+2}}\left(1+\frac{1}{2}(x^2+2x+2)^{-\frac{1}{2}}(x^2+2x+2)'\right) \\
&= \frac{1}{2}\sqrt{x^2+2x+2} + \frac{x}{4\sqrt{x^2+2x+2}}(2x+2) + \frac{1}{x+1+\sqrt{x^2+2x+2}}\left(1+\frac{1}{2}\frac{1}{\sqrt{x^2+2x+2}}(2x+2)\right)
\end{aligned}
$$

Př. $y = \ln\left(\frac{2x^2e^{x+1}}{x+1}\right) + \operatorname{arctg} e^{2x}$

$$
\begin{aligned}
y' &= \frac{1}{\frac{2x^2e^{x+1}}{x+1}}\cdot\left(\frac{2x^2e^{x+1}}{x+1}\right)' + \frac{1}{1+(e^{2x})^2}(e^{2x})' \\
&= \frac{x+1}{2x^2e^{x+1}}\cdot\frac{(2x^2e^{x+1})'(x+1) - 2x^2e^{x+1}(x+1)'}{(x+1)^2} + \frac{1}{1+(e^{2x})^2}e^{2x}(2x)' \\
&= \frac{x+1}{2x^2e^{x+1}}\cdot\frac{(4xe^{x+1}+2x^2e^{x+1})(x+1) - 2x^2e^{x+1}}{(x+1)^2} + \frac{1}{1+e^{4x}}e^{2x}2
\end{aligned}
$$

Př. $y = \sqrt[3]{\frac{2x^3\ln(x+1)}{x^2+1}}$

$$
\begin{aligned}
y' &= \frac{1}{3}\left(\frac{2x^3\ln(x+1)}{x^2+1}\right)^{-\frac{2}{3}}\left(\frac{2x^3\ln(x+1)}{x^2+1}\right)' \\
&= \frac{1}{3}\left(\frac{2x^3\ln(x+1)}{x^2+1}\right)^{-\frac{2}{3}}\frac{\left[2x^3\ln(x+1)\right]'(x^2+1) - \left[2x^3\ln(x+1)\right](x^2+1)'}{(x^2+1)^2} \\
&= \frac{1}{3}\left(\frac{2x^3\ln(x+1)}{x^2+1}\right)^{-\frac{2}{3}}\frac{\left[6x^2\ln(x+1)+2x^3\frac{1}{x+1}(x+1)'\right](x^2+1) - \left[2x^3\ln(x+1)\right]2x}{(x^2+1)^2} \\
&= \frac{1}{3}\left(\frac{2x^3\ln(x+1)}{x^2+1}\right)^{-\frac{2}{3}}\frac{\left[6x^2\ln(x+1)+2x^3\frac{1}{x+1}\right](x^2+1) - \left[2x^3\ln(x+1)\right]2x}{(x^2+1)^2}
\end{aligned}
$$

Př. $y = \ln\sqrt{\frac{x^2+x+1}{x^2-x+1}} + \frac{1}{2}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$

$$
\begin{aligned}
y' &= \frac{1}{\sqrt{\frac{x^2+x+1}{x^2-x+1}}}\left(\sqrt{\frac{x^2+x+1}{x^2-x+1}}\right)' + \frac{1}{2}\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\left(\frac{2x+1}{\sqrt{3}}\right)' \\
&= \frac{1}{\sqrt{\frac{x^2+x+1}{x^2-x+1}}}\frac{1}{2}\left(\frac{x^2+x+1}{x^2-x+1}\right)^{-\frac{1}{2}}\left(\frac{x^2+x+1}{x^2-x+1}\right)' + \frac{1}{2}\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\frac{2}{\sqrt{3}} \\
&= \frac{1}{2}\frac{x^2-x+1}{x^2+x+1}\frac{(2x+1)(x^2-x+1)-(x^2+x+1)(2x-1)}{(x^2-x+1)^2} + \frac{1}{2}\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\frac{2}{\sqrt{3}}
\end{aligned}
$$

Př. $y = \dfrac{x\,\mathrm{arctg}(2x)}{e^x} + x^2 \ln\sqrt{x^2+3}$

$$y' = \frac{(x\,\mathrm{arctg}(2x))' \cdot e^x - x\,\mathrm{arctg}(2x)\cdot (e^x)'}{(e^x)^2} + (x^2)' \cdot \ln\sqrt{x^2+3} + x^2 \cdot \left(\ln\sqrt{x^2+3}\right)'$$

$$= \frac{[x' \cdot \mathrm{arctg}(2x) + x \cdot (\mathrm{arctg}(2x))']e^x - x\,\mathrm{arctg}(2x)e^x}{e^{2x}} + 2x \ln\sqrt{x^2+3} + x^2 \frac{1}{\sqrt{x^2+3}}\left(\sqrt{x^2+3}\right)'$$

$$= \frac{[1 \cdot \mathrm{arctg}(2x) + x\frac{1}{1+(2x)^2}(2x)']e^x - x\,\mathrm{arctg}(2x)e^x}{e^{2x}} + 2x \ln\sqrt{x^2+3} + x^2 \frac{1}{\sqrt{x^2+3}} \frac{1}{2}(x^2+3)^{-\frac{1}{2}}(x^2+3)'$$

$$= \frac{[\mathrm{arctg}(2x) + x\frac{1}{1+(2x)^2}2]e^x - x\,\mathrm{arctg}(2x)e^x}{e^{2x}} + 2x \ln\sqrt{x^2+3} + x^2 \frac{1}{\sqrt{x^2+3}} \frac{1}{2}(x^2+3)^{-\frac{1}{2}}2x$$ ∎

Př. $y = 2\,\mathrm{arctg}\,\dfrac{x+1}{\sqrt{x^2+1}} - x \ln\dfrac{1}{3x+5}$

$$y' = 2\frac{1}{1+\frac{(x+1)^2}{x^2+1}} \cdot \frac{1\cdot\sqrt{x^2+1} - (x+1)\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x^2+1} - 1 \cdot \ln\frac{1}{3x+5} - x\cdot(3x+5)\cdot(-1)(3x+5)^{-2}\cdot 3$$ ∎

Vypočtěte $y'(x)$ a upravte:
Find y' and simplify:

69. $y = 2x\arcsin\dfrac{x^2+1}{\sqrt{x}} + \sqrt{\ln\dfrac{2x-1}{4}}$

70. $y = 2x\arcsin\dfrac{x^2+1}{\sqrt{x}} + \sqrt{\ln\dfrac{2x-1}{4x+1}}$

71. $y = \dfrac{1}{6}\ln\dfrac{(x-1)^2}{x^2+x+1} - \dfrac{1}{\sqrt{3}}\,\mathrm{arctg}\,\dfrac{2x+1}{\sqrt{3}}$

72. $y = -\dfrac{\cos x}{\sin^2 x} + \ln\sqrt{\dfrac{1+\cos x}{\sin x}}$

73. $y = \dfrac{\arccos x}{x} + \dfrac{1}{2}\ln\dfrac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$

74. $y = \ln\dfrac{1+\sqrt{\sin x}}{1-\sqrt{\sin x}} + 2\,\mathrm{arctg}\,\sqrt{\sin x}$

75. $y = \dfrac{\arcsin x}{\sqrt{1-x^2}} + \ln\sqrt{\dfrac{1-x}{1+x}}$

76. $y = \mathrm{arctg}\sqrt{\dfrac{1-x}{1+x}}$

77. $y = x^{(x^a)} + a^{(x^x)}, a > 0$

78. $y = x^{\sin x + \cos x}$

79. $y = (1+x^2)^{3x+\sin x}$

80. $y = \mathrm{arctg}\,\dfrac{3-2x}{2\sqrt{3x-x^2}}$

81. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2}$

82. $y = x\sin x\,\mathrm{arctg}\,x - \dfrac{x\sin x}{\cos x}$

83. $y = \ln(x + \sqrt{1+x^2})$

84. $y = 2\sin(\ln\dfrac{x^2+1}{x}) + \sqrt{1-x^2}\arccos x$

85. $y = \arccos\dfrac{1}{1+\sqrt{1-x^2}} - \cos(\ln\dfrac{\mathrm{tg}\,x}{\sqrt{1-x^2}})$

86. $y = \mathrm{arctg}\,e^{x^2} - \ln\sqrt{\dfrac{e^{2x}}{3x}}$

87. $y = \mathrm{arctg}\,e^x - \ln\sqrt{\dfrac{e^{2x}}{e^{2x}+1}}$

88. $y = x - \ln\sqrt{1+e^{2x}} + e^{-x}\,\mathrm{arctg}\,e^x$

89. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2}\arcsin x$

90. $y = \mathrm{arctg}\,\dfrac{1}{1+\sqrt{1-x^2}}$

91. $y = \dfrac{\arccos x}{x} + \ln\sqrt{\dfrac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}}$

92. $y = \dfrac{2+x}{2}\sqrt{4x+x^2} - 2\ln(x+2+\sqrt{4x+x^2})$

93. $y = \dfrac{x}{2}\sqrt{x^2+2x+2} + \ln(x+1+\sqrt{x^2+2x+2})$

94. $y = -\dfrac{x}{1+8x^3} + \dfrac{1}{12}\ln\dfrac{(1+2x)^2}{1-2x+4x^2} + \dfrac{\sqrt{3}}{6}\,\mathrm{arctg}\,\dfrac{4x-1}{\sqrt{3}}$

Vypočtěte $y''(x)$:

Find y'':

95. $y = \dfrac{x-2}{\sqrt{x^2+1}}$

96. $y = \dfrac{x-2}{x^3}$

97. $y = \dfrac{x^2}{1-x}$

98. $y = (1-x^2)e^x$

99. $y = (1-x^2)e^{-x}$

100. $y = \dfrac{e^{-x}}{1-x}$

Výsledky: , **70**: $\dfrac{1}{x^3-1}$, **73**: $-\dfrac{\arccos x}{x^2}$, **74**: $\frac{2}{\cos x\sqrt{\sin x}}$, **75**: $\frac{x\arcsin x}{(1-x^2)^{3/2}}$, **83**: $\frac{1}{\sqrt{1+x^2}}$, **86**: $\dfrac{2xe^{x^2}}{1+e^{2x^2}} - \dfrac{2x-1}{2x}$, **87**: $\frac{e^x-1}{1+e^{2x}}$, **89**: $(\arcsin x)^2$, **90**: $\frac{1}{2\sqrt{1-x^2}}$, **91**: $-\dfrac{\arccos x}{x^2}$, **95**: $\frac{-4x^2-3x+2}{(x^2+1)^2\sqrt{x^2+1}}$, **96**: $6\dfrac{x-4}{x^5}$, **97**: $\frac{2}{(1-x)^3}$, **98**: $-e^x(x^2+4x+1)$, **99**: $-e^{-x}(1-4x+x^2)$, **100**: $\frac{e^{-x}(x^2+1)}{(1-x)^3}$ (bez záruky)