Příklad 1: Vypočítejte metodou substituce (substituce $\varphi(x) = t$)

a) $\int x \,\mathrm{tg}\left(1-x^2\right)dx = \frac{1}{2}\ln\left|\cos\left(1-x^2\right)\right| + C$

b) $\int \frac{dx}{x\sqrt{1-\ln^2 x}} = \arcsin\left(\ln x\right) + C$

c) $\int \sin(\ln x)\cdot\frac{1}{x}\,dx = -\cos(\ln x) + C$

d) $\int \frac{2x^2}{\cos^2\left(x^3+1\right)}dx = \frac{2}{3}\mathrm{tg}\left(x^3+1\right) + C$

e) $\int \frac{x}{\cos^2\left(x^2\right)}dx = \frac{1}{2}\mathrm{tg}\, x^2 + C$

f) $\int x\sin\left(1-x^2\right)dx = \frac{1}{2}\cos\left(1-x^2\right) + C$

g) $\int \frac{\mathrm{e}^{2x}}{\mathrm{e}^x - 1}dx = \mathrm{e}^x + \ln\left|\mathrm{e}^x - 1\right| + C$

h) $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx = \frac{1}{2}\arcsin^2 x + C$

Příklad 2: Vypočtěte

a) $\int \frac{\sin x}{\sqrt{\cos^3 x}}dx = \frac{2}{\sqrt{\cos x}} + C$

b) $\int \frac{\cos x}{\sin^2 x + 2}dx = \frac{1}{\sqrt{2}}\mathrm{arctg}\frac{\sin x}{\sqrt{2}} + C$

c) $\int \frac{\sin x}{\sqrt{2+\cos x}}dx = -2\sqrt{2+\cos x} + C$

d) $\int \sin^5 x\,dx = -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

e) $\int \frac{\sin^3 x}{2+\cos x}dx = \frac{1}{2}\cos^2 x - 2\cos x + 3\ln\left|\cos x + 2\right| + C$

f) $\int \sin x(1+3\cos^2 x)\,dx = -\cos x - \cos^3 x + C$

g) $\int \frac{\cos x}{\sin^2 x - 2\sin x}dx = -\frac{1}{2}\ln\left|\frac{\sin x}{2-\sin x}\right| + C$

h) $\int \frac{\cos^3 x}{\sin^2 x}dx = -\frac{1}{\sin x} - \sin x + C$

Příklad 3: Metodou substituce vypočtěte

a) $\int \frac{2}{x\sqrt{x+3}}dx = -\frac{2}{\sqrt{3}}\ln\left|\frac{\sqrt{3}+\sqrt{x+3}}{\sqrt{3}-\sqrt{x+3}}\right| + C$

b) $\int \frac{\sqrt{x-3}}{2x}dx = \sqrt{x-3} - \frac{3}{\sqrt{3}}\mathrm{arctg}\sqrt{\frac{x-3}{3}} + C$

c) $\int \frac{1}{1+\sqrt{x+1}}dx = 2\sqrt{x+1} - 2\ln\left(1+\sqrt{x+1}\right) + C$

d) $\int \frac{1}{(1-x)\sqrt{x-5}}dx = -\mathrm{arctg}\frac{\sqrt{x-5}}{2} + C$

e) $\int \frac{1}{(2+x)\sqrt{1+x}}dx = 2\mathrm{arctg}\sqrt{1+x} + C$

f) $\int \frac{\sqrt{x}}{x+\sqrt{x}}dx = 2\sqrt{x} - 2\ln\left|\sqrt{x}+1\right| + C$