I. Vypočtěte integrály přímou integrací, pomocí vlastností integrálu a úpravami integrované funkce

1. $\int_{-1}^{2}\left(2x^3-3x^2+4x\right)dx=\frac{9}{2}$

2. $\int_{0}^{1}\left(x-2\cdot\sqrt[3]{x}-2\mathrm{e}^x\right)dx=1-2\mathrm{e}$

3. $\int_{1}^{4}\frac{3x-2}{\sqrt{x}}dx=18$

4. $\int_{\frac{\pi}{2}}^{\pi}\frac{\cos x}{\sin x+1}dx=-\ln 2$

5. $\int_{1}^{2}\frac{x^2}{x^3-2}dx=\frac{1}{3}\ln 6$

6. $\int_{-1}^{3}\frac{1}{\sqrt{2x+3}}dx=2$

7. $\int_{1}^{2}\frac{1}{\sqrt{2x+x^2}}dx=\ln\left(3+\sqrt{8}\right)-\ln\left(2+\sqrt{3}\right)$

8. $\int_{-1}^{0}\frac{1}{\sqrt{1-2x-x^2}}dx=\frac{\pi}{4}$

9. $\int_{-1}^{1}\frac{4}{x^2+2x+5}dx=\frac{\pi}{2}$

10. $\int_{-1}^{1}\frac{2x-1}{x-2}dx=4-3\ln 3$

11. $\int_{0}^{1}\frac{x^3}{x^2-2}dx=\frac{1}{2}-\ln 2$

12. $\int_{0}^{1}\frac{2+x}{2-x^2}dx=\frac{1}{\sqrt{2}}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{1}{2}\ln 2$

II. Vypočtěte metodou substituce :

1. $\int_{1}^{\sqrt{\mathrm{e}}}\frac{1}{x\sqrt{1-\ln^2 x}}dx=\frac{\pi}{6}$

2. $\int_{-1}^{0}x\cdot\mathrm{e}^{1-x^2}dx=\frac{1}{2}\left(1-\mathrm{e}\right)$

3. $\int_{3}^{8}\frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx=9+4\ln 2$

4. $\int_{0}^{7}\frac{x-3}{\sqrt[3]{x+1}}dx=\frac{3}{5}$

5. $\int_{3}^{4}\frac{\sqrt{x-3}}{x-2}dx=2-\frac{\pi}{2}$

6. $\int_{2}^{4}\frac{1}{x\sqrt{x-1}}dx=\frac{\pi}{6}$

7. $\int_{0}^{\frac{\pi}{2}}\sin x\cos^2 x\,dx=\frac{1}{3}$

8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x}{\sin^3 x}dx=\frac{1}{2}$

9. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x + \cos^3 x}{\sin x} dx = -2\ln\frac{\sqrt{2}}{2} - \frac{1}{4}$

10. $\int_{0}^{\frac{\pi}{4}} \mathrm{tg}^3 x\, dx = \ln\frac{\sqrt{2}}{2} + \frac{1}{2}$

III. Vypočtěte metodou per partes :

1. $\int_{0}^{1} x \cdot \mathrm{e}^{-2x}\, dx = \frac{1}{4} - \frac{3}{4}\mathrm{e}^{-2}$

2. $\int_{0}^{\pi} x \sin 2x\, dx = -\frac{\pi}{2}$

3. $\int_{1}^{3} x^2 \ln\left(x^3\right) dx = 9\ln 27 - \frac{26}{3}$

4. $\int_{0}^{\sqrt{3}} x \cdot \mathrm{arctg} x\, dx = \frac{4\pi - 3\sqrt{3}}{6}$

5. $\int_{0}^{1} \mathrm{arctg}\sqrt{x}\, dx = \frac{\pi}{2} - 1$

6. $\int_{-1}^{0} (2x+1)^2 \cdot \mathrm{e}^x\, dx = 5 - 13\mathrm{e}^{-1}$

7. $\int_{\pi}^{2\pi} x^2 \cos x\, dx = 6\pi$

8. $\int_{-1}^{\mathrm{e}-2} \ln(x+2)\, dx = 1$

9. $\int_{0}^{1} \arccos x\, dx = 1$

10. $\int_{0}^{\frac{\pi}{3}} x \cdot tg^2 x\, dx = \frac{\pi}{\sqrt{3}} - \frac{\pi^2}{18} - \ln 2$