Poznámka: V následujících příkladech budeme předpokládat, že řešíme pro x, pro která má funkce smysl.

I. Vypočtěte derivaci a výsledek upravte

1) $y = x^4 + x^3 + 1$ $\quad$ $y' = x^2(4x+3)$

2) $y = 3x^2 - \sqrt[3]{x} + \sqrt[4]{x}$ $\quad$ $y' = 6x - \dfrac{1}{3\cdot\sqrt[3]{x^2}} + \dfrac{1}{4\cdot\sqrt[4]{x^3}}$

3) $y = x\sqrt{x} + \dfrac{1}{x^2} - \dfrac{1}{x}$ $\quad$ $y' = \dfrac{3}{2}\sqrt{x} - \dfrac{1}{x^3} + \dfrac{1}{x^2}$

4) $y = 2x(x^2 + x + 1)$ $\quad$ $y' = 6x^2 + 4x + 2$

5) $y = (x^2 + x)(x^3 - 4)$ $\quad$ $y' = 5x^4 + 4x^3 - 8x - 4$

6) $y = x \ln x$ $\quad$ $y' = \ln x + 1$

7) $y = x^3 \ln x$ $\quad$ $y' = x^2(3\ln x + 1)$

8) $y = \mathrm{e}^x(x^2 - 2x + 2)$ $\quad$ $y' = \mathrm{e}^x x^2$

9) $y = (x^2 + 1)\cdot \mathrm{arctg} x$ $\quad$ $y' = 2x\,\mathrm{arctg} x + 1$

10) $y = 2\arcsin x \cdot \arccos x$ $\quad$ $y' = \dfrac{2(\arccos x - \arcsin x)}{\sqrt{1-x^2}}$

11) $y = x^2 \cdot \sin x \cdot \cos x$ $\quad$ $y' = x\sin 2x + x^2(\cos^2 x - \sin^2 x) = x\sin 2x + x^2\cos 2x$

12) $y = \dfrac{x^5 + x}{3}$ $\quad$ $y' = \dfrac{1}{3}(5x^4 + 1)$

13) $y = \dfrac{2}{x^2 + 1}$ $\quad$ $y' = \dfrac{-4x}{(x^2+1)^2}$

14) $y = \dfrac{\mathrm{arctg} x}{x^2 + 1}$ $\quad$ $y' = \dfrac{1 - 2x\,\mathrm{arctg} x}{(x^2+1)^2}$

15) $y = \dfrac{3^x}{x}$ $\quad$ $y' = \dfrac{3^x(\ln 3 \cdot x - 1)}{x^2}$

16) $y = \dfrac{x^4 - x}{x + 1}$ $\quad$ $y' = \dfrac{3x^4 + 4x^3 - 1}{(x+1)^2}$

17) $y = \frac{\ln x}{x}$ $\qquad$ $y' = \frac{1 - \ln x}{x^2}$

18) $y = \frac{x^2}{\ln x}$ $\qquad$ $y' = \frac{x(2\ln x - 1)}{\ln^2 x}$

19) $y = \frac{2 + \sqrt{x}}{x}$ $\qquad$ $y' = -\frac{4 + \sqrt{x}}{2x^2}$

20) $y = \frac{\sin x}{1 - \cos x}$ $\qquad$ $y' = \frac{-1}{1 - \cos x} = \frac{1}{\cos x - 1}$

II. Derivujte složenou funkci a výsledek upravte

1) $y = \mathrm{e}^{3x^2+2}$ $\qquad$ $y' = \mathrm{e}^{3x^2+2} \cdot 6x$

2) $y = \operatorname{arccotg} \frac{1}{x^2}$ $\qquad$ $y' = \frac{2x}{x^4 + 1}$

3) $y = \operatorname{tg}^2 \frac{x}{2}$ $\qquad$ $y' = \frac{\sin \frac{x}{2}}{\cos^3 \frac{x}{2}}$

4) $y = \operatorname{arctg} \frac{x-1}{x+1}$ $\qquad$ $y' = \frac{1}{x^2 + 1}$

5) $y = \arccos \sqrt{2x+1}$ $\qquad$ $y' = \frac{-1}{\sqrt{-4x^2 - 2x}}$

6) $y = \sin^2 x - 3\cos^2 x$ $\qquad$ $y' = 4 \cdot \sin 2x$

7) $y = \sqrt{9 - x^2} - \arcsin \frac{x}{3}$ $\qquad$ $y' = \frac{-x-3}{\sqrt{9 - x^2}}$

8) $y = \sqrt{\frac{x^2+1}{x^2-1}}$ $\qquad$ $y' = \sqrt{\frac{x^2-1}{x^2+1}} \cdot \frac{-2x}{\left(x^2-1\right)^2}$

9) $y = \arcsin \frac{x}{1+x}$ $\qquad$ $y' = \frac{1}{(1+x)\sqrt{1+2x}}$

10) $y = \ln \sqrt{\frac{x+1}{x}}$ $\qquad$ $y' = \frac{-1}{2x(x+1)}$

11) $y = \arccos\dfrac{x}{\sqrt{x^2+1}}$ $\qquad y' = \dfrac{-1}{x^2+1}$

12) $y = \ln\dfrac{1}{x+\sqrt{x^2-1}}$ $\qquad y' = -\dfrac{1}{\sqrt{x^2-1}}$

13) $y = \operatorname{arctg}\dfrac{x}{2} + \ln\sqrt{\dfrac{x-2}{x+2}}$ $\qquad y' = \dfrac{4x^2}{x^4-16}$

14) $y = \ln\operatorname{arctg}\dfrac{1}{1+x}$ $\qquad y' = \dfrac{1}{\operatorname{arctg}\dfrac{1}{1+x}} \cdot \dfrac{-1}{x^2+2x+2}$

15) $y = \arcsin\sqrt{\dfrac{1-x}{1+x}}$ $\qquad y' = -\dfrac{1}{(1+x)\sqrt{2x(1-x)}}$

16) $y = \dfrac{1}{\sqrt{3}}\operatorname{arctg}\dfrac{x\sqrt{3}}{1-x^2}$ $\qquad y' = \dfrac{1+x^2}{1+x^2+x^4}$

17) $y = \sqrt{x+1} - \ln\left(1+\sqrt{x+1}\right)$ $\qquad y' = \dfrac{1}{2\left(1+\sqrt{x+1}\right)}$

18) $y = \dfrac{\sin x - x\cos x}{\cos x + x\sin x}$ $\qquad y' = \dfrac{x^2}{(\cos x + x\sin x)^2}$

19) $y = \ln\sqrt{\dfrac{1+\sin x}{\cos x}}$ $\qquad y' = \dfrac{1}{2\cos x}$

20) $y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2}\arcsin x$ $\qquad y' = (\arcsin x)^2$

III. Vypočtěte hodnotu 1.derivace funkce v daném bodě

1) $y = \dfrac{1+\cos x}{1-\cos x}$, $x_0 = \dfrac{\pi}{2}$ $\qquad y'\left(\dfrac{\pi}{2}\right) = -2$

2) $y = x\mathrm{e}^{1-\cos^2 x}$, $x_0 = 0$ $\qquad y'(0) = 1$

3) $y = \ln\sqrt{\dfrac{1-\sin x}{1+\sin x}}$, $x_0 = \pi$ $\qquad y'(\pi) = 1$

4) $y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1}$, $x_0 = 4$ $\qquad y'(4) = -\dfrac{1}{2}$

5) $y = \arcsin\dfrac{x}{\sqrt{4-x^2}}$, $x_0 = 1$ $\qquad y'(1) = \dfrac{4}{3\sqrt{2}}$

6) $y = 2\mathrm{arctg}\sqrt{\dfrac{2-x}{x}}$, $x_0 = 1$ $\qquad y'(1) = -1$

7) $y = \sqrt{\mathrm{e}^{\left(x^2\right)}}$, $x_0 = 1$ $\qquad y'(1) = \sqrt{\mathrm{e}}$

8) $y = \dfrac{8}{x} - \dfrac{3x^2}{\sqrt[3]{x}+1}$, $x_0 = 8$ $\qquad y'(8) = -\dfrac{1033}{72}$

9) $y = \dfrac{\cos^3 x + 1}{\sqrt{x}} - \dfrac{2}{x^2}$, $x_0 = \pi$ $\qquad y'(\pi) = \dfrac{4}{\pi^3}$

10) $y = \dfrac{1}{\mathrm{arctg}\sqrt{x}} + x\dfrac{2-\sqrt[4]{x}}{x+1}$, $x_0 = 1$ $\qquad y'(1) = \dfrac{1}{8} - \dfrac{4}{\pi^2}$

IV. Vypočtěte derivace vyšších řádů

1) $y = \dfrac{1}{7}(4x^3 - x^4)$, $y'' = ?$ $\qquad y'' = \dfrac{12}{7}x(2-x)$

2) $y = \dfrac{2x-1}{x^2}$, $y'' = ?$ $\qquad y'' = \dfrac{4x-6}{x^4}$

3) $y = \dfrac{2x-1}{(x-1)^3}$, $y'' = ?$ $\qquad y'' = \dfrac{12x}{(x-1)^5}$

4) $y = \mathrm{e}^x(1-2x)$, $y''' = ?$ $\qquad y''' = \mathrm{e}^x(-2x-5)$

5) $y = (x^2+3)\mathrm{e}^{-x}$, $y''' = ?$ $\qquad y''' = \mathrm{e}^{-x}(-x^2+6x-9)$

6) $y = x\ln x$, $y^{(4)} = ?$ $\qquad y^{(4)} = \dfrac{2}{x^3}$

7) $y = \mathrm{arctg}x$, $y^{(4)} = ?$ $\qquad y^{(4)} = \dfrac{12x-4x^3}{(1+x^2)^3}$