Oscilační kriteria pro pololineární diferenciální rovnice neutrálního typu

Brno 28. 4. 2014

$$\left[r(t)\,\Phi(z'(t))\right]' + c(t)\,\Phi(x(\sigma(t))) = 0$$

$$z(t) = x(t) + p(t)\,x(\tau(t))$$

$\Phi(x) = |x|^{\alpha-1}\cdot x$

$\alpha \ge 1$, $\quad \sigma(\tau(t)) = \tau(\sigma(t))$, $\quad \int^{\infty} r^{-1/\alpha}(s)\,ds = \infty$

SROVNÁVACÍ METODA (BACULÍKOVÁ, DŽURINA)

$$[r(t)\Phi(z'(t))]' + c(t)\,\Phi(x(\sigma(t))) = 0$$

$$\frac{1}{\tau'(t)}[r(\tau(t))\Phi(z'(\tau(t)))]' + c(\tau(t))\,\Phi(x(\sigma(\tau(t)))) = 0$$

$$[r(t)\Phi(z'(t))]' + c(t)\,\Phi(x(\sigma(t))) = 0$$

$$\frac{p_0^\alpha}{\tau_0}[r(\tau(t))\Phi(z'(\tau(t)))]' + c(\tau(t))\,\Phi(p_0 x(\sigma(\tau(t)))) \le 0$$

$+$

$$y(t) := r(t)z'(t) + \frac{p_0^\alpha}{\tau_0} r(\tau(t))\,\Phi(z'(\tau(t)))$$

$$y'(t) + \min\{c(t), c(\tau(t))\}\left[\Phi(x(\sigma(t))) + \Phi(p_0 x(\sigma(\tau(t))))\right] \le 0$$

SROVNÁVACÍ METODA (BACULÍKOVÁ, DŽURINA)

$$y(t) = r(t) z'(t) + \frac{p_0^{\alpha}}{\tau_0} r(\tau(t)) \phi(z'(\tau(t)))$$

$$y'(t) + \min\{c(t), c(\tau(t))\} \left[\phi(x(\sigma(t))) + \phi(p_0 x(\sigma(\tau(t)))) \right] \le 0$$

$\leftrightarrow \quad \sigma \circ \tau = \tau \circ \sigma$

$$y'(t) + \min\{c(t), c(\tau(t))\} \quad 2^{1-\alpha} \phi(z(\sigma(t))) \le 0$$

$$y'(t) + \min\{c(t), c(\tau(t))\} \, 2^{1-\alpha} \left[\int_{t_0}^{\sigma(t)} r^{-1/\alpha}(s) \, ds \right]^{\alpha} \frac{\tau_0}{p_0^{\alpha} + \tau_0} y(\tau^{-1}(\sigma(t))) \le 0$$

METODA RICCATIHO ROVNICE

$$(*) \quad \boxed{x'' + c(t)x = 0} \qquad w = \frac{x'}{x} \qquad \boxed{w' + c(t) + w^2 = 0}$$

$$\rho > 0, \quad w > 0 \quad (\text{j.e.-h. } c > 0 \text{ a } x > 0)$$

$$(\rho w)' + \rho w^2 + \rho c - \rho' w = 0$$

$$(\rho w)' + \cancel{\rho\left(w - \frac{\rho'}{2\rho}\right)^2} + \rho c - \frac{(\rho')^2}{4\rho} \leq 0$$

$$\rho(t)w(t) - \rho(t_1)w(t_1) + \int_{t_1}^{t} \rho(s)c(s) - \frac{[\rho'(s)]^2}{4\rho(s)}\, ds \leq 0$$

$$\int^{\infty} \rho(s)c(s) - \frac{(\rho'(s))^2}{4\rho(s)}\, ds = \infty \Rightarrow (*) \text{ osc.}$$

Riccatiho metoda pro $\left[r\phi(z')\right]' + c\phi(x\circ\sigma) = 0$, $z = x + p\cdot x\circ\tau$

$$\omega(t) = \rho(t)\,\frac{r(t)\phi(z'(t))}{\phi(z(\sigma(t)))}$$

$$\omega'(t) = \frac{\rho'(t)}{\rho(t)}\omega(t) + \rho(t)\frac{\left[r(t)\phi(z'(t))\right]'}{\phi(z(\sigma(t)))} - \alpha\,\rho(t)\,\frac{r(t)\phi(z'(t))}{\phi(z(\sigma(t)))}\,\frac{\sigma'(t)\,z'(\sigma(t))}{z(\sigma(t))}$$

$$\left.\begin{array}{l} \sigma(t) \le t \\ r(t)\phi(z'(t)) \text{ klesá} \end{array}\right\} \quad z'(\sigma(t)) \ge \left[\frac{r(t)}{r(\sigma(t))}\right]^{1/\alpha} z'(t)$$

$$\omega'(t) \le \frac{\rho'(t)}{\rho(t)}\omega(t) + \rho(t)\frac{\left[r(t)\phi(z'(t))\right]'}{\phi(z(\sigma(t)))} - \frac{\alpha\,\sigma'(t)}{\rho^{1/\alpha}(t)\,r^{1/\alpha}(t)}\left|\omega(t)\right|^{\frac{\alpha+1}{\alpha}}$$

RICCATIHO METODA PRO $\left[r\Phi(z')\right]' + c\Phi(x\circ\sigma) = 0$, $z = x + p\cdot x\circ\tau$

$$w(t) = \rho(t)\,\frac{r(t)\Phi(z'(t))}{\Phi(z(\sigma(t)))}$$

$$w'(t) \le \frac{\rho'(t)}{\rho(t)}\,w(t) + \rho(t)\,\frac{\left[r(t)\phi(z'(t))\right]'}{\phi(z(\sigma(t)))} - \frac{\alpha\,\sigma'(t)}{\rho^{1/\alpha}(t)\,r^{1/\alpha}(t)}\,|w(t)|^{\frac{\alpha+1}{\alpha}}$$

$$v(t) = \rho(t)\,\frac{r(\tau(t))\,\phi(z'(\tau(t)))}{\phi(z(\sigma(t)))}$$

$$v'(t) \le \frac{\rho'(t)}{\rho(t)}\,v(t) + \rho(t)\,\frac{\left[r(\tau(t))\phi(z'(\tau(t)))\right]'}{\phi(z(\sigma(t)))} - \frac{\alpha\,\sigma'(t)}{\rho^{1/\alpha}(t)\,r^{1/\alpha}(t)}\,|v(t)|^{\frac{\alpha+1}{\alpha}}$$

INOVACE KLASICKÉHO POSTUPU

PŮVODNÍ | F+M (2014)

1) Součet rovnic $\Longrightarrow$ vhodná lin. kombinace

2) $\left(\frac{1}{2}x + \frac{1}{2}y\right)^\alpha \leq \frac{1}{2}x^\alpha + \frac{1}{2}y^\alpha \Longrightarrow \left(\frac{1}{\ell}x + \frac{1}{\ell^*}y\right)^\alpha \leq \frac{1}{\ell}x^\alpha + \frac{1}{\ell^*}y^\alpha$

3) $\min\{c(t), c(\tau(t))\} \Longrightarrow \min\{c(t), \varphi \cdot c(\tau(t))\}$

4) $\sigma(\tau(t)) = \tau(\sigma(t)) \Longrightarrow \sigma(\tau(t)) \geq \tau(\sigma(t))$, pokud x roste

Věta: Jestliže existuje $\ell > 1$ a $\varphi > 0$ takové, že

$$\limsup_{t\to\infty} \int_{t_0}^{t} \rho(s) \min\{c(s), \varphi c(\tau(s))\} - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{\rho(s)\, r(\delta(s))}{[\delta'(s)]^{\alpha}} \cdot \left[\ell^{\alpha-1} + (\ell^{*})^{\alpha-1} \frac{P_0^{\alpha}\varphi}{\tau_0}\right] \left[\frac{\rho'_{+}(s)}{\rho(s)}\right]^{\alpha} ds = \infty,$$

potom rovnice

$$\left[r(t)\, \Phi(z'(t))\right]' + c(t)\, \phi\left(x(\sigma(t))\right) = 0, \quad z(t) = x(t) + p(t)\, x(\tau(t))$$

osciluje.

Použití: 1) φ: $\quad c(t) = \varphi \cdot c(\tau(t))$

2) ρ: druhý člen v limsup je $\int 1/s \, ds$

3) ℓ: $[\ell^{\alpha-1} \dots \ell^{*}] \longrightarrow \min$

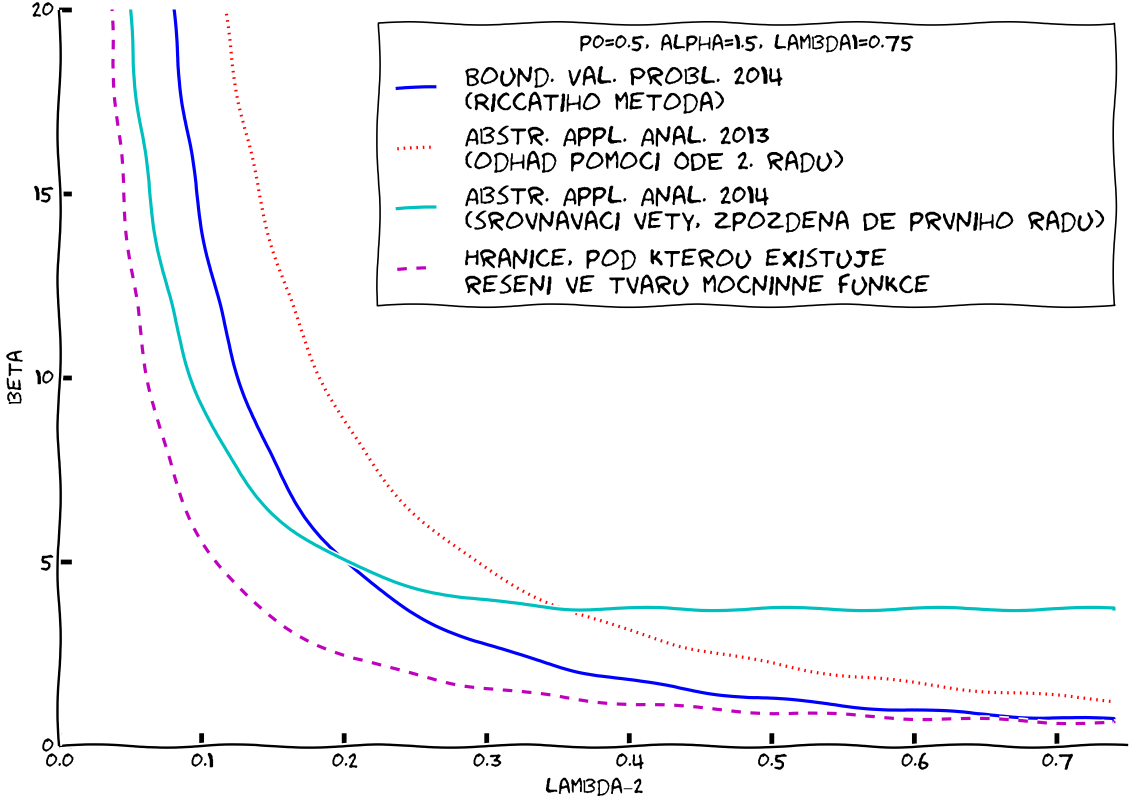
POROVNÁNÍ METOD

ZOBECNĚNÁ NEUTRÁLNÍ EULEROVA ROVNICE

$$\left[\Phi\left(z'(t)\right)\right]' + \frac{\beta}{t^{\alpha+1}}\Phi\left(x(\lambda_2 t)\right) = 0$$

$$z(t) = x(t) + p_0\, x(\lambda_1 t), \qquad 0 < \lambda_2 < \lambda_1 < 1$$

$$\Phi(x) = |x|^{\alpha-1}x, \qquad \alpha \geq 1$$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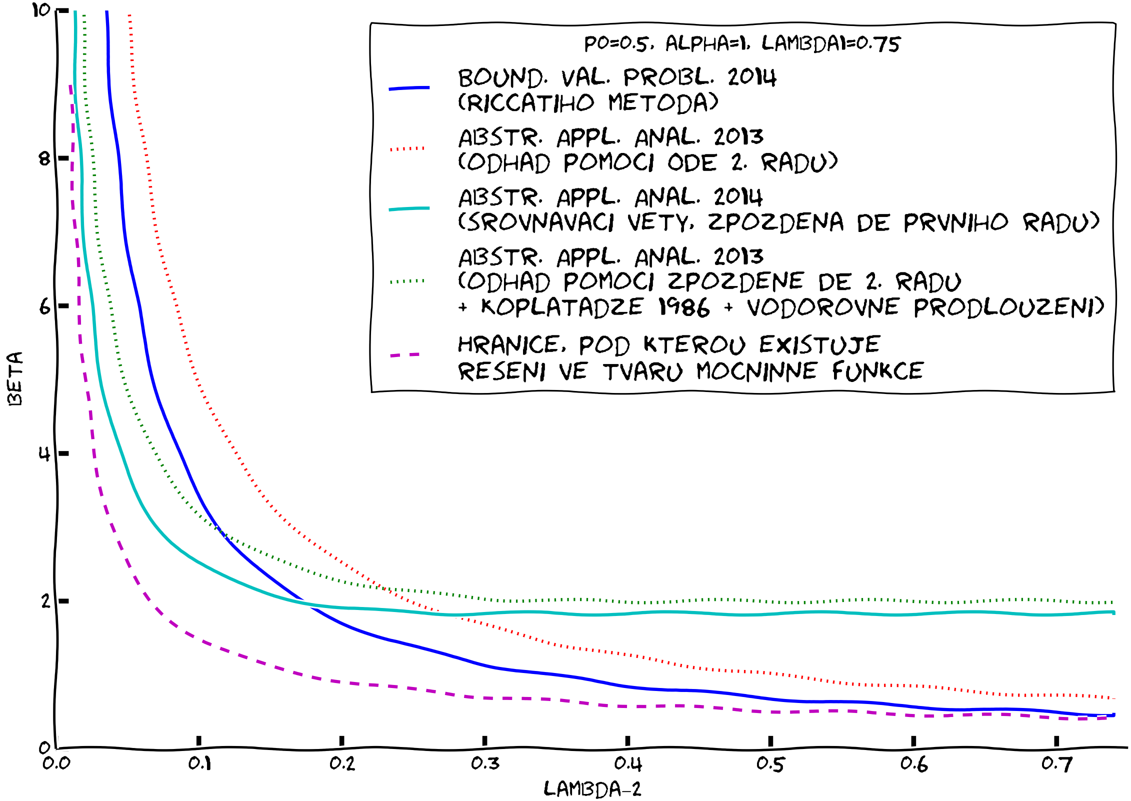
BOUND. VAL. PROBL. 2014
(RICCATIHO METODA)
ABSTR. APPL. ANAL. 2013
(ODHAD POMOCI ODE 2. RADU)
ABSTR. APPL. ANAL. 2014
(SROVNAVACI VETY, ZPOZDENA DE PRVNIHO RADU)
HRANICE, POD KTEROU EXISTUJE
RESENI VE TVARU MOCNINNE FUNKCE
BETA
0
5
10
15
20
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
LAMBDA-2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BOUND. VAL. PROBL. 2014
(RICCATIHO METODA)
ABSTR. APPL. ANAL. 2013
(ODHAD POMOCI ODE 2. RADU)
ABSTR. APPL. ANAL. 2014
(SROVNAVACI VETY, ZPOZDENA DE PRVNIHO RADU)
ABSTR. APPL. ANAL. 2013
(ODHAD POMOCI ZPOZDENE DE 2. RADU
+ KOPLATADZE 1986 + VODOROVNE PRODLOUZENI)
HRANICE, POD KTEROU EXISTUJE
RESENI VE TVARU MOCNINNE FUNKCE
BETA
0
2
4
6
8
10
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
LAMBDA-2