

Cvičení 3.1

- $\int (2x^3 - x^2 + 3x + 2) dx = \frac{1}{2}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$
- $\int (x^5 - 3x^2 - x + 5) dx = \frac{1}{6}x^6 - x^3 - \frac{1}{2}x^2 + 5x$
- $\int \left(2\sqrt[3]{x} + 3\sqrt{x} - \frac{1}{\sqrt[3]{x}}\right) dx = \frac{3}{2}x^{4/3} + 2x^{3/2} - \frac{3}{2}x^{2/3}$
- $\int \sqrt{x} \left(3\sqrt{x} + 2x - \frac{5}{\sqrt{x}}\right) dx = \frac{3}{2}x^2 + \frac{4}{5}x^{5/2} - 5x$
- $\int (x+1)^2 (\sqrt{x}-1) dx =$
 $= \frac{2}{3}x^{3/2} - x + \frac{4}{5}x^{5/2} - x^2 + \frac{2}{7}x^{7/2} - \frac{1}{3}x^3$
- $\int (x^{2/3} + 2x^{1/3}) \sqrt[3]{x} dx = \frac{1}{2}x^2 + \frac{6}{5}x^{5/3}$

Cvičení 3.2

- $\int \left(\frac{1}{x+1} + \frac{4}{2x-1}\right) dx = \ln|x+1| + 2\ln|2x-1|$
- $\int \left(\frac{1}{3x+2} - \frac{3}{x+2}\right) dx = \frac{1}{3}\ln|3x+2| - 3\ln|x+2|$
- $\int (e^{2x+1} + e^{1-x}) dx = \frac{1}{2}e^{2x+1} - e^{1-x}$
- $\int (2^{x+5} + 3^{2x}) dx = \frac{2^{x+5}}{\ln 2} + \frac{1}{2} \frac{3^{2x}}{\ln 3}$
- $\int \left(\frac{1}{x-1} + e^{x-1}\right) dx = \ln|x-1| + e^{x-1}$
- $\int \left(\frac{1}{2-x} + 2^{x-2}\right) dx = -\ln|2-x| + \frac{2^{x-2}}{\ln 2}$

Cvičení 3.3

- $\int (\sin(3x+1) - \cos(3x-1)) dx =$
 $= -\frac{1}{3}\cos(3x+1) - \frac{1}{3}\sin(3x-1)$
- $\int \left(\frac{3}{\cos^2 x} + \frac{2}{\sin^2(2x+1)}\right) dx = 3\operatorname{tg} x - \operatorname{cotg}(2x+1)$
- $\int \left(-\sin(x-1) + \frac{1}{\cos^2(x-1)}\right) dx =$
 $= \cos(x-1) + \operatorname{cotg}(x-1)$
- $\int \left(\frac{4}{\sin^2(4x)} + \cos(4x)\right) dx = -\operatorname{cotg}(4x) + \frac{1}{4}\sin(4x)$

Cvičení 3.4

- $\int \left(\frac{1}{x^2+4} + \frac{3}{9x^2+1}\right) dx = \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + \operatorname{arctg}(3x)$
- $\int \left(\frac{1}{\sqrt{x^2+1}} + \frac{1}{\sqrt{4-x^2}}\right) dx =$
 $= \ln(x + \sqrt{x^2+1}) + \arcsin\left(\frac{x}{2}\right)$
- $\int \left(\frac{1}{\sqrt{16-9x^2}} + \frac{1}{16+9x^2}\right) dx =$
 $= \frac{1}{3}\arcsin\left(\frac{3x}{4}\right) + \frac{1}{12}\operatorname{arctg}\left(\frac{3x}{4}\right)$
- $\int \left(\frac{1}{2x^2+3} + \frac{1}{\sqrt{2x^2+3}}\right) dx =$
 $= \frac{1}{\sqrt{6}}\operatorname{arctg}\left(\frac{x\sqrt{6}}{3}\right) + \frac{1}{\sqrt{2}}\ln(\sqrt{2}x + \sqrt{2x^2+3})$

Cvičení 3.5

- $\int \frac{x^2}{x^3+5} dx = \frac{1}{3}\ln|x^3+5|$
- $\int \frac{x+2}{x^2+4x-1} dx = \frac{1}{2}\ln|x^2+4x-1|$
- $\int \frac{1}{x \ln x} dx = \ln|\ln x|$

- $\int \frac{\cos x \sin x}{\cos^2 x + 2} dx = -\frac{1}{2}\ln(\cos^2 x + 2)$
- $\int \frac{\cos x}{\sin x + 1} dx = \ln(\sin x + 1)$
- $\int 2\operatorname{cotg} x dx = 2\ln|\sin x|$

Cvičení 3.6

- $\int \frac{(\sqrt{x})^3 + 1}{\sqrt{x} + 1} dx = \frac{1}{2}x^2 - \frac{2}{3}x^{3/2} + x$
- $\int \frac{e^{2x}-1}{e^x-1} dx = e^x + x$
- $\int \frac{\sin(2x)}{\sin^2 x} dx = \ln \sin^2 x$
- $\int \frac{\cos(2x)}{\sin^2 x} dx = -\operatorname{cotg} x - 2x$
- $\int \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2 dx = x + \cos x$
- $\int \sin^2 x dx = -\frac{1}{4}\sin(2x) + \frac{1}{2}x$

Cvičení 3.7

- $\int \frac{1}{\sqrt{5-4x-x^2}} dx = \arcsin \frac{x+2}{3}$
- $\int \frac{1}{\sqrt{-2x-x^2}} dx = \arcsin(1+x)$
- $\int \frac{4}{\sqrt{3+4x-4x^2}} dx = 2\arcsin \frac{2x-1}{2}$
- $\int \frac{1}{\sqrt{2x+x^2}} dx = \ln|1+x + \sqrt{2x+x^2}|$
- $\int \frac{1}{\sqrt{9x^2-6x+4}} dx = \frac{1}{3}\ln|3x-1 + \sqrt{9x^2-6x+4}|$
- $\int \frac{1}{\sqrt{x^2+4x+11}} dx = \ln|x+2 + \sqrt{x^2+4x+11}|$

Cvičení 3.8

- $\int \frac{2x^2}{1+x} dx = -2x + x^2 + 2\ln|1+x|$
- $\int \frac{x^2-3}{x^2+1} dx = x - 4\operatorname{arctg} x$
- $\int \frac{4x^6-3x^2+5}{2x^2+2} dx = \frac{2}{5}x^5 - \frac{2}{3}x^3 + \frac{1}{2}x + 2\operatorname{arctg} x$
- $\int \frac{2x^4-x^2-2}{x^3-2x} dx = x^2 + \ln|x| + \ln|x^2-2|$

Cvičení 3.9

- $\int \frac{1}{2x+3} dx = \frac{1}{2}\ln|2x+3|$
- $\int \frac{3x-5}{x-1} dx = 3x - 2\ln|x-1|$
- $\int \frac{1}{(x+2)^3} dx = -\frac{1}{2(x+2)^2}$
- $\int \frac{1}{(2x-4)^2} dx = -\frac{1}{2} \frac{1}{2x-4}$

Cvičení 3.10

- $\int \frac{1}{x^2+2x+5} dx = \frac{1}{2}\operatorname{arctg} \frac{x+1}{2}$
- $\int \frac{1}{x^2+4x+29} dx = \frac{1}{5}\operatorname{arctg} \frac{x+2}{5}$
- $\int \frac{x+1}{x^2-2x+5} dx = \frac{1}{2}\ln|x^2-2x+5| + \operatorname{arctg} \frac{x-1}{2}$
- $\int \frac{x-2}{x^2+6x+10} dx = \frac{1}{2}\ln|x^2+6x+10| - 5\operatorname{arctg}(x+3)$
- $\int \frac{7x+4}{5x^2-3x+1} dx =$
 $= \frac{7}{10}\ln|5x^2-3x+1| + \frac{61}{55}\sqrt{11}\operatorname{arctg}\left(\frac{10x-3}{\sqrt{11}}\right)$

$$6. \int \frac{5x^4 - 7x^3}{x^2 + x + 1} dx = \frac{5}{3}x^3 - 6x^2 + 7x + \frac{5}{2} \ln|x^2 + x + 1| - \frac{19}{\sqrt{3}} \operatorname{arctg} \left(\frac{2x + 1}{\sqrt{3}} \right)$$

Cvičení 3.11

$$1. \int \frac{5x + 1}{x^2 + x - 2} dx = 3 \ln|x + 2| + 2 \ln|x - 1|$$

$$2. \int \frac{x - 6}{x^2 - 4} dx = -\ln|x - 2| + 2 \ln|x + 2|$$

$$3. \int \frac{1}{x^3 - 2x^2 - 3x} dx = -\frac{1}{3} \ln|x| + \frac{1}{4} \ln|x + 1| + \frac{1}{12} \ln|-3 + x|$$

$$4. \int \frac{x + 1}{x - 2x^2} dx = \ln|x| - \frac{3}{2} \ln|2x - 1|$$

$$5. \int \frac{x^4 - 1}{x(x^2 - 4)} dx = \frac{1}{2}x^2 + \frac{1}{4} \ln|x| + \frac{15}{8} \ln|x - 2| + \frac{15}{8} \ln|x + 2|$$

$$6. \int \frac{x^3}{x^2 - 1} dx = \frac{1}{2}x^2 + \frac{1}{2} \ln|x - 1| + \frac{1}{2} \ln|x + 1|$$

Cvičení 3.12

$$1. \int \frac{3x^2 + 4x + 12}{x^3 + 4x} dx = 3 \ln|x| + 2 \operatorname{arctg} \frac{x}{2}$$

$$2. \int \frac{x^2}{(1 + 2x^2)(x^2 + 1)} dx = -\frac{1}{\sqrt{2}} \operatorname{arctg}(x\sqrt{2}) + \operatorname{arctg} x$$

$$3. \int \frac{3x}{x^3 - 1} dx = \ln|x - 1| - \frac{1}{2} \ln(x^2 + x + 1) + \sqrt{3} \operatorname{arctg} \left(\frac{2x + 1}{\sqrt{3}} \right)$$

$$4. \int \frac{1}{x^3 + 1} dx = \frac{1}{3} \ln|x + 1| - \frac{1}{6} \ln(x^2 - x + 1) + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{2x - 1}{\sqrt{3}} \right)$$

Cvičení 3.13

$$1. \int \frac{x^2 - 3x + 2}{x(x + 1)^2} dx = 2 \ln|x| + \frac{6}{x + 1} - \ln|x + 1|$$

$$2. \int \frac{x^3 + 1}{x^3 - x^2} dx = x + \frac{1}{x} - \ln|x| + 2 \ln|x - 1|$$

$$3. \int \frac{x + 2}{x^3 - 2x^2 + x} dx = 2 \ln|x| - \frac{3}{x - 1} - 2 \ln|x - 1|$$

$$4. \int \frac{x^4 + 3x^2 - 1}{(x + 1)^2} dx = \frac{1}{3}x^3 - x^2 + 6x - \frac{3}{x + 1} - 10 \ln|x + 1|$$

Cvičení 3.14

$$1. \int x e^{x^2} dx = \frac{1}{2} e^{x^2}$$

$$2. \int \frac{\sin \frac{1}{x}}{x^2} dx = \cos \frac{1}{x}$$

$$3. \int \frac{1}{x \ln x} dx = \ln|\ln|x||$$

$$4. \int x \sin(3x^2 + 2) dx = -\frac{1}{6} \cos(3x^2 + 2)$$

$$5. \int \frac{x}{x^4 + 16} dx = \frac{1}{8} \operatorname{arctg} \frac{x^2}{4}$$

$$6. \int \sqrt[5]{(8 - 3x)^4} dx = \frac{5}{27} (-8 + 3x)^{9/5}$$

Cvičení 3.15

$$1. \int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx = \sqrt{x}(4 - \sqrt{x}) - 4 \ln(\sqrt{x} + 1)$$

$$2. \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = -2 \cos \sqrt{x}$$

$$3. \int \frac{e^{2\sqrt{x}}}{\sqrt{x}} dx = e^{2\sqrt{x}}$$

$$4. \int \frac{1}{(x + 1)\sqrt{x}} dx = 2 \operatorname{arctg} \sqrt{x}$$

Cvičení 3.16

$$1. \int \frac{\sqrt{x + 1} + 1}{\sqrt{x + 1} - 1} dx = 4 \ln|\sqrt{x + 1} - 1| + x + 1 + 4\sqrt{x + 1}$$

$$2. \int \frac{1}{1 + \sqrt{x - 1}} dx = 2\sqrt{x - 1} - 2 \ln|\sqrt{x - 1} + 1|$$

$$3. \int \frac{\sqrt{2x + 1}}{x} dx = 2\sqrt{2x + 1} + \ln \left| \frac{\sqrt{2x + 1} - 1}{\sqrt{2x + 1} + 1} \right|$$

$$4. \int \frac{1}{(3 - x)\sqrt{x + 1}} dx = \frac{1}{2} \ln \left| \frac{2 + \sqrt{x + 1}}{2 - \sqrt{x + 1}} \right|$$

Cvičení 3.17

$$1. \int \frac{x^{1/3} + 1}{\sqrt{x}} dx = \frac{6}{5} x^{5/6} + 2\sqrt{x}$$

$$2. \int \frac{1}{\sqrt{x - 1} + (x - 1)^{1/4}} dx = 2\sqrt{x + 1} - 4\sqrt[4]{x + 1} + 4 \ln(\sqrt[4]{x + 1} + 1)$$

$$3. \int \frac{\sqrt[3]{x}}{x(\sqrt{x} + \sqrt[3]{x})} dx = 6 \ln \frac{\sqrt[6]{x}}{1 + \sqrt[6]{x}}$$

$$4. \int \frac{1}{\sqrt{x + 1} + (x + 1)^{1/3}} dx = \frac{\sqrt{x + 1}}{3} + \frac{\sqrt[3]{x + 1}}{2} + \sqrt[6]{x + 1} - \ln(1 + \sqrt[6]{x + 1})$$

Cvičení 3.18

$$1. \int \frac{\sin^3 x}{\cos^2 x + 1} dx = \cos x - 2 \operatorname{arctg} \cos x$$

$$2. \int \cos^5 x dx = -\frac{1}{5} \sin^5 x + \frac{2}{5} \sin^3 x - \sin x$$

$$3. \int \sin^2 x \cos^3 x dx = -\frac{1}{5} \cos^4 x \sin x + \frac{1}{15} \cos^2 x \sin x + \frac{2}{15} \sin x$$

$$4. \int \frac{\sin 2x}{1 + \sin^2 x} dx = \ln(1 + \sin^2 x)$$

$$5. \int \cot^3 x dx = -\ln|\sin x| - \frac{1}{2 \sin^2 x}$$

$$6. \int \frac{\sin x \cos x}{1 + \sin^4 x} dx = \frac{1}{2} \operatorname{arctg}(\sin^2 x)$$

Cvičení 3.19

$$1. \int \frac{1 - \sin x}{\cos x} dx = \ln(1 + \sin x)$$

$$2. \int \frac{1}{\sin x} dx = -\frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x}$$

$$3. \int \frac{\cos^2 x}{\sin x} dx = \cos x - \frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x}$$

$$4. \int \frac{\sin^4 x}{\cos^3 x} dx = \sin x - \frac{3}{4} \ln \frac{1 + \sin x}{1 - \sin x} - \frac{1}{4} \frac{1}{1 + \sin x} + \frac{1}{4} \frac{1}{1 - \sin x}$$

$$5. \int \frac{1}{\sin^2 x \cos x} dx = -\frac{1}{\sin x} + \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x}$$

$$6. \int \frac{1}{(2 + \cos x) \sin x} dx = \frac{1}{6} \ln \frac{(2 + \cos x)^2 (1 - \cos x)}{(1 - \cos x)^3}$$

Cvičení 3.20

$$1. \int x \sin x dx = \sin x - x \cos x$$

$$2. \int x \operatorname{arctg} x dx = \frac{1}{2} x^2 \operatorname{arctg} x - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg} x$$

$$3. \int (3x + 2) \cos x dx = 3 \cos x + (3x + 2) \sin x$$

4. $\int (3x - 1) e^{2x} dx = -\frac{5}{4} e^{2x} + \frac{3}{2} e^{2x} x$
5. $\int x \ln(x + 1) dx =$
 $= -\frac{1}{2} \ln(x + 1) + \frac{1}{2} x + \frac{1}{2} \ln(x + 1) x^2 - \frac{1}{4} x^2$
6. $\int (x + 1) 2^x dx = \frac{x 2^x}{\ln 2} + \frac{2^x}{\ln 2} - \frac{2^x}{\ln^2 2}$

Cvičení 3.21

1. $\int (x^2 - 2x + 1) e^x dx = e^x (x^2 - 4x + 5)$
2. $\int (x^2 + 1) \cos x dx = (x^2 - 1) \sin x + 2x \cos x$
3. $\int x^2 \sin(2x) dx =$
 $= -\frac{1}{2} \cos(2x) x^2 + \frac{1}{4} \cos(2x) + \frac{1}{2} x \sin(2x)$
4. $\int (x^2 - x) e^{-x} dx = -e^{-x} (x^2 + x + 1)$

Cvičení 3.22

1. $\int \ln x dx = -x + x \ln x$
2. $\int \arcsin x dx = x \arcsin x + \sqrt{1 - x^2}$
3. $\int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - 2x + 2 \operatorname{arctg} x$
4. $\int \operatorname{arctg}(x - 1) dx =$
 $= (x - 1) \operatorname{arctg}(x - 1) - \frac{1}{2} \ln(1 + (x - 1)^2)$

Cvičení 3.23

1. $\int e^x \cos x dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x$
2. $\int \frac{\ln x}{x} dx = \frac{1}{2} \ln x^2$
3. $\int 2^x \sin x dx = -\frac{2^x \cos x}{1 + \ln^2 2} + \frac{2^x \sin x \ln 2}{1 + \ln^2 2}$
4. $\int \sin(\ln x) dx = \frac{1}{2} x (\sin(\ln x) - \cos(\ln x))$

Cvičení 3.24

1. $\int \ln x^2 dx = \ln x^2 x - 2x \ln x + 2x$
2. $\int \frac{\ln x}{\sqrt{x}} dx = 4\sqrt{x} \ln \sqrt{x} - 4\sqrt{x}$

3. $\int x^2 \sqrt{x + 1} dx =$
 $= \frac{2}{3} x^2 (x + 1)^{\frac{3}{2}} - \frac{8}{15} x (x + 1)^{\frac{5}{2}} + \frac{16}{105} (x + 1)^{\frac{7}{2}}$
4. $\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$

Cvičení 3.25

1. $\int \frac{1}{\sin x} dx = -\frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x}$
2. $\int \frac{1 + \sin x}{1 - \sin x} dx = \frac{4}{1 - \operatorname{tg} \frac{x}{2}} - x$
3. $\int \frac{1}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \ln \frac{\sqrt{2} - 1 + \operatorname{tg} \frac{x}{2}}{\sqrt{2} + 1 - \operatorname{tg} \frac{x}{2}}$
4. $\int \frac{1}{5 - 3 \cos x} dx = \frac{1}{2} \operatorname{arctg} \left(2 \operatorname{tg} \frac{x}{2} \right)$

Cvičení 3.26

1. $\int \frac{x^3}{\sqrt{2 - 2x - x^2}} dx =$
 $= \frac{1}{6} (-2x^2 + 5x - 23) \sqrt{2 - 2x - x^2} - \frac{11}{2} \arcsin \frac{x + 1}{\sqrt{3}}$
2. $\int \sqrt{4 - x^2} dx = \frac{1}{2} x \sqrt{4 - x^2} + 2 \arcsin \frac{x}{2}$
3. $\int \sqrt{x^2 + x + 1} dx =$
 $= \frac{1}{4} (2x + 1) \sqrt{x^2 + x + 1} + \frac{3}{8} \ln \left(x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right)$
4. $\int x \arcsin(x + 1) dx =$
 $= \left(-\frac{3}{4} + \frac{1}{2} x^2 \right) \arcsin(x + 1) + \frac{x - 3}{4} \sqrt{-x(x + 2)}$