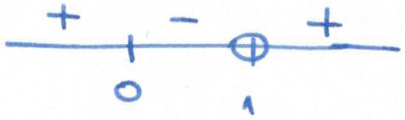
Cvičení 2.5.2019

Př.: f: $y = \frac{x^3}{x-1}$ $\quad D(f) = \mathbb{R} \setminus \{1\}$

Nulové body: $\frac{x^3}{x-1} = 0 \Rightarrow x^3 = 0 \Rightarrow x = 0$

Znaménko:



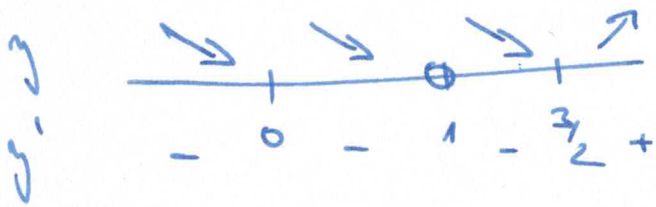
Limity v $\pm\infty$ a bodech nespojitosti:

$$\lim_{x \to \pm\infty} \frac{x^3}{x-1} = \frac{\infty}{\infty} = \lim_{x \to \pm\infty} \frac{3x^2}{1} = \infty$$

$$\lim_{x \to 1^{\pm}} \frac{x^3}{x-1} = \pm\infty$$

Monotonie: $y' = \frac{(2x-3)x^2}{(x-1)^2} = 0 \Rightarrow (2x-3)x^2 = 0 \Rightarrow$

$\Rightarrow 2x-3 = 0$ nebo $x^2 = 0 \Rightarrow x = \frac{3}{2}$ a $x = 0$

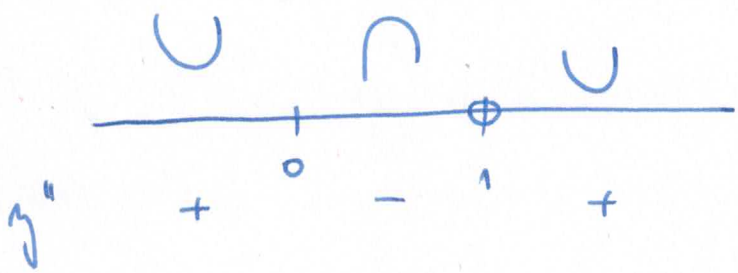


lok. min v bodě $x = \frac{3}{2}$

Konvexnost / konkávnost:

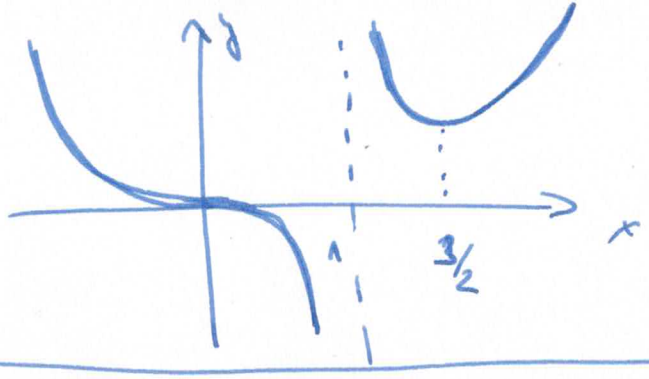
$$y'' = \frac{2(x^2-3x+3)x}{(x-1)^3} = 0 \Rightarrow (x^2-3x+3)x = 0 \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow x^2-3x+3 = 0 \text{ nebo } x = 0 \\ x_{1,2} = \frac{3 \pm \sqrt{9-4\cdot 3}}{2} = \frac{3 \pm \sqrt{-3}}{2} \notin \mathbb{R} \end{array}\right\} \Rightarrow x = 0$$



infl. bod $x = 0$

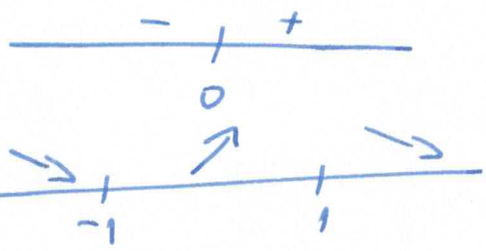
Graf:



Př 2:

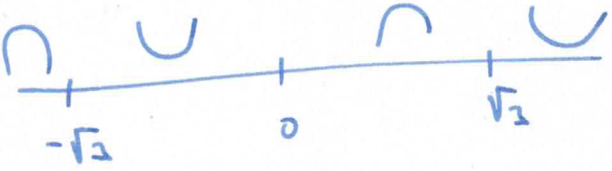
$y = \frac{x}{x^2+1}$ $\quad D(f) = \mathbb{R}$ $\quad \lim_{x \to \pm\infty} \frac{x}{x^2+1} = \lim_{x \to \pm\infty} \frac{1}{2x} = 0$

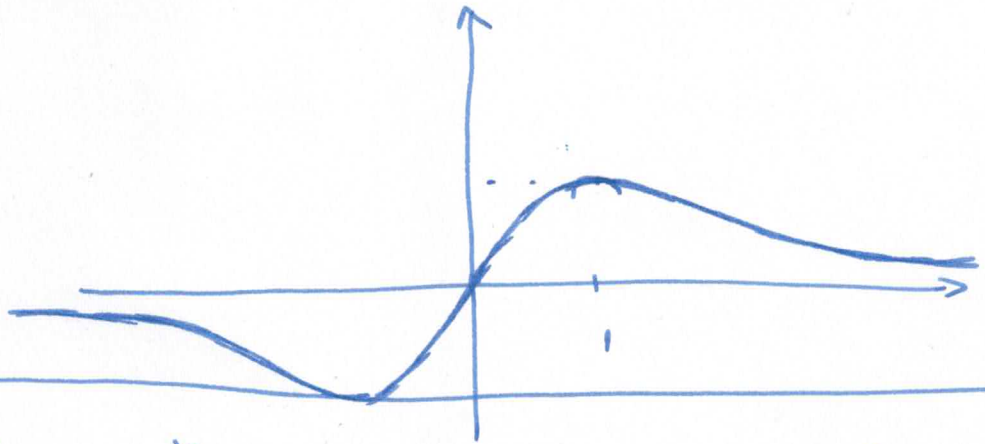
$y' = \frac{1-x^2}{(x^2+1)^2}$



lichá fce

$y'' = \frac{2x(x^2-3)}{(x^2+1)^3}$



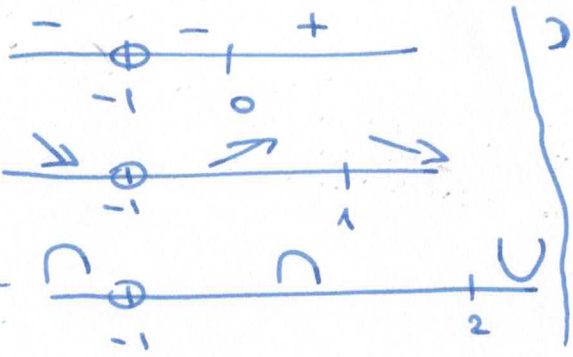


Př 3:

$y = \frac{x}{(x+1)^2}$

$y' = -\frac{x-1}{(x+1)^3}$

$y'' = \frac{2(x-2)}{(x+1)^4}$



$D(f) = \mathbb{R} \setminus \{-1\}$

$\lim_{x \to \pm\infty} \frac{x}{(x+1)^2} = 0$

$\lim_{x \to -1^{\pm}} \frac{x}{(x+1)^2} = -\infty$