Př:

$$x_1 + 2x_2 + 2x_3 = 3$$
$$2x_1 + 2x_2 - x_3 = 1$$
$$2x_1 + 3x_2 + x_3 = -1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 2 & 2 & -1 & 1 \\ 2 & 3 & 1 & -1 \end{array}\right) \begin{matrix} -2 \\ \swarrow \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & -2 & -5 & -5 \\ 0 & -1 & -3 & -7 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3 & 7 \\ 0 & 2 & 5 & 5 \end{array}\right) \begin{matrix} (-2) \\ \swarrow \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & -1 & -9 \end{array}\right)$$

$$x_1 + 2x_2 + 2x_3 = 3$$
$$x_2 + 3x_3 = 7$$
$$-x_3 = -9 \quad \Rightarrow x_3 = 9$$

$$x_2 + 27 = 7$$
$$x_2 = -20$$

$$x_1 - 40 + 18 = 3$$
$$x_1 = 40 + 3 - 18 =$$
$$= 25$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 25 \\ -20 \\ 9 \end{pmatrix}$$

$$3x_1 - x_2 - x_3 - x_4 = 0$$
$$2x_1 + x_2 + x_3 - 2x_4 = -4$$
$$x_1 - 2x_2 - 2x_3 + x_4 = 4$$
$$3x_1 - x_2 - x_3 + x_4 = 6$$

$$\left(\begin{array}{cccc|c} 3 & -1 & -1 & -1 & 0 \\ 2 & 1 & 1 & -2 & -4 \\ 1 & -2 & -2 & 1 & 4 \\ 3 & -1 & -1 & 1 & 6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -2 & 1 & 4 \\ 0 & 5 & 5 & -4 & -12 \\ 0 & 5 & 5 & -4 & -12 \\ 0 & 5 & 5 & -2 & -6 \end{array}\right)$$

(-2)(-3), (-1)

$$\sim \left(\begin{array}{cccc|c} 1 & -2 & -2 & 1 & 4 \\ 0 & 5 & 5 & -4 & -12 \\ 0 & 5 & 5 & -2 & -6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -2 & 1 & 4 \\ 0 & 5 & 5 & -4 & -12 \\ 0 & 0 & 0 & 2 & 6 \end{array}\right)$$

$$x_1 - 2x_2 - 2x_3 + x_4 = 4$$
$$5x_2 + 5x_3 - 4x_4 = -12$$
$$2x_4 = 6 \Rightarrow x_4 = 3$$

$$5x_2 + 5x_3 - 12 = -12$$
$$x_3 = t$$
$$5x_2 = -5t$$
$$x_2 = -t$$

$$x_1 - 2x_2 - 2x_3 + x_4 = 4$$
$$x_1 - 2(-t) - 2 \cdot t + 2 = 4$$
$$x_1 + 2 = 4$$
$$x_1 = 2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ t \\ -t \\ 3 \end{pmatrix}$$

1. Vlastní čísla a vektory 2 x 2

1) $A = \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}$ $\qquad |A - \lambda| = \begin{vmatrix} 3-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda) - 1 =$

$= \lambda^2 - 4\lambda + 2$

$\lambda^2 - 4\lambda + 2 = 0$

$\lambda_{1,2} = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2} = \begin{cases} 3,414 \\ 0,589 \end{cases}$

a) $\lambda = 0,589$

$A - \lambda I = \begin{pmatrix} 3-0,589 & -1 \\ -1 & 1-0,589 \end{pmatrix} = \begin{pmatrix} 2,411 & -1 \\ -1 & 0,411 \end{pmatrix}$

$\begin{pmatrix} 2,411 & -1 \\ -1 & 0,411 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow u_1 = 1$

$u_2 = 2,411$

$\vec{u} = \begin{pmatrix} 1 \\ 2,411 \end{pmatrix} \qquad |\vec{u}| = \sqrt{1 + (2,411)^2}$

$\vec{e}_1 = \frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} 0,3831 \\ 0,9237 \end{pmatrix}$

b) $\vec{e}_2 \perp \vec{e}_1 \Rightarrow e_2 =$

2) $P = \begin{pmatrix} 0,3831 & -0,9237 \\ 0,9237 & 0,3831 \end{pmatrix}$

$P^{-1} A \cdot P =$

$$= \begin{pmatrix} 0,3831 & 0,9237 \\ -0,9237 & 0,3831 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0,3831 & -0,9237 \\ 0,9237 & 0,3831 \end{pmatrix} =$$

$$= \begin{pmatrix} 0,3831 \cdot 3 + 0,9237 \cdot (-1) & 0,3831 \cdot (-1) + 0,9237 \cdot 1 \\ -0,9237 \cdot 3 + 0,3831(-1) & -0,9237(-1) + 0,3831 \cdot 1 \end{pmatrix} \begin{pmatrix} 0,3831 & -0,9237 \\ 0,9237 & 0,3831 \end{pmatrix}$$

$$= \begin{pmatrix} 0,2256 & 0,5406 \\ -3,1542 & 1,3068 \end{pmatrix} \begin{pmatrix} 0,3831 & -0,9237 \\ 0,9237 & 0,3831 \end{pmatrix}$$

$$= \begin{pmatrix} 0,5858 & -0,0013 \\ -0,0013 & 3,4142 \end{pmatrix} \approx \begin{pmatrix} 0,59 & 0 \\ 0 & 3,41 \end{pmatrix}$$

Vlastní čísla a vektory 3x3 matice

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{vmatrix} = (5-\lambda)(2-\lambda)(3-\lambda) - (5-\lambda) =$$

$$\begin{matrix} 5-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \end{matrix}$$

$$= (5-\lambda)\left((2-\lambda)(3-\lambda) - 1\right) =$$

$$= (5-\lambda)(\lambda^2 - 5\lambda + 5)$$

$$\lambda_1 = 5$$

$$\lambda_{2,3} = \frac{5 \pm \sqrt{25-20}}{2} = \frac{5 \pm \sqrt{5}}{2} = \begin{cases} 3,6180 \\ 1,3820 \end{cases}$$

a) $\lambda_1 = 5$ $\quad \vec{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

b) $\lambda_2 = 3,6180$

$$\begin{pmatrix} 5-3,6180 & 0 & 0 \\ 0 & 2-3,6180 & 1 \\ 0 & 1 & 3-3,6180 \end{pmatrix} = \begin{pmatrix} 1,\ldots & 0 & 0 \\ 0 & -1,6180 & 1 \\ 0 & 1 & -0,618 \end{pmatrix}$$

$$\vec{u_2} = \begin{pmatrix} 0 \\ +1 \\ +1,618 \end{pmatrix} \qquad \vec{e_2} = \frac{\vec{u_2}}{\|\vec{u_2}\|}$$

... vlastní vektor $\qquad$ ... jednotkový vlastní vektor

c) $\lambda_3 = 1,3820$

$$\begin{pmatrix} 5-1,3820 & 0 & 0 \\ 0 & 2-1,3820 & 1 \\ 0 & 1 & 3-1,3820 \end{pmatrix} = \begin{pmatrix} 3,6180 & 0 & 0 \\ 0 & 0,6180 & 1 \\ 0 & 1 & 1,6180 \end{pmatrix}$$

$$\vec{u_3} = \begin{pmatrix} 0 \\ 1,6180 \\ -1 \end{pmatrix} \qquad \vec{e_3} = \frac{\vec{u_3}}{\|\vec{u_3}\|}$$