Cvičení Matematika 11.4.2019

Př (Násobení matic)

$$A \cdot B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -2 & 2 \\ -1 & 2 & -1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -3 & 13 \\ -1 & 2 & -1 \\ 0 & 0 & -6 \end{pmatrix}$$

$1 \cdot 2 + (-2) \cdot (-1) + 3 \cdot 0 = 4$

$1 \cdot (-2) + (-2) \cdot (2) + 3 \cdot 1 = -3$

$1 \cdot 2 + (-2)(-1) + 3 \cdot 3 = 13$

$0 \cdot 2 + 1 \cdot (-1) + 0 \cdot 0 = -1$

$0 \cdot (-2) + 1 \cdot 2 + 0 \cdot 1 = 2$

$0 \cdot 2 + 1 \cdot (-1) + 0 \cdot 3 = -1$

$\vdots$

$$B \cdot A = \begin{pmatrix} 2 & -2 & 2 \\ -1 & 2 & -1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 2 & -1 \\ 3 & 7 & -6 \end{pmatrix}$$

$$B \cdot C = \begin{pmatrix} 2 & -2 & 2 \\ -1 & 2 & -1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 2 & -6 & 8 \\ -1 & 6 & -4 \\ 0 & 3 & 12 \end{pmatrix}$$

(násobky sloupců matice B)

$$C \cdot B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & -2 & 2 \\ -1 & 2 & -1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 2 \\ -3 & 6 & -3 \\ 0 & 4 & 12 \end{pmatrix}$$

(násobky řádků matice B)

Př: (Soustava jako maticová rovnice)

$$\begin{pmatrix} 2 & -3 & 2 \\ 2 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 21 \\ 0 \end{pmatrix}$$

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Př: Matice rotace

$$R_{-\theta} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$R_{-\theta} \cdot R_{\theta} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} c^2+s^2 & -cs+cs \\ -cs+cs & s^2+c^2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ kde $c = \cos\theta$ a $s = \sin\theta$

a proto $c^2+s^2 = \cos^2\theta + \sin^2\theta = 1$

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Př: (Matice posunutí)

$$P_{ab} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \\ 1 \end{pmatrix}$$

$(x,y) \longrightarrow (x+a, y+b)$ tj. jde o posunutí

$$\underbrace{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}}_{P} \underbrace{\begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}}_{P^{-1}} = \begin{pmatrix} 1 & 0 & -a+a \\ 0 & 1 & -b+b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}}_{P^{-1}} \cdot \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Př: (Matice zachovávající směry)

1)
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ d \\ g \end{pmatrix}$$

$(1,0,0)$ a $(a,d,g)$ mají stejný směr pokud existuje $k \neq 0$ takové, že $k(1,0,0) = (k,0,0) = (a,d,g)$

odsud: $d = 0$, $g = 0$, $a \neq 0$

$$A = \begin{pmatrix} a & b & c \\ 0 & e & f \\ 0 & h & i \end{pmatrix}$$

2)
$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & e & f \\ 0 & f & i \end{pmatrix}$$

3)
$$\begin{pmatrix} a & 0 & 0 \\ 0 & e & f \\ 0 & f & i \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ e \\ f \end{pmatrix} \Rightarrow$$ směr pokud $f = 0$

$(0,1,0)$ a $(0,e,f)$ mají stejný

$$\begin{pmatrix} a & 0 & 0 \\ 0 & e & f \\ 0 & f & i \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix} \Rightarrow$$ — " —

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{pmatrix}$$ Matice musí být diagonální

Př (determinanty)

1) $D_1 = \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix} = 2 \cdot 3 - (-1) \cdot 4 = 6 + 4 = 10$

2) $D_2 = \begin{vmatrix} 2 & -1 \\ x-4 & y-3 \end{vmatrix} = 2(y-3) - (-1) \cdot (x-4) = x + 2y - 10$

3) $D_3 = \begin{vmatrix} 2-\lambda & -1 \\ 4 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - (-1) \cdot 4 = \lambda^2 - 2\lambda - 3\lambda + 6 + 4 =$

$= \lambda^2 - 5\lambda + 10$

4) $D_4 = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ -1 & -1 & 2 \end{vmatrix} = 1 \cdot 3 \cdot 2 + 2 \cdot (-1) \cdot 0 + (-1)(-1) 1$

$\begin{matrix} 1 & -1 & 0 \\ 2 & 3 & 1 \end{matrix}$

$- (0 \cdot 3 \cdot (-1) + 1 \cdot (-1) \cdot 1 + 2(-1) \cdot 2) =$

$= 6 + 1 - (-1 - 4) = 12$

5) $D_5 = \begin{vmatrix} a & -1 & 0 \\ 2 & 3 & 1 \\ -1 & -1 & 2 \end{vmatrix} = a \cdot 3 \cdot 2 + 2 \cdot (-1) \cdot 0 + (-1)(-1) \cdot 1 -$

$\begin{matrix} a & -1 & 0 \\ 2 & 3 & 1 \end{matrix}$

$- (0 \cdot 3(-1) + a \cdot (-1) \cdot 1 + 2(-1)2) =$

$= 6a + 1 - (-a - 4) = 7a + 5$

6) $D_6 = \begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda)(7-\lambda)$

**Př** (Matice projekce)

$$P = \begin{pmatrix} c^2 & c\cdot s \\ c\cdot s & s^2 \end{pmatrix} \quad \text{kde } c = \cos\alpha,\ s = \sin\alpha$$

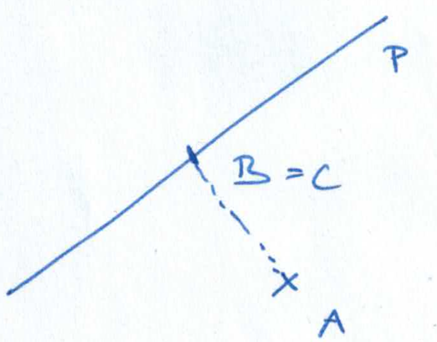
1) $$P^2 = \begin{pmatrix} c^2 & c\cdot s \\ c\cdot s & s^2 \end{pmatrix}\begin{pmatrix} c^2 & c\cdot s \\ c\cdot s & s^2 \end{pmatrix} = \begin{pmatrix} c^4 + c^2s^2 & c^3s + cs^3 \\ c^3s + cs^3 & c^2s^2 + s^4 \end{pmatrix} =$$

$$= \begin{pmatrix} c^2(c^2+s^2) & cs(c^2+s^2) \\ cs(c^2+s^2) & s^2(c^2+s^2) \end{pmatrix} = \begin{pmatrix} c^2 & cs \\ cs & s^2 \end{pmatrix} = P$$

protože $c^2 + s^2 = \cos^2\alpha + \sin^2\alpha = 1$

2) $$|P| = \begin{vmatrix} c^2 & cs \\ cs & s^2 \end{vmatrix} = c^2\cdot s^2 - c\cdot s\cdot c\cdot s =$$

$$= c^2s^2 - c^2s^2 = 0$$




$A \Rightarrow B = C$
$B \Rightarrow C$

2 různé body se zobrazí do jednoho $\Rightarrow$

$\Rightarrow$ tedy nemůže mít inverzní zobrazení.

Př (Matice derivování)

$$x^2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad A x^2 = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = 2x$$

$$x^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad A \cdot x = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 1$$

$$1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad A \cdot 1 = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0$$

tj. opravdu platí $(x^2)' = 2x$, $(x)' = 1$, $(1)' = 0$

Pro polynom $ax^2 + bx + c$ dostaneme

$$A(ax^2 + bx + c) = A\,ax^2 + A\,bx + A \cdot c =$$
$$= a \cdot (A x^2) + b(Ax) + c(A \cdot 1) =$$
$$= a \cdot 2x + b + 0 = 2ax + b$$

$A^2$ ... druhá derivace
$$A^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$A^3$ .. třetí derivace
$$A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$