(1) $\int (x+1)\cos x\,dx = \boxed{\begin{array}{ll} u = x+1 & u' = 1 \\ v' = \cos x & v = \sin x \end{array}} =$

$= (x+1)\sin x - \int \sin x\,dx = (x+1)\sin x + \cos x + C$

(2) $\int (x-1)e^{-x}\,dx = \boxed{\begin{array}{ll} u = x-1 & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array}} =$

$= -(x-1)e^{-x} - \int -e^{-x}\,dx = -(x-1)e^{-x} + \int e^{-x}\,dx =$

$= -(x-1)e^{-x} - e^{-x} = x e^{-x} + C$

(3) $\int \sqrt{x}\ln x\,dx \quad \boxed{\begin{array}{ll} u = \ln x & u' = 1/x \\ v' = \sqrt{x} & v = \frac{2}{3}x^{3/2} \end{array}} =$

$= \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}\cdot x^{3/2}\cdot\frac{1}{x}\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$

$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\cdot\frac{2}{3}\cdot x^{3/2}$

(4) $\int x^2\sin x\,dx = \boxed{\begin{array}{ll} u = x^2 & u' = 2x \\ v' = \sin x & v = -\cos x \end{array}} =$

$= -x^2\cos x - \int 2x(-\cos x)\,dx = -x^2\cos x + 2\int x\cos x\,dx =$

$\boxed{\begin{array}{ll} u = x & u' = 1 \\ v' = \cos x & v = \sin x \end{array}} = -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right] =$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

(5) $\int x \sin x^2 \, dx \quad \boxed{\begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}dt \end{array}} = \int \frac{1}{2} \sin t \; dt =$

$= -\frac{1}{2} \cos t = -\frac{1}{2} \cos(x^2) + c$

(6) $\int \cos x \cdot \sqrt{\sin x} \, dx = \boxed{\begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array}} = \int \sqrt{t} \, dt$

$= \frac{2}{3} t^{3/2} = \frac{2}{3} (\sin x)^{3/2} = \frac{2}{3} \cdot \sin x \cdot \sqrt{\sin x}$

(7) $\int \frac{\operatorname{arctg} x}{x^2+1} dx = \int \operatorname{arctg} x \cdot \frac{1}{x^2+1} dx \quad \boxed{\begin{array}{l} \operatorname{arctg} x = t \\ \frac{1}{1+x^2} dx = dt \end{array}} =$

$= \int t \cdot dt = \frac{1}{2} t^2 = \frac{1}{2} (\operatorname{arctg} x)^2 + c$

(8) $\int \sin x \cos^5 x \, dx \quad \boxed{\begin{array}{l} \cos x = t \\ -\sin x\,dx = dt \\ \sin x\,dx = -dt \end{array}} = -\int t^5 dt = -\frac{1}{6} t^6$

$= -\frac{1}{6} \cos^6 x + c$

(9) $\int \frac{1}{1+\sqrt{x}} dx \quad \boxed{\begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array}} = \int \frac{1}{1+t} \cdot 2t \, dt =$

$= 2 \int \frac{t}{t+1} = 2 \int \frac{t+1}{t+1} - \frac{1}{t+1} dt = 2 \int 1 - \frac{1}{t+1} dt =$

$= 2(t - \ln|t+1|) = 2t - 2\ln|t+1| =$

$= 2\sqrt{x} - 2\ln(\sqrt{x}+1) + c$

(10) $\displaystyle\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + c$

(11) $\displaystyle\int \frac{3}{5x-1}\,dx = \frac{3}{5}\int \frac{5}{5x-1}\,dx = \frac{3}{5}\ln|5x-1| + c$

(12) $\displaystyle\int \frac{x^2}{x^3-1}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3-1}\,dx = \frac{1}{3}\ln|x^3-1| + c$

(14) $\displaystyle\int x e^{x^2}\,dx \quad \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}\,dt \end{array} \right| = \frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t =$

$\displaystyle = \frac{1}{2}e^{x^2} + c$

(15) $\displaystyle\int x^2 e^x\,dx = \left| \begin{array}{ll} u = x^2 & u' = 2x \\ v' = e^x & v = e^x \end{array} \right| = x^2 e^x - \int 2x e^x\,dx \left| \begin{array}{ll} u = 2x & u' = 2 \\ v' = e^x & v = e^x \end{array} \right| =$

$\displaystyle = x^2 e^x - \left[2x e^x - 2\int e^x\,dx\right] = x^2 e^x - 2x e^x + 2\int e^x\,dx =$

$\displaystyle = x^2 e^x - 2x e^x + 2e^x + c$

(13) $\displaystyle\int \frac{x+1}{x^2+1}\,dx = \int \frac{x}{x^2+1} + \frac{1}{x^2+1}\,dx =$

$\displaystyle = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx =$

$\displaystyle = \frac{1}{2}\ln(x^2+1) + \operatorname{arctg} x + c$

Střední hodnota:

(1)

$$\frac{1}{3}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\int_1^4 x^{1/2}\,dx = \frac{1}{3}\cdot\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4 =$$

$$= \frac{1}{3}\left[\frac{2}{3}\cdot 4^{\frac{3}{2}} - \frac{2}{3}\,1^{\frac{3}{2}}\right] = \frac{1}{3}\left[\frac{2}{3}\cdot 8 - \frac{2}{3}\right] = \frac{2}{9}\cdot 7 = \frac{14}{9}$$

(2)

$$\frac{1}{\pi}\int_0^\pi \sin x\,dx = \frac{1}{\pi}\left[-\cos x\right]_0^\pi = \frac{1}{\pi}\left[-\cos\pi - (-\cos 0)\right] =$$

$$= \frac{1}{\pi}\left[-(-1) - (-1)\right] = \frac{1}{\pi}\cdot 2 = \frac{2}{\pi}$$

(3)

$$\frac{1}{2\pi}\int_0^{2\pi} \sin x\,dx = \frac{1}{2\pi}\left[-\cos x\right]_0^{2\pi} = \frac{1}{2\pi}\left[-\cos 2\pi - (-\cos 0)\right] = 0$$

(4)

$$\frac{1}{1}\cdot\int_0^1 ax^2\,dx = \left[a\frac{x^3}{3}\right]_0^1 = a\cdot\frac{1}{3} - a\cdot\frac{0}{3} = a\cdot\frac{1}{3}$$

$$\frac{1}{1}\cdot\int_0^1 ax^2\,dx = 1$$

$$a\cdot\frac{1}{3} = 1$$

$$\underline{\underline{a = 3}}$$

Populace a přežívání

$N(t)$ .. populace v čase t

$N(0) = 5600$, $\frac{dN}{dt} = R(t)$

Bez umírání:

$$N(t) = N(0) + \int_0^{10} R(t)dt = 5600 + \int_0^{10} 720 \cdot e^{0,1t} dt =$$

$$= 5600 + \left[7200\, e^{0,1t}\right]_0^{10} \dots = \dots \doteq 18000$$

Se umíráním:

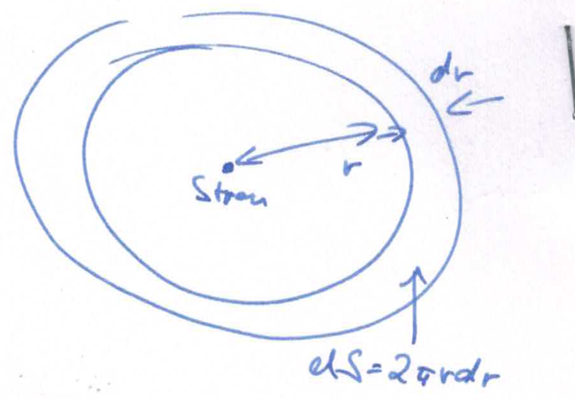
V čase t přibude $R(t) = 720 e^{0,1t}$ podílů na časovou podmínku a tělo musí přežít do doby $10-t$. Zlomek, který přežije po dobu $10-t$ je $S(10-t)$

$$N(t) = N(0) \cdot S(10) + \int_0^{10} R(t)\, S(10-t)\, dt =$$

$$= 5600 \cdot e^{-0,2 \cdot 10} + \int_0^{10} 720 \cdot e^{0,1t} \cdot e^{-0,2(10-t)} dt =$$

$$= 5600 \cdot e^{-2} + 720 \int_0^{10} e^{-2} \cdot e^{0,3t} dt = \dots = 7000$$

Mezihvězdný oblak:



$S = \pi r^2$ .. obsah kruhu

$dS = 2\pi r\, dr$

$$M = \int_0^R D(r) \cdot 2\pi r\, dr = 2\pi D_0 \int_0^R r \cdot e^{-r^2} dr$$

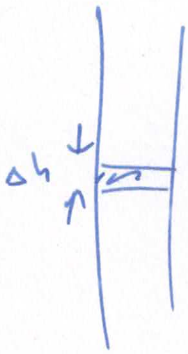
Dá se integrovat substitucí $-r^2 = t$.

$$M = -\pi D_0 \int_0^{-R^2} e^t dt = -\pi D_0 \left(e^{-R^2} - e^0\right) = \pi D_0 \left(e^0 - e^{-R^2}\right)$$

$$= \pi D_0 \left(1 - e^{-R^2}\right)$$

Hmotnost dřeva : Výška trámu .. H

průřez " ... S



$\Delta V = S \cdot \Delta h$

$\Delta m = \Delta V \cdot \rho = S \cdot \Delta h \cdot \rho_0 (1 + k h)$

$$m = \int_0^H S \cdot \rho_0 (1 + k h)\, dh$$

$$= S\rho_0 \int_0^H (1 + k h)\, dh = S \cdot \rho_0 \left[\int_0^H 1\, dh + k \int_0^H h\, dh \right] =$$

$$= S \cdot \rho_0 \left[H + k \cdot \left[\tfrac{1}{2} h^2 \right]_0^H \right] = S \cdot \rho_0 H + S \cdot \rho_0 \tfrac{1}{2} H^2$$

$$= \underbrace{S \cdot H}_{\text{objem}} \cdot \underbrace{\left(\rho_0 + \tfrac{1}{2} H \right)}_{\text{hustota uprostřed}}$$

Vitamín A v mrkvi.

$f(x) = \sqrt{14-x}$

Mrkev vznikne sečtením válců o poloměru podstavy $f(x)$ a výšce Δx

$$\Delta V = \pi f^2(x)\,\Delta x \Rightarrow V = \int_0^{14} \pi f^2(x)\,dx$$

Obsah vitamínu A

$$\Delta m = \pi f^2(x)\Delta x \cdot c(x) \Rightarrow m = \int_0^{14} \pi f^2(x)\cdot c(x)\,dx$$

Průměrná koncentrace $\tilde{c} = \frac{m}{V}$

Pesticidy v játrech

$$c = e^{-0{,}25T}\left[-\overbrace{\frac{0{,}32}{0{,}64}}^{\frac{1}{2}} e^{-0{,}64t}\right]_0^T = e^{-0{,}25T}\left[-\frac{1}{2}e^{-0{,}64T} - \left(-\frac{1}{2}e^0\right)\right]$$

$$= \frac{1}{2}e^{-0{,}25T} - \frac{1}{2}e^{-0{,}89T}$$

$$\frac{dc}{dT} = \frac{1}{2}(-0{,}25)e^{-0{,}25T} + \frac{1}{2}\,0{,}89\cdot e^{-0{,}89T}$$

$\frac{dc}{dT} = 0$ pokud $\frac{1}{2}\,0{,}89\,e^{-0{,}89T} = \frac{1}{2}\,0{,}25e^{-0{,}25T}$

$$\frac{0{,}89}{0{,}25} = e^{0{,}64T}$$

$$0{,}64T = \ln\frac{0{,}89}{0{,}25}$$

$$T = \frac{1}{0{,}64}\cdot\ln\frac{0{,}89}{0{,}25} \doteq 1{,}98$$