(1) $\int x^2 + 2x \, dx = \frac{1}{3} x^3 + x^2 + c$

(2) $\int \sqrt{x}\,(x + \sqrt{x}) \, dx = \int x^{3/2} + x \, dx = \frac{2}{5} x^{\frac{5}{2}} + \frac{1}{2} x^2 + c$

(3) $\int \frac{1}{\sqrt{x}} + \sqrt{x} \, dx = \int x^{-\frac{1}{2}} + x^{\frac{1}{2}} \, dx = 2 x^{\frac{1}{2}} + \frac{2}{3} x^{\frac{3}{2}} + c$

(4) $\int \frac{x^2 - 1}{x} \, dx = \int x - \frac{1}{x} \, dx = \frac{1}{2} x^2 + \ln|x| + c$

(5) $\int e^x + e^{2x} \, dx = e^x + \frac{1}{2} e^{2x} + c$

(6) $\int \sin\left(x + \frac{\pi}{3}\right) dx = -\cos\left(x + \frac{\pi}{3}\right) + c$

(7) $\int \frac{1}{4x^2} \, dx = \frac{1}{4} \int x^{-2} \, dx = \frac{1}{4} \cdot \frac{1}{-1} \cdot x^{-1} = -\frac{1}{4x} + c$

(8) $\int \frac{1}{4 + x^2} \, dx = \frac{1}{2} \operatorname{arctg} \frac{x}{2} + c$

(9a) $\int \frac{1}{1 + 4x^2} \, dx = \int \frac{1}{1 + (2x)^2} \, dx = \frac{1}{2} \operatorname{arctg}(2x) + c$

(9b) $\int \frac{1}{1 + 4x^2} \, dx = \frac{1}{4} \int \frac{1}{\frac{1}{4} + x^2} \, dx = \frac{1}{4} \cdot 2 \cdot \operatorname{arctg}\left(\frac{x}{\frac{1}{2}}\right) + c$

$= \frac{1}{4} \cdot 2 \cdot \operatorname{arctg}(2x) + c = \frac{1}{2} \operatorname{arctg}(2x) + c$

(10) $\int \frac{1}{r^2} - \frac{1}{r^6} \, dr = \int r^{-2} - r^{-6} \, dr =$

$= -r^{-1} - \frac{1}{-5} r^{-5} = -\frac{1}{r} + \frac{1}{5r^5} + c$

(11) $\int_0^{\frac{\pi}{2}} \cos x \, dx = \left[+\sin x\right]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$

(12) $$\int_0^1 (x-1)^3\,dx = \left[\frac{(x-1)^4}{4}\right]_0^1 = \frac{0^4}{4} - \frac{(-1)^4}{4} = -\frac{1}{4}$$

(13) $$\int_{-1}^{1} 3x^2 + x^5\,dx = \left[x^3 + \frac{1}{6}x^6\right]_{-1}^{1} =$$

$$= 1 + \frac{1}{6} - \left(-1 + \frac{1}{6}\right) = 1 + \frac{1}{6} + 1 - \frac{1}{6} = 2$$

(14) $$\int_0^{10} e^{-0,1t}\,dt = \left[\frac{1}{-0,1}\,e^{-0,1t}\right]_0^{10} =$$

$$= -\frac{1}{0,1}\cdot e^{-1} - \left(\frac{1}{-0,1}\,e^{0}\right) =$$

$$= -10\cdot e^{-1} + 10\cdot 1 = 10\left(1 - e^{-1}\right)$$

(15) $$\int_{-a}^{a} u^3\,du = \left[\frac{1}{4}u^4\right]_{-a}^{a} = \frac{1}{4}a^4 - \left(\frac{1}{4}(-a)^4\right) = 0$$

Pružina:

$$F = k \cdot x$$

$$W = \int_0^1 F\,dx = \int_0^1 kx\,dx = k \cdot \left[\frac{1}{2}x^2\right]_0^1 = k \cdot \frac{1}{2}$$

$$W = \int_0^l F\,dx = \int_0^l kx\,dx = k\left[\frac{1}{2}x^2\right]_0^l = \frac{1}{2}kl^2$$

Numericky v různých podmodelech

a) centimetry:

$$W = \int_0^{10} \underset{\uparrow\ N/m}{10} \cdot \underset{\uparrow\ cm}{x}\, \underset{\uparrow\ cm}{dx} = \left[5 \cdot x^2\right]_0^{10} = 500\ N.cm = 5\ N.m = 5\ J$$

b) decimetry

$$W = \int_0^1 100 \cdot x\,dx = \left[50x^2\right]_0^1 = 50\ N.dm = 5\ N.m$$

c) metry

$$W = \int_0^{0,1} 1000\ x\,dx = \left[500x^2\right]_0^{0,1} = 5\ N.m$$

Výpočet π:

$$\int_0^1 x^n\,dx = \left[\frac{1}{n+1}x^{n+1}\right]_0^1 = \frac{1}{n+1}$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\operatorname{arctg} x\right]_0^1 = \operatorname{arctg} 1 - \operatorname{arctg} 0 = \frac{\pi}{4}$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \dots$$

Napouštění nádrže

$$\int_0^{20} 180 + 3 \cdot t \, dt = 180 \cdot 20 + \left[3 \cdot \frac{t^2}{2} \right]_0^{20} =$$

$$= 180 \cdot 20 + 3 \cdot \frac{400}{2} - 3 \cdot \frac{0}{2} = 4200$$

Prostá Zendikace

$$\int_4^6 c'(t) \, dt = \int_4^6 10^3 (t - 7) \, dt = 10^3 \int_4^6 t - 7 \, dt =$$

$$= 10^3 \left[\frac{1}{2} t^2 - 7t \right]_4^6 = 10^3 \left[\frac{1}{2} \cdot 6^2 - 7 \cdot 6 - \left(\frac{1}{2} \cdot 4^2 - 7 \cdot 4 \right) \right] =$$

$$= 10^3 (18 - 42 - 8 + 28) = -4000$$

Rychlost učení

$$w(t) = \int w'(t) \, dt = \int \frac{4t}{100} - 3 \left(\frac{t}{100} \right)^2 dt =$$

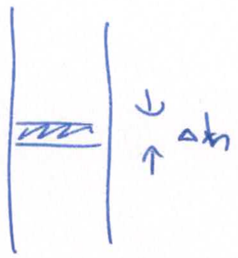
$$= \frac{1}{25} \cdot \frac{1}{2} t^2 - \frac{3}{100^2} \cdot \frac{1}{3} t^3 + c = \frac{1}{50} t^2 - \frac{1}{10000} t^3 + c$$

pro $t = 0$ je $w = 0$ $\Rightarrow$ $c = 0$

$$w(t) = \frac{1}{50} t^2 - \frac{1}{10000} t^3$$

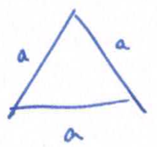
Hmotnost vzduchu odpovídající sloupečku $\quad H = 340$

$$m = V \cdot \rho \implies \Delta m = S \cdot \Delta h \cdot \rho$$



$$m = \int_0^{340} S \cdot \rho \, dh = \int_0^{340} S \cdot \rho_0 \, e^{-\frac{\rho_0 g h}{p_0}} dh$$

$$= S \cdot \rho_0 \left[-\frac{p_0}{\rho_0 g} e^{-\frac{\rho_0 g h}{p_0}} \right]_0^{340} =$$



$$S = \frac{1}{2} a \cdot a \cdot \sin 60^\circ = a^2 \cdot \frac{\sqrt{3}}{4}$$

$$= S \cdot \rho_0 \left[-\frac{p_0}{\rho_0 g} e^{-\frac{\rho_0 g \cdot 340}{p_0}} + \frac{p_0}{\rho_0 g} e^0 \right] =$$

$$= \frac{S \cdot p_0}{g} \left[1 - e^{-\frac{\rho_0 g \cdot 340}{p_0}} \right]$$

Pro zadané hodnoty:

$$m = 1590 \text{ kg}$$

Pokud by se hustota neměnila s výškou:

$$m = S \cdot H \cdot \rho = \ldots = 1623 \text{ kg}$$

Urči integrál s parametrem:

$$\int_0^{10} a \sqrt{x} \, dx = \left[a \cdot \frac{2}{3} x^{\frac{3}{2}} \right]_0^{10} = a \cdot \frac{2}{3} \cdot 10^{\frac{3}{2}} = \frac{2}{3} a \sqrt{1000}$$

$$\frac{2}{3} a \sqrt{1000} = 2019 \implies a = \frac{3 \cdot 2019}{2 \sqrt{1000}} = \frac{3 \cdot 2019}{20 \sqrt{10}}$$