Cv. cvič. Matematika 7.3.2019

Základní lineární aproximace

$\sin x$

$\sin(0) = 0$

$\frac{d}{dx}(\sin x) = \cos x\Big|_{x=0} = 1$

$\sin x \approx 0 + 1.(x-0) = x$

$\boxed{\sin x \approx x}$

$\cos x$

$\cos(0) = 1$

$\frac{d}{dx}\cos(x) = -\sin x\Big|_{x=0} = 0$

$\cos x \approx 1 + 0.(x-0) = 1$

$\boxed{\cos x \approx 1}$

$(1+x)^m$

$(1+0)^m = 1$

$\frac{d}{dx}(1+x)^m = m(1+x)^{m-1}\Big|_{x=0} = m.(1+0)^{m-1} = m$

$(1+x)^m \approx 1 + m(x-0) = 1+mx$

$\boxed{(1+x)^m \approx 1+mx}$

Kinolza M+M

$f(x) = \frac{ax}{b+x}$, $f(0) = \frac{a.0}{b+0} = 0$

$\frac{df}{dx} = \frac{ab}{(b+x)^2}$, $\frac{df(0)}{dx} = \frac{ab}{(b+0)^2} = \frac{ab}{b^2} = \frac{a}{b}$

$f(x) \approx 0 + \frac{a}{b}(x-0) = \frac{a}{b}.x$

$\boxed{f(x) \approx \frac{a}{b}.x}$

Krab. cíla z A4

Rozměry: $(x) \times (297 - 2x) \times (210 - 2x)$

$V = x \cdot (297 - 2x)(210 - 2x)$

$V = 4x^3 - 2x^2(210 + 297) + x \cdot 210 \cdot 297$

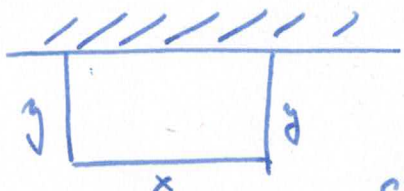
$\frac{dV}{dx} = 12x^2 - 4x(210 + 297) + 210 \cdot 297$

$\frac{dV}{dx} = 0$ pro $12x^2 - 4 \cdot x(210 + 297) + 210 \cdot 297 = 0$

$x_{1,2} = \ldots = \begin{cases} 40{,}4 \\ 128{,}6 \end{cases}$

Čtverce o straně 40 mm maximalizují objem krabičky.

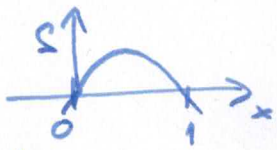
Plot ze tří stran



$x > 0, y > 0$

(1)

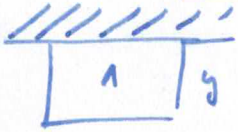
$y = \frac{1-x}{2}$, $S = xy = \frac{1}{2}x(1-x)$ ...



maximum pro $x = \frac{1}{2}$, $y = \frac{1-\frac{1}{2}}{2} = \frac{1}{4} = \frac{1}{2}x$

poměr stran 2 : 1

(2)



$y = \frac{1}{x}$, $L = x + 2y = x + \frac{2}{x} \to \min.$

$\frac{dL}{dx} = 1 - \frac{2}{x^2}$; $\frac{dL}{dx} = 0$ pro $1 - \frac{2}{x^2} = 0$ |·

$x^2 = 2$ a $x = \sqrt{2}$. Pokud $y = \frac{1}{x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}x$

poměr stran 2 : 1

Řešení prot. příkladu

$$f(x) = \frac{(x+1)^3}{x}$$

$$\frac{df}{dx} = \frac{3(x+1)^2 \cdot x - (x+1)^3 \cdot 1}{x^2} = \frac{(x+1)^2}{x^2}\left(3x - (x+1)\right) =$$

$$= \frac{(x+1)^2}{x^2}(2x-1)$$

$\frac{df}{dx} = 0$ pro $x = \frac{1}{2}$ (Řešení $x = -1$ nemá význam.)

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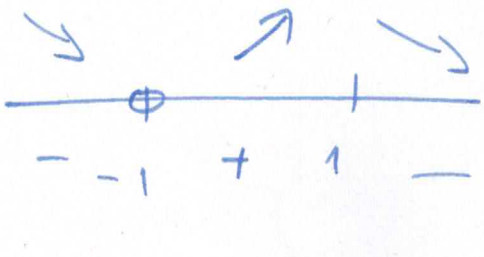
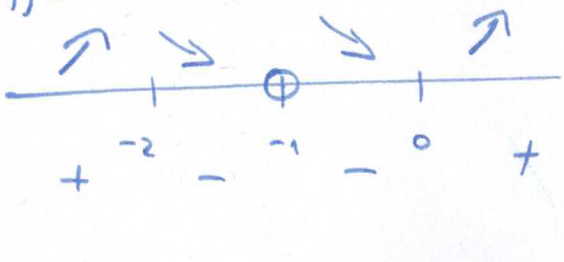
1) $y = \frac{x}{(x+1)^2}$, $y' = \frac{1-x}{(x+1)^3}$



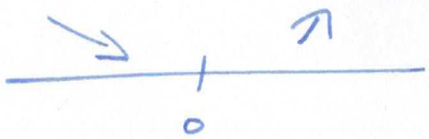
lok. Max. v bodě $x = 1$

2) $y = \frac{x^2}{x+1}$, $y' = \frac{x(x+2)}{(x+1)^2} = \frac{1}{(x+1)^2} \cdot x(x+2)$



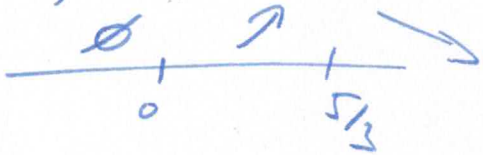
lok. Min. v bodě $x = 0$
lok. Max v bodě $x = -2$

3) $y = \frac{x^2}{x^2+1}$, $y' = \frac{2x}{(x^2+1)^2}$



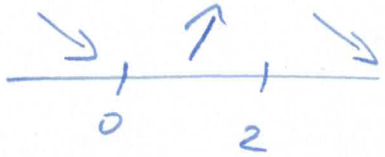
lok. Min. v bodě $x = 0$

4) $y = (5 - x)\sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}(5 - 3x)$



lok. Max v bodě $x = \frac{5}{3}$

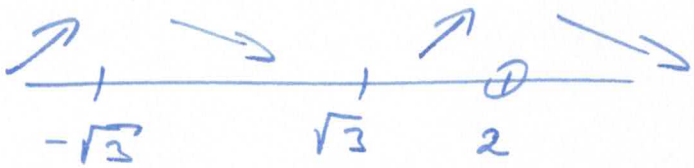
4) $y = x^2 e^{-x}$ $\quad y' = -(x-2)x\, e^{-x}$



loc. min pro $x = 0$

loc. max pro $x = 2$

6) $y' = (x^2+3)\,\dfrac{x^2-3}{2-x}$ $\qquad (2 = \sqrt{4})$



loc. Maximum v bodě $x = -\sqrt{3}$

loc. Minimum v bodě $x = \sqrt{3}$