Vypočtěte následující limity pomocí l'Hopitalova pravidla:

Typ $\frac{0}{0}$ a $\frac{\pm\infty}{\pm\infty}$.

1. $\lim_{x\to\infty} \frac{x \ln x}{x^2+x+1} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to\infty} \frac{\ln x + 1}{2x+1} ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2} \overset{\text{dosadit}}{=} 0$

2. $\lim_{x\to 0} \frac{\operatorname{arctg} 2x}{\arcsin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\frac{2}{1+4x^2}}{\frac{1}{\sqrt{1-x^2}}} \overset{\text{uprava}}{=} \lim_{x\to 0} \frac{2\sqrt{1-x^2}}{1+4x^2} \overset{\text{dosadit}}{=} 2$

3. $\lim_{x\to 1} \frac{x-1}{\ln x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{1}{\frac{1}{x}} \overset{\text{dosadit}}{=} 1$

4. $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = ||\tfrac{0}{0}|| \overset{\text{uprava}}{=} \frac{1}{2} \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$

5. $\lim_{x\to 0} \frac{\arcsin x}{1-e^x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{-e^x} \overset{\text{dosadit}}{=} -1$

6. $\lim_{x\to e} \frac{\ln x - 1}{\ln^2 x + \ln x - 2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to e} \frac{\frac{1}{x}}{2\frac{1}{x}\ln x + \frac{1}{x}} \overset{\text{uprava}}{=} \lim_{x\to e} \frac{1}{2\ln x + 1} \overset{\text{dosadit}}{=} \frac{1}{3}$

7. $\lim_{x\to 1} \frac{\ln x}{\operatorname{cotg} \frac{x\pi}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 \frac{\pi x}{2}}\frac{\pi}{2}} \overset{\text{dosadit}}{=} -\frac{2}{\pi}$

8. $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0^+} \frac{\frac{1}{e^x-1}e^x}{\frac{1}{x}} \overset{\text{uprava}}{=} \lim_{x\to 0^+} \frac{xe^x}{e^x-1} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0^+} \frac{e^x + xe^x}{e^x} \overset{\text{dosadit}}{=} 1$

9. $\lim_{x\to 0} \frac{e^x - 1}{\sin 2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x}{2\cos 2x} \overset{\text{dosadit}}{=} \frac{1}{2}$

10. $\lim_{x\to 0} \frac{x - \sin x}{\sin^3 x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{1-\cos x}{3\sin^2 x \cos x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\sin x}{6\sin x \cos^2 x - 3\sin^3 x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$= \lim_{x\to 0} \frac{\cos x}{6\cos^3 x - 6\cdot 2\cdot \sin^2 x \cos x - 9\sin^2 x \cos x} \overset{\text{dosadit}}{=} \frac{1}{6}$

11. $\lim_{x\to 3} \frac{x-3}{x^2 - 8x + 15} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 3} \frac{1}{2x-8} \overset{\text{dosadit}}{=} -\frac{1}{2}$

12. $\lim_{x\to 1} \frac{1 + \cos \pi x}{(x-1)^2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{-\pi \sin \pi x}{2(x-1)} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{-\pi^2 \cos \pi x}{2} \overset{\text{dosadit}}{=} \frac{\pi^2}{2}$

13. $\lim_{x\to\infty} \frac{x^2}{e^{x^2}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to\infty} \frac{2x}{2xe^{x^2}} \overset{\text{uprava}}{=} \lim_{x\to\infty} \frac{1}{e^{x^2}} \overset{\text{dosadit}}{=} 0$

14. $\lim_{x\to 1} \frac{1-x}{\operatorname{cotg} \frac{\pi x}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{-1}{-\frac{\pi}{2}\sin^{-2}\frac{\pi x}{2}} \overset{\text{dosadit}}{=} \frac{2}{\pi}$

15. $\lim_{x\to 0} \frac{\sin x - x}{x \sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x\cos x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{-\sin x}{\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} 0$

16. $\lim_{x\to \frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim_{x\to \frac{\pi}{2}} \frac{\frac{3}{\cos^2 3x}}{\frac{1}{\cos^2 x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{uprava}}{=} 3 \lim_{x\to \frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 3x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} 3 \lim_{x\to \frac{\pi}{2}} \frac{-2\sin x \cos x}{-6\sin 3x \cos 3x} = ||\tfrac{0}{0}||$

$\overset{\text{uprava}}{=} 3\cdot\frac{2}{6}\cdot \lim_{x\to \frac{\pi}{2}} \frac{\sin x}{\sin 3x} \cdot \lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cos 3x} \overset{\text{uprava}}{=} -\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cos 3x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} -\lim_{x\to \frac{\pi}{2}} \frac{-\sin x}{-3\sin 3x} \overset{\text{dosadit}}{=} \frac{1}{3}$

16B. $$\lim_{x\to\frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} = ||\tfrac{\infty}{\infty}|| \overset{\text{uprava}}{=} \lim_{x\to\frac{\pi}{2}} \frac{\sin 3x \cos x}{\sin x \cos 3x} \overset{\text{uprava}}{=} \lim_{x\to\frac{\pi}{2}} \frac{\sin 3x}{\sin x} \cdot \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\cos 3x} = -\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\cos 3x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$$
$$= -\lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{-3\sin 3x} \overset{\text{dosadit}}{=} \frac{1}{3}$$

17. $$\lim_{x\to 0} \frac{x - \operatorname{arctg} x}{x^3} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{1-\frac{1}{1+x^2}}{3x^2} = ||\tfrac{0}{0}|| \overset{\text{uprava}}{=} \lim_{x\to 0} \frac{1}{3(1+x^2)} \overset{\text{dosadit}}{=} \frac{1}{3}$$

18. $$\lim_{x\to 0} \frac{xe^x + x - 2e^x + 2}{x^3} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x + xe^x + 1 - 2e^x}{3x^2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x + e^x + xe^x - 2e^x}{6x} \overset{\text{uprava}}{=}$$
$$= \lim_{x\to 0} \frac{xe^x}{6x} \overset{\text{uprava}}{=} \lim_{x\to 0} \frac{e^x}{6} \overset{\text{dosadit}}{=} \frac{1}{6}$$

19. $$\lim_{x\to 0} \frac{xe^x + x - 2e^{-x} + 2}{x^3} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x + xe^x + 1 + 2e^x}{3x^2} \overset{\text{dosadit}}{=} \frac{4}{+0} = \infty$$

20. $$\lim_{x\to 0} \frac{\cos 2x - 1}{2\sin^2 x + 2x \sin 2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{-2\sin 2x}{4\sin x \cos x + 2\sin 2x + 4x\cos 2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$$
$$= \lim_{x\to 0} \frac{-4\cos 2x}{4\cos x \cos x - 4\sin x \sin x + 4\cos 2x + 4\cos 2x - 8x\sin 2x} \overset{\text{dosadit}}{=} -\frac{1}{3}$$

21. $$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$$
$$= \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x} \overset{\text{dosadit}}{=} 2$$

22. $$\lim_{x\to 0} \frac{1 - \cos x^2}{x^2 \sin x^2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{2x \sin x^2}{2x\sin x^2 + 2xx^2\cos x^2} \overset{\text{uprava}}{=}$$
$$= \lim_{x\to 0} \frac{\sin x^2}{\sin x^2 + x^2 \cos x^2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{2x\cos x^2}{2x\cos x^2 + 2x\cos x^2 - 2xx^2 \sin x^2} = ||\tfrac{0}{0}|| \overset{\text{uprava}}{=}$$
$$= \lim_{x\to 0} \frac{\cos x^2}{\cos x^2 + \cos x^2 - x^2 \sin x^2} \overset{\text{dosadit}}{=} \frac{1}{2}$$

23. $$\lim_{x\to 0} \frac{x \operatorname{cotg} x - 1}{x^2} = \lim_{x\to 0} \frac{x\cos x - \sin x}{x^2 \sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\cos x - x\sin x - \cos x}{2x\sin x + x^2 \cos x} \overset{\text{uprava}}{=}$$
$$= -\lim_{x\to 0} \frac{\sin x}{2\sin x + x\cos x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\cos x}{2\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} -\frac{1}{3}$$

24. $$\lim_{x\to 0} \frac{x\cos x - \sin x}{x \sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\cos x - x\sin x - \cos x}{\sin x + x\cos x} \overset{\text{uprava}}{=} -\lim_{x\to 0} \frac{-x\sin x}{\sin x + x\cos x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$$
$$= -\lim_{x\to 0} \frac{\sin x + x\cos x}{\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} 0$$

25. $$\lim_{x\to 0^+} \frac{e^x - 1 - x}{(e^x - 1)x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0^+} \frac{e^x - 1}{e^x - 1 - xe^x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0^+} \frac{e^x}{e^x - xe^x - e^x} \overset{\text{dosadit}}{=} \frac{1}{2}$$

26. $$\lim_{x\to 1} \frac{x\ln x - x + 1}{(x-1)\ln x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{\ln x + x\frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} \overset{\text{uprava}}{=} \lim_{x\to 1} \frac{\ln x}{\frac{x-1}{x} + \ln x} \overset{\text{uprava}}{=} \lim_{x\to 1} \frac{x\ln x}{(x-1) + x\ln x} =$$
$$= ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 1} \frac{1 + \ln x}{2 + \ln x} \overset{\text{dosadit}}{=} \frac{1}{2}$$

27. $$\lim_{x\to 0} \frac{x - \sin x}{\sin^3 x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{1 - \cos x}{3\sin^2 x \cos x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim_{x\to 0} \frac{\sin x}{6\sin x \cos^2 x - 3\sin^3 x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$$
$$= \lim_{x\to 0} \frac{\cos x}{6\cos^3 x - 6\cdot 2\cdot \sin^2 x \cos x - 9\sin^2 x \cos x} \overset{\text{dosadit}}{=} \frac{1}{6}$$

28. $\lim\limits_{x\to 0+} \dfrac{\ln(e^x+x)}{x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0+} \dfrac{\frac{1}{e^x+x}(e^x+1)}{1} \overset{\text{dosadit}}{=} 2$

29. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\ln\sin x}{\operatorname{cotg} x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to \frac{\pi}{2}} \dfrac{\frac{\cos x}{\sin x}}{-\frac{1}{\sin^2 x}} \overset{\text{uprava}}{=} -\lim\limits_{x\to \frac{\pi}{2}} \cos x \sin x \overset{\text{dosadit}}{=} 0$

30. $\lim\limits_{x\to 0} \dfrac{\ln(\arcsin x)}{\operatorname{cotg} x} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{\arcsin x}\cdot\frac{1}{\sqrt{1-x^2}}}{-\frac{1}{\sin^2 x}} \overset{\text{uprava}}{=} -\lim\limits_{x\to 0} \dfrac{1}{\sqrt{1-x^2}} \cdot \lim\limits_{x\to 0} \dfrac{\sin^2 x}{\arcsin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$= -\lim\limits_{x\to 0} \dfrac{2\sin x\cos x}{\frac{1}{\sqrt{1-x^2}}} \overset{\text{dosadit}}{=} 0$

31. $\lim\limits_{x\to 0^+} \dfrac{\ln(1+\operatorname{tg}^2\sqrt{x})}{2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{1+\operatorname{tg}^2\sqrt{x}}\cdot 2\operatorname{tg}\sqrt{x}\cdot\frac{1}{\cos^2\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}{2} \overset{\text{uprava}}{=}$

$= \dfrac{1}{2}\lim\limits_{x\to 0^+} \dfrac{1}{(1+\operatorname{tg}^2\sqrt{x})\cos^3\sqrt{x}} \cdot \lim\limits_{x\to 0^+} \dfrac{\sin\sqrt{x}}{\sqrt{x}} = \dfrac{1}{2}\lim\limits_{x\to 0^+} \dfrac{\sin\sqrt{x}}{\sqrt{x}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \ \dots\ = \dfrac{1}{2}$

Typ $0\cdot(\pm\infty)$**.**

32. $\lim\limits_{x\to 0^+} x\ln x = ||0\times\infty|| \overset{\text{uprava}}{=} \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} \overset{\text{uprava}}{=} \lim\limits_{x\to 0^+} -x \overset{\text{dosadit}}{=} 0$

33. $\lim\limits_{x\to\infty} x^2 e^{-x^2} = ||0\times\infty|| \overset{\text{uprava}}{=} \lim\limits_{x\to\infty} \dfrac{x^2}{e^{x^2}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \dfrac{2x}{2xe^{x^2}} \overset{\text{uprava}}{=} \lim\limits_{x\to\infty} \dfrac{1}{e^{x^2}} \overset{\text{dosadit}}{=} 0$

34. $\lim\limits_{x\to 1} (1-x)\operatorname{tg}\dfrac{\pi x}{2} = ||0\times\infty|| \overset{\text{uprava}}{=} \lim\limits_{x\to 1} \dfrac{1-x}{\operatorname{cotg}\frac{\pi x}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \dfrac{-1}{-\frac{\pi}{2}\sin^{-2}\frac{\pi x}{2}} \overset{\text{dosadit}}{=} \dfrac{2}{\pi}$

34B. $\lim\limits_{x\to 1} (1-x)\operatorname{tg}\dfrac{\pi x}{2} = ||0\times\infty|| \overset{\text{uprava}}{=} \lim\limits_{x\to 1} \dfrac{(1-x)\sin\frac{\pi x}{2}}{\cos\frac{\pi x}{2}} = \lim\limits_{x\to 1}\sin\dfrac{\pi x}{2}\cdot\lim\limits_{x\to 1}\dfrac{1-x}{\cos\frac{\pi x}{2}} = 1\cdot\lim\limits_{x\to 1}\dfrac{1-x}{\cos\frac{\pi x}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$= \lim\limits_{x\to 1} \dfrac{-1}{-\frac{\pi}{2}\sin\frac{\pi x}{2}} \overset{\text{dosadit}}{=} \dfrac{2}{\pi}$

35. $\lim\limits_{x\to 1^-} \ln x\ln(1-x) = ||0\times\infty|| \overset{\text{uprava}}{=} \lim\limits_{x\to 1^-} \dfrac{\ln(1-x)}{\frac{1}{\ln x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1^-} \dfrac{\frac{-1}{1-x}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}} \overset{\text{uprava}}{=} \lim\limits_{x\to 1^-} \dfrac{x\ln^2 x}{1-x} = ||\tfrac{0}{0}||$

$\overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1^-} \dfrac{\ln^2 x + 2\ln x}{-1} \overset{\text{dosadit}}{=} 0$

36. $\lim\limits_{x\to\infty} x^2\ln\cos\dfrac{a}{x} = ||\infty\times 0|| \overset{\text{uprava}}{=} \lim\limits_{x\to 0} \dfrac{\ln\cos\frac{a}{x}}{x^{-2}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{\cos\frac{a}{x}}\cdot(-\sin\frac{a}{x})\cdot a\cdot(-1)\cdot x^{-2}}{-2x^{-3}} \overset{\text{uprava}}{=}$

$= -\dfrac{a}{2}\cdot\lim\limits_{x\to\infty} \dfrac{\operatorname{tg}\frac{a}{x}}{x^{-1}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} -\dfrac{a}{2}\cdot\lim\limits_{x\to\infty} \dfrac{\frac{1}{\cos^2\frac{a}{x}}\cdot a\cdot(-1)\cdot x^{-2}}{-x^{-2}} \overset{\text{uprava}}{=} -\dfrac{a}{2}\lim\limits_{x\to\infty}\dfrac{a}{\cos^2\frac{a}{x}} \overset{\text{dosadit}}{=} -\dfrac{a^2}{2}$

37. $\lim\limits_{x\to\infty} x(\operatorname{arctg} x - \dfrac{\pi}{2}) = ||\infty\times 0|| \overset{\text{uprava}}{=} \lim\limits_{x\to\infty} \dfrac{\operatorname{arctg} x - \frac{\pi}{2}}{\frac{1}{x}} \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = ||\tfrac{0}{0}|| \overset{\text{uprava}}{=}$

$= \lim\limits_{x\to\infty} \dfrac{-x^2}{1+x^2} =$ … limita podílu polynomů … $= -1$

Typ $\pm\infty\mp\infty$**.**

38. $\lim\limits_{x\to 0}\Big(\dfrac{1}{x} - \dfrac{1}{\sin x}\Big) = \lim\limits_{x\to 0} \dfrac{\sin x - x}{x\sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin x + x\cos x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$= \lim\limits_{x\to 0} \dfrac{-\sin x}{\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} 0$

39. $\displaystyle\lim_{x\to0}\Big(\frac{1}{x^3}-\frac{1}{\sin^3 x}\Big)=-\infty$

40. $\displaystyle\lim_{x\to0}\Big(\operatorname{cotg} x-\frac{1}{x}\Big)=\lim_{x\to0}\frac{x\cos x-\sin x}{x\sin x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}\lim_{x\to0}\frac{\cos x-x\sin x-\cos x}{\sin x+x\cos x}\overset{\text{uprava}}{=}$

$\displaystyle=-\lim_{x\to0}\frac{-x\sin x}{\sin x+x\cos x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}-\lim_{x\to0}\frac{\sin x+x\cos x}{\cos x+\cos x-x\sin x}\overset{\text{dosadit}}{=}0$

41. $\displaystyle\lim_{x\to0+}\Big(\frac{1}{x}-\frac{1}{e^x-1}\Big)=\lim_{x\to0+}\frac{e^x-1-x}{(e^x-1)x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}\lim_{x\to0+}\frac{e^x-1}{e^x-1-xe^x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}$

$\displaystyle=\lim_{x\to0+}\frac{e^x}{e^x-xe^x-e^x}\overset{\text{dosadit}}{=}\frac{1}{2}$

42. $\displaystyle\lim_{x\to1}\Big(\frac{x}{x-1}-\frac{1}{\ln x}\Big)=\lim_{x\to1}\frac{x\ln x-x+1}{(x-1)\ln x}\overset{\text{l'H.p.}}{=}\lim_{x\to1}\frac{\ln x}{\frac{x-1}{x}+\ln x}\overset{\text{uprava}}{=}\lim_{x\to1}\frac{x\ln x}{(x-1)+x\ln x}=$

$\displaystyle=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}\lim_{x\to1}\frac{1+\ln x}{2+\ln x}\overset{\text{dosadit}}{=}\frac{1}{2}$

Exponenciální výrazy.

43. $\displaystyle\lim_{x\to0}x^x=||0^0||=e^L$, kde $\displaystyle L=\lim_{x\to0}x\ln x\overset{\text{viz. výše}}{=}0$; výsledek: $e^L=e^0=1$

44. $\displaystyle\lim_{x\to0+}(e^x+x)^{\frac{1}{x}}=||1^\infty||=e^{\lim_{x\to0+}\frac{1}{x}\ln(e^x+x)}=e^L$, kde $\displaystyle L=\lim_{x\to0+}\frac{\ln(e^x+x)}{x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}$

$\displaystyle=\lim_{x\to0+}\frac{\frac{1}{e^x+x}(e^x+1)}{1}\overset{\text{dosadit}}{=}2$; výsledek: $e^L=e^2$

45. $\displaystyle\lim_{x\to\frac{\pi}{2}}(\sin x)^{\operatorname{tg} x}=||1^\infty||=e^{\lim_{x\to\frac{\pi}{2}}\operatorname{tg} x\ln\sin x}=e^L$, kde $\displaystyle L=\lim_{x\to\frac{\pi}{2}}\frac{\ln\sin x}{\operatorname{cotg} x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}\lim_{x\to\frac{\pi}{2}}\frac{\frac{\cos x}{\sin x}}{-\frac{1}{\sin^2 x}}\overset{\text{uprava}}{=}$

$\displaystyle=-\lim_{x\to\frac{\pi}{2}}\cos x\sin x\overset{\text{dosadit}}{=}0$; výsledek: $e^L=e^0=1$

46. $\displaystyle\lim_{x\to a}\Big(\frac{\sin x}{\sin a}\Big)^{\frac{1}{x-a}}=||1^\infty||=e^{\lim_{x\to a}\frac{\ln\frac{\sin x}{\sin a}}{x-a}}=e^L$, kde $\displaystyle L=\lim_{x\to a}\frac{\ln\frac{\sin x}{\sin a}}{x-a}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}\lim_{x\to a}\frac{\frac{\cos x}{\sin x}}{1}\overset{\text{dosadit}}{=}$

$=\operatorname{cotg} a$; výsledek: $e^L=e^{\operatorname{cotg} a}$

47. $\displaystyle\lim_{x\to0}\Big(\frac{\operatorname{arctg} x}{x}\Big)^{\frac{1}{x^2}}=||1^\infty||=e^L$, kde $\displaystyle L=\lim_{x\to0}\frac{\ln\frac{\operatorname{arctg} x}{x}}{x^2}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}$ atd ... ; výsledek: $e^L=e^{-\frac{1}{3}}$

48. $\displaystyle\lim_{x\to0^+}\big(\ln\frac{1}{x}\big)^x=||\infty^0||=e^L$, kde $\displaystyle L=\lim_{x\to0^+}x\ln\ln\frac{1}{x}\overset{\text{subst.}}{=}\lim_{t\to\infty}\frac{1}{t}\ln\ln t=\lim_{t\to\infty}\frac{\ln\ln t}{t}=||\tfrac{\infty}{\infty}||\overset{\text{l'H.p.}}{=}$

$\displaystyle=\lim_{t\to\infty}\frac{\frac{1}{\ln t}\cdot\frac{1}{t}}{1}\overset{\text{dosadit}}{=}0$; výsledek: $e^L=e^0=1$

49. $\displaystyle\lim_{x\to0}(\arcsin x)^{\operatorname{tg} x}=||\tfrac{0}{0}||=e^L$, kde $\displaystyle L=\lim_{x\to0}\frac{\ln(\arcsin x)}{\operatorname{cotg} x}=||\tfrac{\infty}{\infty}||\overset{\text{l'H.p.}}{=}\lim_{x\to0}\frac{\frac{1}{\arcsin x}\cdot\frac{1}{\sqrt{1-x^2}}}{-\frac{1}{\sin^2 x}}\overset{\text{uprava}}{=}$

$\displaystyle=-\lim_{x\to0}\frac{1}{\sqrt{1-x^2}}\cdot\lim_{x\to0}\frac{\sin^2 x}{\arcsin x}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}-\lim_{x\to0}\frac{2\sin x\cos x}{\frac{1}{\sqrt{1-x^2}}}=0$; výsledek: $e^L=1$

50. $\displaystyle\lim_{x\to\infty}\Big(\frac{x}{x+2}\Big)^{2x}=||1^\infty||=e^{\lim_{x\to\infty}2x\ln\frac{x}{x+2}}=e^L$, kde $\displaystyle L=2\lim_{x\to\infty}\frac{\ln\frac{x}{x+2}}{\frac{1}{x}}=||\tfrac{0}{0}||\overset{\text{l'H.p.}}{=}$

$\displaystyle=2\lim_{x\to\infty}\frac{\frac{x+2}{x}\cdot\frac{x+2-x}{(x+2)^2}}{-x^{-2}}\overset{\text{uprava}}{=}-4\lim_{x\to\infty}\frac{x^2}{x(x+2)}=-4$; výsledek: $e^L=e^{-4}$

51. $\displaystyle\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}} = ||1^{\infty}|| = e^L$, kde $\displaystyle L = \lim_{x\to 0}\frac{\sin x \ln\frac{\sin x}{x}}{x-\sin x} \overset{\text{uprava}}{=} \lim_{x\to 0}\frac{-\ln\frac{x}{\sin x}}{\frac{x}{\sin x}-1} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$\displaystyle = \lim_{x\to 0}\frac{-\frac{\sin x}{x}\left(\frac{x}{\sin x}\right)'}{\left(\frac{x}{\sin x}\right)'} \overset{\text{uprava}}{=} \lim_{x\to 0} -\frac{\sin x}{x} = -1$; výsledek: $\displaystyle e^L = \frac{1}{e}$

52. $\displaystyle\lim_{x\to 0^+}\left(1+\operatorname{tg}^2\sqrt{x}\right)^{\frac{1}{2x}} = ||1^{\infty}|| = e^L$, kde $\displaystyle L = \lim_{x\to 0^+}\frac{\ln(1+\operatorname{tg}^2\sqrt{x})}{2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$\displaystyle = \lim_{x\to 0^+}\frac{\frac{1}{1+\operatorname{tg}^2\sqrt{x}}\cdot 2\operatorname{tg}\sqrt{x}\cdot\frac{1}{\cos^2\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}{2} \overset{\text{uprava}}{=} \frac{1}{2}\lim_{x\to 0^+}\frac{1}{(1+\operatorname{tg}^2\sqrt{x})\cos^3\sqrt{x}}\cdot\lim_{x\to 0^+}\frac{\sin\sqrt{x}}{\sqrt{x}} =$

$\displaystyle = \frac{1}{2}\lim_{x\to 0^+}\frac{\sin\sqrt{x}}{\sqrt{x}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \ \dots\ = \frac{1}{2}$; výsledek: $e^L = \sqrt{e}$

53. $\displaystyle\lim_{x\to\frac{\pi}{2}^-}(\operatorname{tg} x)^{2x-\pi} = 1$

54. $\displaystyle\lim_{x\to 0^+}(e^x-1)^{\frac{1}{\ln x}} = 1$

55. $\displaystyle\lim_{x\to 1} x^{\operatorname{tg}\frac{\pi}{2x}} = e^{-\frac{2}{\pi}}$

56. $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x^2+1}{x}} = e$

57. $\displaystyle\lim_{x\to\infty}\left(\cos\frac{1}{x}\right)^{x^2} = e^{-\frac{1}{2}}$