Definition 2.1 (Excellent number)

Let n be positive integer. The number n is said to be *excellent*, if the last digit of the number α defined by the relation

$$\alpha = n^2 + \int_0^{2\pi} \sin x \mathrm{d}x \tag{1}$$

equals 1.

Definition 2.2 (Happy number)

Let n be positive integer. The number n is said to be *happy*, if the last digit of the number n equals 1.

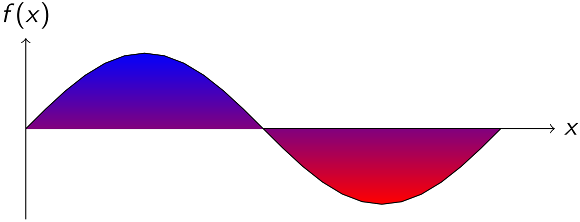

Figure: Sine curve

Theorem 2.4

Let $f(x)$ be integrable in the sense of Riemann on $[a, b]$. Let $F(x)$ be a function continuous on $[a, b]$ which is an antiderivative of the function f on the interval (a, b). Then

$$\int_a^b f(x)\mathrm{d}x = [F(x)]_a^b = F(b) - F(a)$$

holds.

$$\int_0^{2\pi} \sin x \mathrm{d}x = 0. \tag{2}$$

Theorem 2.5 (Characterization of excellent numbers)

The positive integer n is excellent if and only if the last digit of the number n is either 1 or 9.

Theorem 2.6 (Relationship between happy and excellent numbers)

Each happy number is excellent.