Inequalities

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Look at three or four or twenty my quizzes and then fill in my   please!

To create your own test from based on this one you will need free AcroTeXeDucation bundle, the TeX source attached here ⬚ and to follow instruction on home site.

1. Theory

Věta 1 (continuity of elementary functions) *Every elementary function is continuous on its domain.*

Věta 2 (Bolzano) *Let f be a function defined and continuous on $[a, b]$. If $f(a).f(b) < 0$ holds (i.e. the values $f(a)$ and $f(b)$ have different signs), then there exists a zero of the function f on the interval (a, b), i.e. there exists $c \in (a, b)$ such that $f(c) = 0$.*

General nonlinear inequality When solving one of the inequalities

$$f(x) > 0, \quad f(x) \geq 0, \quad f(x) \leq 0, \quad \text{and} \quad f(x) < 0,$$

we can proceed in the following steps.

1. We find all points of discontinuity of the function $f(x)$.

2. We find all of the solutions of the equation $f(x) = 0$.

3. We use the points from the last two steps and divide the real axis subintervals. The function f preserves its sign on each subinterval. We choose arbitrary (convenient) number ξ form each subinterval, evaluate $f(\xi)$ and mark the sign of this value to the subinterval. We do this step for all subintervals.

4. Performing the preceding step for all subintervals, we assign the sign of the function $f(x)$ to each subinterval. Now it is clear where $f(x) > 0$ holds and where the inequality is opposite.

2. Test

- Given an inequality, find the domain and x-intercepts of the function. Write your answer as comma separated list of numbers or write the word `none` or `empty`. The order of the numbers in your answer does not matter.

- Write the sign chart for the function. You have to order the x-intercepts and poins of discontinuity and place them on the real axis. Then find intervals with positive and negative signs and the intervals which do not belong to the domain of the function.

- Finally, write the solution of the inequality. Use `[a,b]` for closed interval, `(a,b)` for open interval, `U` for union, `inf` for ∞ and `-inf` for $-\infty$. The set $\mathbb{R} \setminus (1, 2]$ can be written as `(-inf,1]U(2,inf)`.

Example how to fill-in the quiz

1. Solve $\dfrac{x^2 + 1}{x^2 - 1} \le 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{x^2 + 1}{x^2 - 1} = 0$

(c) Complete the sign chart

(i) positive
negative
undefined

(iii) positive
negative
undefined

(v) positive
negative
undefined

(ii) x-intercept
discontinuity

(iv) x-intercept
discontinuity

(d) Solution:

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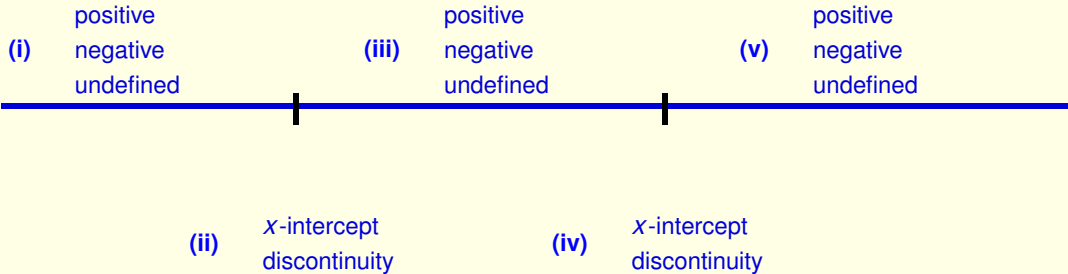Page *4* of *17*

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2. Solve $\dfrac{x - 1}{x + 2} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{x - 1}{x + 2} = 0$

(c) Complete the sign chart

(i)
positive
negative
undefined

(iii)
positive
negative
undefined

(v)
positive
negative
undefined

(ii)
x-intercept
discontinuity

(iv)
x-intercept
discontinuity

(d) Solution:

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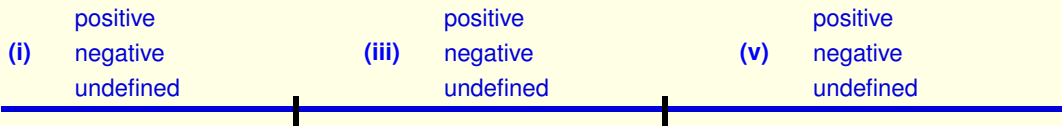
3. Solve $x^2 - 3x + 2 > 0$

(a) Find point(-s) of discontinuity

(b) Solve $x^2 - 3x + 2 = 0$

(c) Complete the sign chart

(i) positive negative undefined

(iii) positive negative undefined

(v) positive negative undefined

(ii) x-intercept discontinuity

(iv) x-intercept discontinuity

(d) Solution:

4. Solve $x^2 - 4 \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $x^2 - 4 = 0$

(c) Complete the sign chart

(i) positive
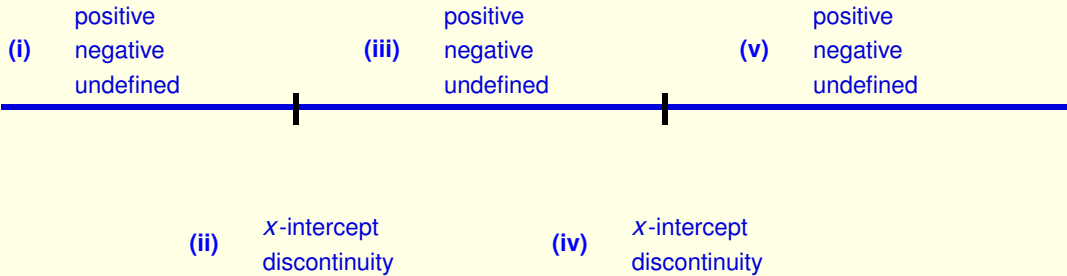   negative
    undefined

(iii) positive
    negative
    undefined

(v) positive
    negative
    undefined

(ii) x-intercept
    discontinuity

(iv) x-intercept
    discontinuity
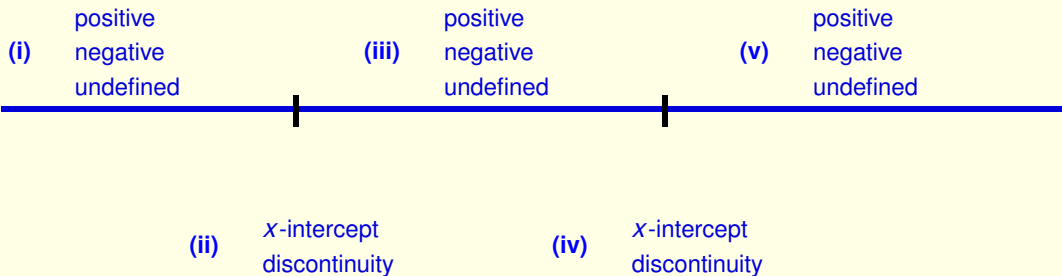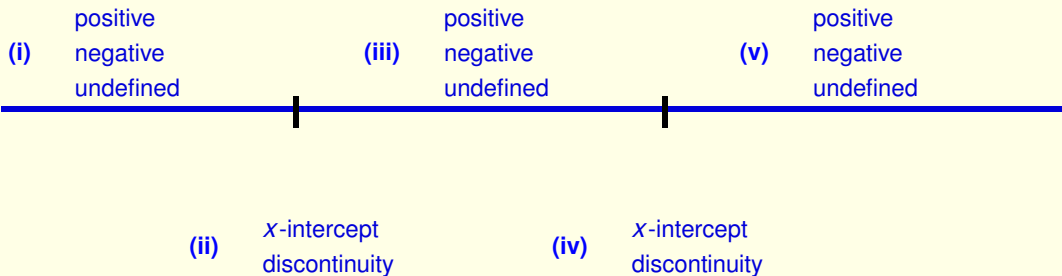
(d) Solution:

5. Solve $\dfrac{x^2 - 7x}{x^2 + 4} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{x^2 - 7x}{x^2 + 4} = 0$
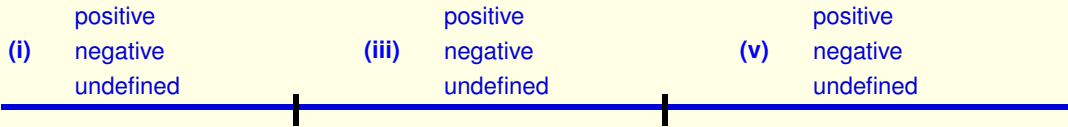
(c) Complete the sign chart

(i) positive
negative
undefined

(iii) positive
negative
undefined

(v) positive
negative
undefined

(ii) x-intercept
discontinuity

(iv) x-intercept
discontinuity

(d) Solution:

6. Solve $\dfrac{xe^x}{(x+1)^3} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{xe^x}{(x+1)^3} = 0$

(c) Complete the sign chart

(i)   positive   **(iii)**   positive   **(v)**   positive

     negative   negative   negative

     undefined   undefined   undefined

(ii) x-intercept   **(iv)** x-intercept

      discontinuity   discontinuity
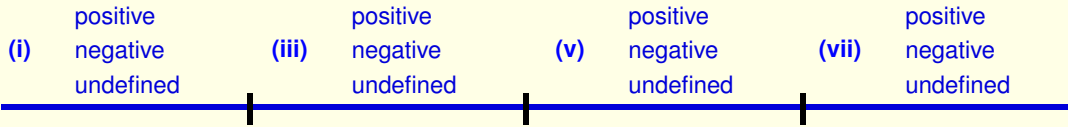
(d) Solution:

7. Solve $\dfrac{(x-2)\ln(x)}{x} \leq 0$

   (a) Find point(-s) of discontinuity

   (b) Solve $\dfrac{(x-2)\ln(x)}{x} = 0$

   (c) Complete the sign chart

(i)   positive   **(iii)**   positive   **(v)**   positive   **(vii)**   positive
        negative   negative   negative   negative
        undefined   undefined   undefined   undefined

(ii) x-intercept   **(iv)** x-intercept   **(vi)** x-intercept
         discontinuity   discontinuity   discontinuity

   (d) Solution:

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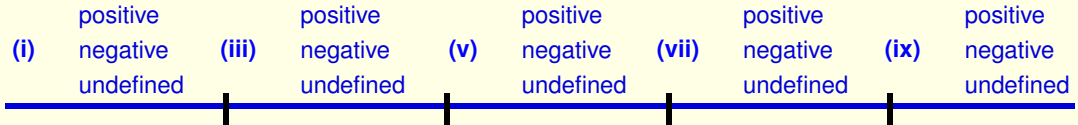
Theory

Test

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8. Solve $\dfrac{\ln^2 x - \ln x}{x - 6} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{\ln^2 x - \ln x}{x - 6} = 0$

(c) Complete the sign chart

| **(i)** | positive
negative
undefined | **(iii)** | positive
negative
undefined | **(v)** | positive
negative
undefined | **(vii)** | positive
negative
undefined | **(ix)** | positive
negative
undefined |

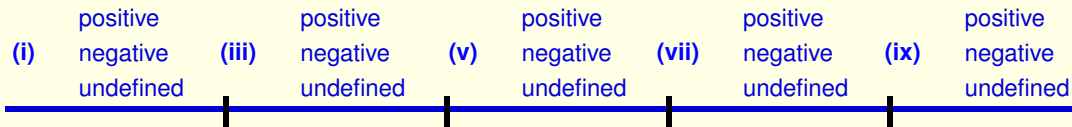| | **(ii)** | x-intercept
discontinuity | **(iv)** | x-intercept
discontinuity | **(vi)** | x-intercept
discontinuity | **(viii)** | x-intercept
discontinuity |

(d) Solution:

9. Solve $\dfrac{1 - \ln^2 x}{1 - x} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{1 - \ln^2 x}{1 - x} = 0$

(c) Complete the sign chart

(d) Solution:

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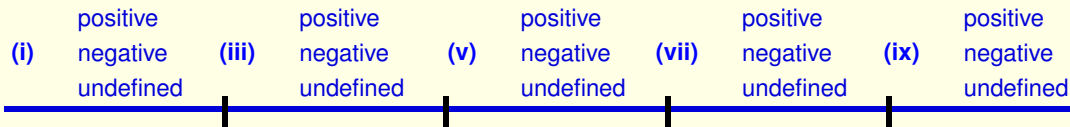
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10. Solve $\dfrac{\ln^2 x + 3\ln x + 2}{x - 1} \geq 0$
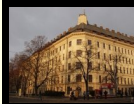
   (a) Find point(-s) of discontinuity

   (b) Solve $\dfrac{\ln^2 x + 3\ln x + 2}{x - 1} = 0$

   (c) Complete the sign chart

| **(i)** | positive
negative
undefined | **(iii)** | positive
negative
undefined | **(v)** | positive
negative
undefined | **(vii)** | positive
negative
undefined | **(ix)** | positive
negative
undefined |

| | **(ii)** | x-intercept
discontinuity | **(iv)** | x-intercept
discontinuity | **(vi)** | x-intercept
discontinuity | **(viii)** | x-intercept
discontinuity |

   (d) Solution:

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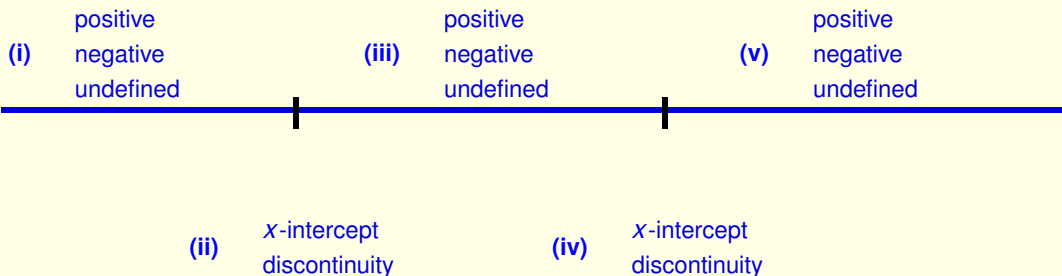
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11. Solve $\dfrac{e^x - 2}{x - 2} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{e^x - 2}{x - 2} = 0$

(c) Complete the sign chart

(i)

positive
negative
undefined

(iii)

positive
negative
undefined

(v)

positive
negative
undefined

(ii) x-intercept
discontinuity

(iv) x-intercept
discontinuity

(d) Solution:

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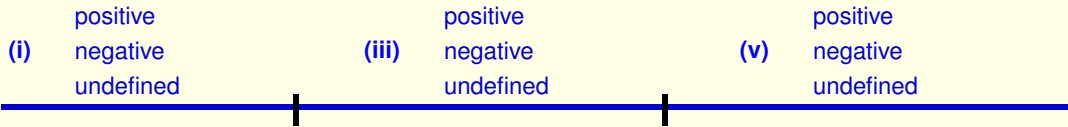
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12. Solve $\dfrac{e^x - 2}{(x+1)^2} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{e^x - 2}{(x+1)^2} = 0$

(c) Complete the sign chart

(i) positive
     negative
     undefined

(iii) positive
      negative
      undefined

(v) positive
     negative
     undefined

(ii) x-intercept
      discontinuity

(iv) x-intercept
      discontinuity

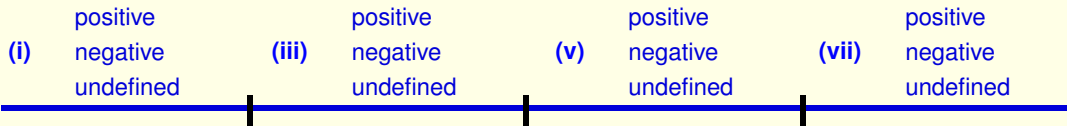(d) Solution:

13. Solve $\dfrac{x(3 - e^x)}{x - 1} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{x(3 - e^x)}{x - 1} = 0$

(c) Complete the sign chart

(i)	positive negative undefined	**(iii)**	positive negative undefined	**(v)**	positive negative undefined	**(vii)**	positive negative undefined

(ii) x-intercept discontinuity   **(iv)** x-intercept discontinuity   **(vi)** x-intercept discontinuity

(d) Solution:

14. Solve $\dfrac{2e^x - 3}{x^2 - x} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\dfrac{2e^x - 3}{x^2 - x} = 0$

(c) Complete the sign chart

(i)   positive
negative
undefined

(iii)   positive
negative
undefined

(v)   positive
negative
undefined

(vii)   positive
negative
undefined

(ii) x-intercept
discontinuity

(iv) x-intercept
discontinuity

(vi) x-intercept
discontinuity

(d) Solution: