First order differential equation with separated variables Interactive tests

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

1. Theory

Definition 1 (ODE with separated variables) *Let f and g be continuous functions. The equation*

$$y' = f(x)g(y) \tag{1}$$

is said to be an ordinary differential equation with separated variables.

Omitting constant solutions, which are solutions of $g(y) = 0$, we can write the equation in the form

$$\frac{y'}{g(y)} = f(x)$$

and replacing $y' = \dfrac{\mathrm{d}y}{\mathrm{d}x}$ and multiplying by $\mathrm{d}x$ we get[1]

$$\frac{1}{g(y)}\,\mathrm{d}y = f(x)\,\mathrm{d}x \tag{2}$$

The words "*separate variables*" mean: convert the equation into the form (2).
The general solution of (1) is

$$\int \frac{1}{g(y)}\,\mathrm{d}y = \int f(x)\,\mathrm{d}x + C,$$

where both integrals denote *one* of the primitive functions.

[1] The functions $f(x)$ and $\dfrac{1}{g(y)}$ are unique, up to a *common* constant multiple.

2. Tests

Quiz Find the general solution of the ODE with separated variables.

$$yy' = 4x$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

- Correct answers to the first question include `ydy = 4 x dx`, `ydy - 4 x dx=0` and any constant multiple.
- Correct answers to the second question include `y^2/2 = 2x^2-C`, `y^2/2 - 2x^2+1=C` and any constant multiple of this equation.

Quiz Find the general solution of the ODE with separated variables.

$$\frac{1}{x+1} - \frac{1}{y-1}y' = 0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$x^2 + y' \cos(y) + 1 = 0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y(y+y')+1=0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$\frac{x^2+1}{x}+\frac{yy'}{y^2-1}=0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$(x-1)y^3 - e^x y' = 0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' = \frac{y-1}{x^2 y^2}$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$2(1+e^x)yy' = e^x$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$(y^2-1)+yy'(x+1)=0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' = \frac{2x+1}{2(y-1)}$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' = \frac{2x-1}{x^2}y$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' = 2\sqrt{y}\ln x$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' e^{x^2+y} = -x/y$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' \cos^2 x = (1 + \cos^2 x)\sqrt{1 - y^2}$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' + xy = y$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y' = x^2(1+y^2)$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate:

Quiz Find the general solution of the ODE with separated variables.

$$y \ln y + xy' = 0$$

1. Find constant solution(-s). Write the number or numbers separated by commas. Write the word `empty` if there si no constant solution,
2. Separate variables:
3. Integrate: