



Inequalities

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Inequalities
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Theory

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Look at three or four or twenty
my quizzes and then fill in my
please!



1. Theory



Theorem 1 (continuity of elementary functions) *Every elementary function is continuous on its domain.*

Theorem 2 (Bolzano) *Let f be a function defined and continuous on $[a, b]$. If $f(a) \cdot f(b) < 0$ holds (i.e. the values $f(a)$ and $f(b)$ have different signs), then there exists a zero of the function f on the interval (a, b) , i.e. there exists $c \in (a, b)$ such that $f(c) = 0$.*

General nonlinear inequality When solving one of the inequalities

$$f(x) > 0, \quad f(x) \geq 0, \quad f(x) \leq 0, \quad \text{and} \quad f(x) < 0,$$

we can proceed in the following steps.

1. We find all points of discontinuity of the function $f(x)$.
2. We find all of the solutions of the equation $f(x) = 0$.
3. We use the points from the last two steps and divide the real axis subintervals. The function f preserves its sign on each subinterval. We choose arbitrary (convenient) number ξ from each subinterval, evaluate $f(\xi)$ and mark the sign of this value to the subinterval. We do this step for all subintervals.
4. Performing the preceding step for all subintervals, we assign the sign of the function $f(x)$ to each subinterval. Now it is clear where $f(x) > 0$ holds and where the inequality is opposite.

2. Test

Quiz

Given an inequality, find the domain and x -intercepts of the function. Write your answer as comma separated list of numbers or write the word **none** or **empty**. The order of the numbers in your answer does not matter.

Write the sign chart for the function. You have to order the x -intercepts and points of discontinuity and place them on the real axis. Then find intervals with positive and negative signs and the intervals which do not belong to the domain of the function.

Finally, write the solution of the inequality. Use $[a, b]$ for closed interval, (a, b) for open interval, \cup for union, inf for ∞ and $-\text{inf}$ for $-\infty$. The set $\mathbb{R} \setminus (1, 2]$ can be written as $(-\text{inf}, 1] \cup (2, \text{inf})$.

Example how to fill-in the quiz

The screenshot shows a quiz question in an Adobe Reader application. The question is: "1. Solve $\frac{x^2 + 1}{x^2 - 1} \leq 0$ ".

Part (a) asks for the point(s) of discontinuity, with the answer "1,-1" entered in a text box.

Part (b) asks to solve $\frac{x^2 + 1}{x^2 - 1} = 0$, with the answer "none" entered in a text box.

Part (c) asks to complete the sign chart. The chart shows a horizontal real axis with tick marks at -1 and 1. Above the axis, there are three regions labeled (i), (iii), and (v). Below the axis, there are two regions labeled (ii) and (iv). The sign in each region is indicated by a checked box for "positive" and an unchecked box for "negative" or "undefined".

Part (d) asks for the solution, with the answer "(-1,1)" entered in a text box.

The interface also includes a sidebar with navigation buttons like "Home Page", "Print", "Title Page", "Page 4 of 17", "Go Back", "Full Screen", "Close", and "Quit".

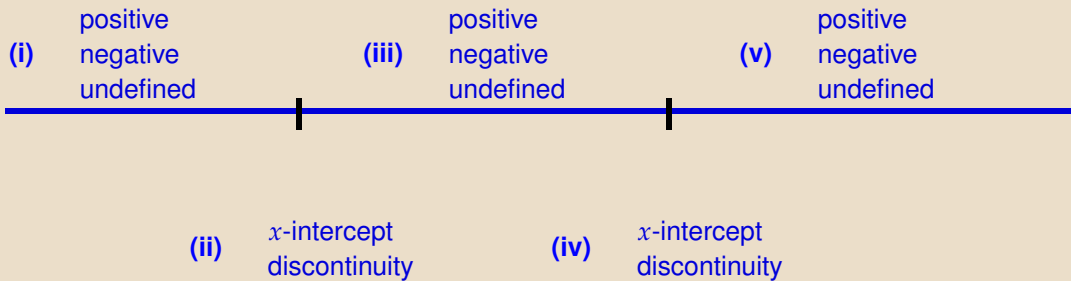


1. Solve $\frac{x^2 + 1}{x^2 - 1} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{x^2 + 1}{x^2 - 1} = 0$

(c) Complete the sign chart



(d) Solution:

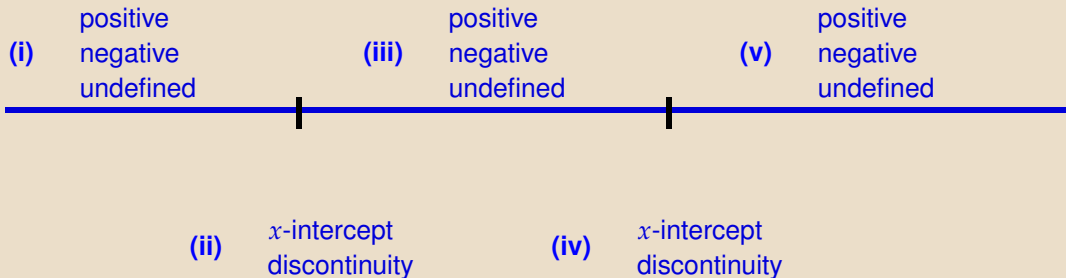


2. Solve $\frac{x-1}{x+2} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{x-1}{x+2} = 0$

(c) Complete the sign chart



(d) Solution:

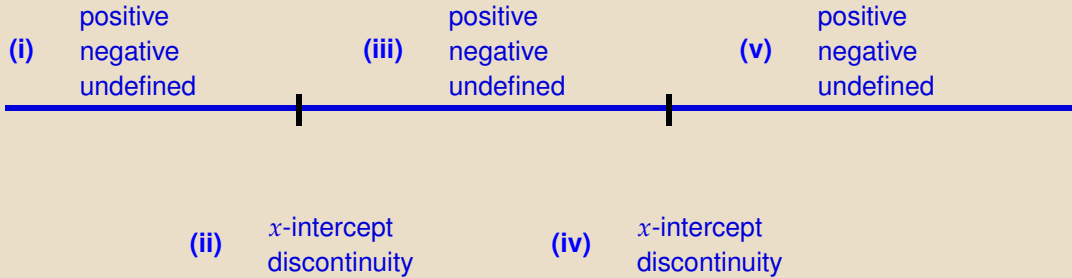


3. Solve $x^2 - 3x + 2 > 0$

(a) Find point(-s) of discontinuity

(b) Solve $x^2 - 3x + 2 = 0$

(c) Complete the sign chart



(d) Solution:

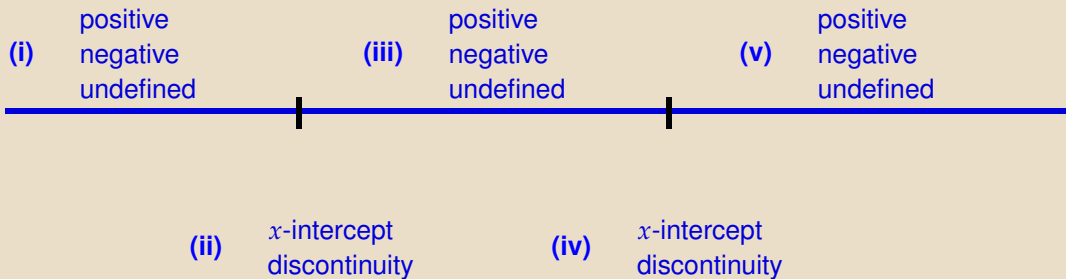


4. Solve $x^2 - 4 \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $x^2 - 4 = 0$

(c) Complete the sign chart



(d) Solution:

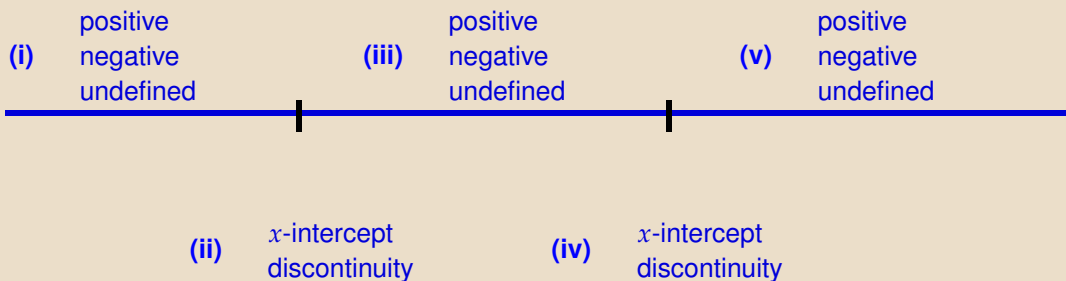


5. Solve $\frac{x^2 - 7x}{x^2 + 4} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{x^2 - 7x}{x^2 + 4} = 0$

(c) Complete the sign chart



(d) Solution:

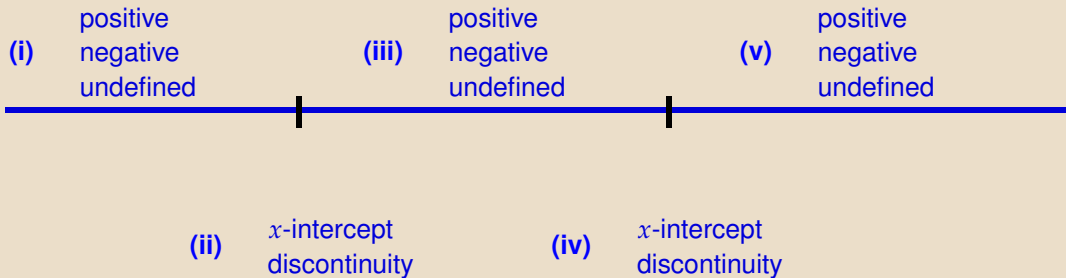


6. Solve $\frac{xe^x}{(x+1)^3} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{xe^x}{(x+1)^3} = 0$

(c) Complete the sign chart



(d) Solution:

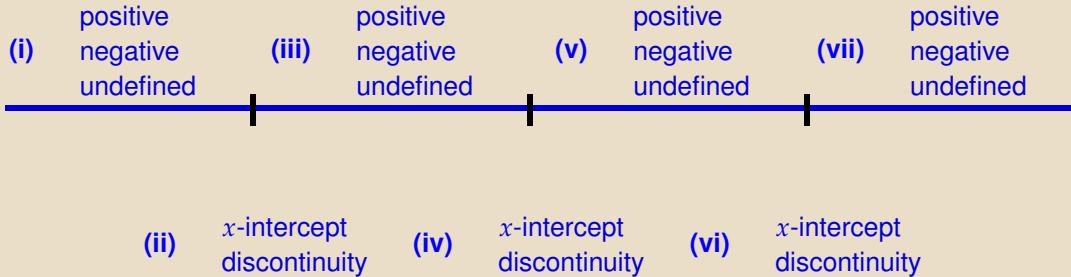


7. Solve $\frac{(x-2)\ln(x)}{x} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{(x-2)\ln(x)}{x} = 0$

(c) Complete the sign chart



(d) Solution:

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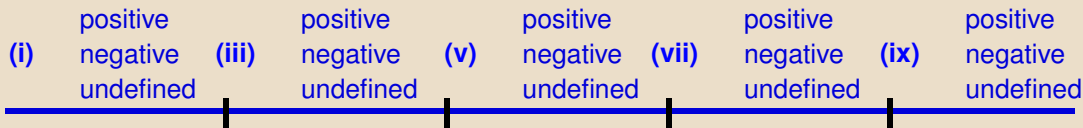


8. Solve $\frac{\ln^2 x - \ln x}{x - 6} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{\ln^2 x - \ln x}{x - 6} = 0$

(c) Complete the sign chart



(ii) x -intercept
discontinuity (iv) x -intercept
discontinuity (vi) x -intercept
discontinuity (viii) x -intercept
discontinuity

(d) Solution:

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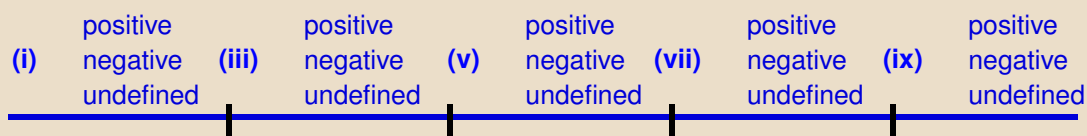


9. Solve $\frac{1 - \ln^2 x}{1 - x} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{1 - \ln^2 x}{1 - x} = 0$

(c) Complete the sign chart



(ii) x -intercept discontinuity (iv) x -intercept discontinuity (vi) x -intercept discontinuity (viii) x -intercept discontinuity

(d) Solution:

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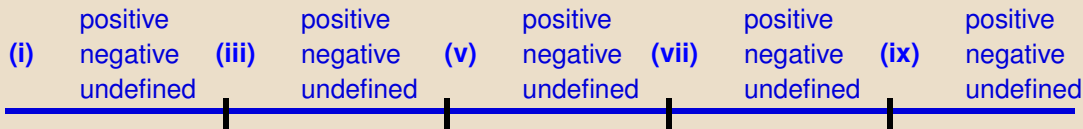


10. Solve $\frac{\ln^2 x + 3 \ln x + 2}{x - 1} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{\ln^2 x + 3 \ln x + 2}{x - 1} = 0$

(c) Complete the sign chart



(ii) x -intercept discontinuity (iv) x -intercept discontinuity (vi) x -intercept discontinuity (viii) x -intercept discontinuity

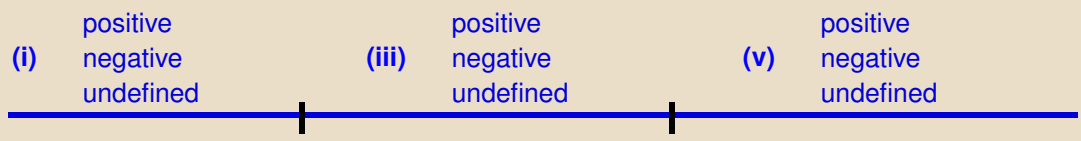
(d) Solution:

11. Solve $\frac{e^x - 2}{x - 2} \leq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{e^x - 2}{x - 2} = 0$

(c) Complete the sign chart



(ii) x -intercept
discontinuity

(iv) x -intercept
discontinuity

(d) Solution:

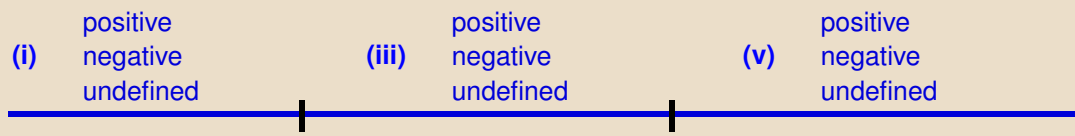


12. Solve $\frac{e^x - 2}{(x + 1)^2} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{e^x - 2}{(x + 1)^2} = 0$

(c) Complete the sign chart



(d) Solution:

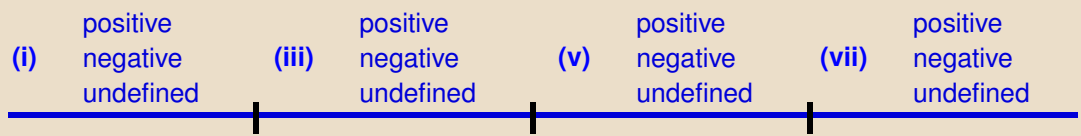


13. Solve $\frac{x(3 - e^x)}{x - 1} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{x(3 - e^x)}{x - 1} = 0$

(c) Complete the sign chart



(ii) x -intercept discontinuity (iv) x -intercept discontinuity (vi) x -intercept discontinuity

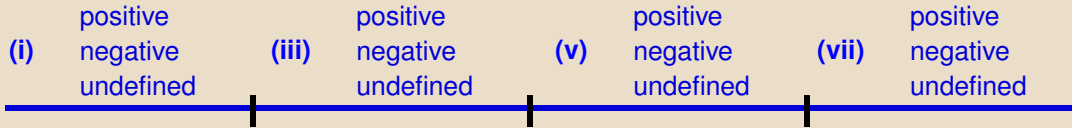
(d) Solution:

14. Solve $\frac{2e^x - 3}{x^2 - x} \geq 0$

(a) Find point(-s) of discontinuity

(b) Solve $\frac{2e^x - 3}{x^2 - x} = 0$

(c) Complete the sign chart



(ii) x -intercept discontinuity (iv) x -intercept discontinuity (vi) x -intercept discontinuity

(d) Solution: