Limits – algebra of limits
Interactive tests

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July 14, 2006

Look at three or four or twenty my quizzes and
then fill in my   please!

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Algebra of limits
file lim2.tex

Theory

Test

Home Page

Print

Title Page

◀◀   ▶▶

◀   ▶

Page 1 of 5

Go Back

Full Screen

Close

Quit

1. Theory

An arbitrary limit belongs to one of the following groups (families).

(a) The limit can be evaluated by investigating the graphs of basic elementary functions and using the rules for algebra of limits and, if necessary, the limit of a composite function.

(b) The limit is one of the indeterminate forms $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $0(\pm\infty)$ or $\pm\infty \mp \infty$. Such a limit can be usually evaluated by l'Hospital rule. Sometimes it may be better to use also a different approach (e.g. for limits of polynomials or rational functions at $\pm\infty$ we have a faster method than the l'Hospital rule available.)

(c) The limit contains an expression of the type $\frac{\text{nonzero value}}{\text{zero}}$. We investigate one-sided limits first. These one-sided limits are infinite and from their mutual relationship conclude whether or not the two-sided limit exists.

An important step is to substitute the value which x approaches and recognise which of these groups the limit belongs to. After this step we can properly choose the method which has to be used for evaluating of the limit. Sometimes the evaluation is easy, sometimes it needs long calculations.

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Quiz Given a limit, establish the family where this limit belongs to and evaluate the limit. Write `infinity` and `-infinity` for $\pm\infty$, write `undefined` for the limit which does not exists.

algebra of limits   indeterminate form $0\infty, \frac{0}{0}$ or ∞/∞ $\dfrac{\text{nonzero}}{\text{zero}}$ your result

1. $\lim\limits_{x\to\infty} xe^x$

2. $\lim\limits_{x\to-\infty} xe^x$

3. $\lim\limits_{x\to 0} xe^x$

4. $\lim\limits_{x\to\infty} \dfrac{x}{e^x}$

5. $\lim\limits_{x\to-\infty} \dfrac{x}{e^x}$

6. $\lim\limits_{x\to 0} \dfrac{e^x}{x}$

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Test

Home Page

Print

Title Page

◀◀　▶▶

◀　▶

Page *3* of *5*

Go Back

Full Screen

Close

Quit

7. $\displaystyle\lim_{x\to 0^+} \frac{\ln x}{x}$

8. $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

9. $\displaystyle\lim_{x\to\infty} \frac{2x^2 + x + 1}{x^2 + 4}$

10. $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{x + 1}$

11. $\displaystyle\lim_{x\to -1^+} \frac{\sin \frac{\pi}{2} x}{x + 1}$

12. $\displaystyle\lim_{x\to\infty} \frac{x + 2}{\operatorname{atan} x}$

13. $\displaystyle\lim_{x\to 0} \frac{\operatorname{atan} x^2}{x}$

14. $\displaystyle\lim_{x\to\infty} e^{\frac{1}{x}}$

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Theory

Test

Home Page

Print

Title Page

◀◀ ▶▶

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Go Back

Full Screen

Close

Quit

15. $\displaystyle\lim_{x\to-\infty}\frac{x^3+x+2}{3x^2-1}$

16. $\displaystyle\lim_{x\to\infty}\frac{e^{\frac{1}{x}}}{x}$

17. $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{2\sin(2x)}{\sin(4x)}$

18. $\displaystyle\lim_{x\to-\infty} x\operatorname{atan} x$

19. $\displaystyle\lim_{x\to\infty} x\operatorname{atan} x$

20. $\displaystyle\lim_{x\to0}\frac{2x\cos x}{\sin x}$

21. $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{2x\cos x}{\sin x}$

22. $\displaystyle\lim_{x\to\pi}\frac{2x\cos x}{\sin x}$

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