

# Local extrema for functions of two variables

## Interactive quizzes

Robert Mařík

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ROBERT MAŘÍK  
Extrema in  $\mathbb{R}^2$   
file le21.tex

Look at three or four or twenty my quizzes and  
then fill in my \_\_\_\_\_ please!



**Quiz** You have to find the first derivatives and all stationary points. Then you get one of the stationary points and you have to find Hessian at this point and distinguish between maximum, minimum or saddle point.

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1. Consider the function  $z = x^2 + y^2$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[0, 0]$

$$H([0, 0]) = \begin{vmatrix} & \\ & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[0, 0]$

min

MAX

saddle

???

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2. Consider the function  $z = x^4 + y^4 - 4xy + 30$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$
$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format [A,B]; [C,D]; [E,F]; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$
$$\frac{\partial^2 z}{\partial x \partial y} =$$
$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[-1, -1]$

$$H([-1, -1]) = \begin{vmatrix} & & & \\ & & & \\ & & & \\ & & & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[-1, -1]$

min

MAX

saddle

???





3. Consider the function  $z = x^2 - y^2$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[0, 0]$

$$H([0, 0]) = \begin{vmatrix} & \\ & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[0, 0]$

min

MAX

saddle

???

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4. Consider the function  $z = x^4 + y^4$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format [A,B]; [C,D]; [E,F]; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[0,0]$

$$H([0,0]) = \begin{vmatrix} & \\ & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[0,0]$

min

MAX

saddle

???





5. Consider the function  $z = 9x - 9y - x^2 - y^2$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $\left[-\frac{9}{2}, \frac{9}{2}\right]$

$$H\left(-\frac{9}{2}, \frac{9}{2}\right) = \left| \begin{array}{cc} & \\ & \end{array} \right| =$$

(e) Establish the type of stationary point  $\left[-\frac{9}{2}, \frac{9}{2}\right]$

min

MAX

saddle

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6. Consider the function  $z = x^2y^2 - x^2 - y^2$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$
$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$
$$\frac{\partial^2 z}{\partial x \partial y} =$$
$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[1, -1]$

$$H([1, -1]) = \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[1, -1]$

min

MAX

saddle

???

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7. Consider the function  $z = \ln(x - y) - x^2 - y$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[\frac{1}{2}, -\frac{1}{2}]$

$$H([\frac{1}{2}, -\frac{1}{2}]) = \left| \begin{array}{cc} & \\ & \end{array} \right| =$$

(e) Establish the type of stationary point  $[\frac{1}{2}, -\frac{1}{2}]$

min

MAX

saddle

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8. Consider the function  $z = 9xy + \frac{1}{x} + \frac{3}{y}$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]; [C, D]; [E, F]; \dots$ )

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $\left[\frac{1}{3}, 1\right]$

$$H\left(\left[\frac{1}{3}, 1\right]\right) = \left| \begin{array}{cc} & \\ & \end{array} \right| =$$

(e) Establish the type of stationary point  $\left[\frac{1}{3}, 1\right]$

min

MAX

saddle

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9. Consider the function  $z = e^{2x}(x + y^2)$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A, B]$ ;  $[C, D]$ ;  $[E, F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $\left[-\frac{1}{2}, 0\right]$

$$H\left(-\frac{1}{2}, 0\right) = \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} =$$

(e) Establish the type of stationary point  $\left[-\frac{1}{2}, 0\right]$

min

MAX

saddle

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10. Consider the function  $z = 3x^3 + 3x^2y - y^3 - 15x$ .

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

(b) Find stationary points (write in format  $[A,B]$ ;  $[C,D]$ ;  $[E,F]$ ; ...)

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} =$$

$$\frac{\partial^2 z}{\partial x \partial y} =$$

$$\frac{\partial^2 z}{(\partial y)^2} =$$

(d) Evaluate Hessian at  $[1,1]$

$$H([1,1]) = \begin{vmatrix} & \\ & \end{vmatrix} =$$

(e) Establish the type of stationary point  $[1,1]$

min

MAX

saddle

???