Local extrema for functions of two variables
Interactive quizzes

Robert Mařík

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

Quiz You have to find the first derivatives and all stationary points. Then you get one of the stationary points and you have to find Hessian at this point and distinguish between maximum, minimum or saddle point.

1. Consider the function $z = x^2 + y^2$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}?$$

$$\frac{\partial z}{\partial y} = \boxed{}?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}?$$

(d) Evaluate Hessian at ???

$$H(\ ???\ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

2. Consider the function $z = x^4 + y^4 - 4xy + 30$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}?$$

$$\frac{\partial z}{\partial y} = \boxed{}?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}?$$

(d) Evaluate Hessian at ???

$$H(\ \ ???\ \ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

3. Consider the function $z = x^2 - y^2$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}?$$

$$\frac{\partial z}{\partial y} = \boxed{}?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}?$$

(d) Evaluate Hessian at ???

$$H(\ ???\ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

4. Consider the function $z = x^4 + y^4$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}?$$

$$\frac{\partial z}{\partial y} = \boxed{}?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}?$$

(d) Evaluate Hessian at ???

$$H(\ ???\ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

5. Consider the function $z = 9x - 9y - x^2 - y^2$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}\,?$$

$$\frac{\partial z}{\partial y} = \boxed{}\,?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}\,?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}\,?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}\,?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}\,?$$

(d) Evaluate Hessian at ???

$$H(\quad ??? \quad) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}\,?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

6. Consider the function $z = x^2y^2 - x^2 - y^2$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}\,?$$

$$\frac{\partial z}{\partial y} = \boxed{}\,?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}\,?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}\,?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}\,?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}\,?$$

(d) Evaluate Hessian at ???

$$H(\quad ??? \quad) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}\,?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

7. Consider the function $z = \ln(x-y) - x^2 - y$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}\,?$$

$$\frac{\partial z}{\partial y} = \boxed{}\,?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}\,?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}\,?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}\,?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}\,?$$

(d) Evaluate Hessian at ???

$$H(\quad ??? \quad) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}\,?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

8. Consider the function $z = 9xy + \frac{1}{x} + \frac{3}{y}$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}\,?$$

$$\frac{\partial z}{\partial y} = \boxed{}\,?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}\,?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}\,?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}\,?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}\,?$$

(d) Evaluate Hessian at ???

$$H(\ ???\ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}\,?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

9. Consider the function $z = e^{2x}(x + y^2)$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}\,?$$

$$\frac{\partial z}{\partial y} = \boxed{}\,?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}\,?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}\,?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}\,?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}\,?$$

(d) Evaluate Hessian at ???

$$H(\quad ??? \quad) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}\,?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???

10. Consider the function $z = 3x^3 + 3x^2y - y^3 - 15x$.

(a) Find the first derivatives

$$\frac{\partial z}{\partial x} = \boxed{}?$$

$$\frac{\partial z}{\partial y} = \boxed{}?$$

(b) Find stationary points (write in format `[A,B]; [C,D]; [E,F]; ...`)

$\boxed{}?$

(c) Find the second derivatives

$$\frac{\partial^2 z}{(\partial x)^2} = \boxed{}?$$

$$\frac{\partial^2 z}{\partial x \partial y} = \boxed{}?$$

$$\frac{\partial^2 z}{(\partial y)^2} = \boxed{}?$$

(d) Evaluate Hessian at ???

$$H(\ ???\ ) = \begin{vmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{vmatrix} = \boxed{}?$$

(e) Establish the type of stationary point ???

☐ min ☐ MAX ☐ saddle ☐ ???