Second order nonhomogeneous linear differential equation Interactive tests

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

1. Theory

Definition 1 (second order linear differential equation) Let p, q and f be functions continuous on the interval I. The equation

$$y'' + p(x)y' + q(x)y = f(x) \tag{1}$$

is said to be a *second order linear differential equation*. Under a *solution* of this equation we understand every function which has the second derivative on the interval I and satisfies (1) for every $x \in I$.

Definition 2 (associated homogeneous equation) Consider nonhomogeneous equation (1). Homogeneous equation

$$y'' + p(x)y' + q(x)y = 0. \tag{2}$$

with the left-hand side identical with equation (1) is called a *homogeneous equation associated to the nonhomogeneous equation* (1).

Theorem 1 (general solution) Let $y_1(x)$ and $y_2(x)$ be fundamental system of solutions of the homogeneous LDE (2) and $y_p(x)$ be an arbitrary particular solution of the nonhomogeneous LDE (1). Then the function

$$y(x) = Ay_1(x) + By_2(x) + y_p(x), \qquad A \in \mathbb{R},\ B \in \mathbb{R} \tag{3}$$

is a general solution of the nonhomogeneous LDE (1).

2. Tests

- Given an equation and the form of the particular solution, solve the equation.
- You have to adjust the real constants in the particular solution to make the equation true after substituting particular solution.
- When you find the particular solution, find also the general solution.
- Use constants A and B in the general solution.
  More precisely, for the equation $y'' + y = 1$ write the solution in any of the following forms

  1. $y =$ `1+A sin(x)+B cos(x)`
  2. $y =$ `1+A*cos(x)+B*sin(x)`
  3. $y =$ `A cos(x)+B sin(x)+1`

  Remark: The answer `1+A*(sin(x)+cos(x))+B*(sin(x)-cos(x))+cos(x)` is marked as correct as well, since neither particular solution nor the fundamental system are unique.

Quiz 1. Solve $y'' + 3y' - 4y = 2$.
Consider particular solution in the form $y_p = a$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. Find the value of the undetermined constant $a =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 2. Solve $y'' + 2y' + y = 5$.
Consider particular solution in the form $y_p = a$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. Find the value of the undetermined constant $a =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 3. Solve $y'' + 2y' + y = 5e^x$.
Consider particular solution in the form $y_p = ae^x$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. Find the value of the undetermined constant $a =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 4. Solve $y'' - 2y' + y = 5e^x$.

Consider particular solution in the form $y_p = ax^2e^x$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. Find the value of the undetermined constant $a =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 5. Solve $y'' + 4y = 5e^{3x}$.

Consider particular solution in the form $y_p = ae^{3x}$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. Find the value of the undetermined constant $a =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 6. Solve $y'' - y = 3e^x$.
Consider particular solution in the form $y_p = axe^x$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. Find the value of the undetermined constant $a =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 7. Solve $y'' + 2y' + y = x + 1$.
Consider particular solution in the form $y_p = ax + b$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : [ ] [?] $\Longrightarrow$ $a =$ [ ] [?]

x^2 : [ ] [?] $b =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 8. Solve $y'' + y = x - 3$.
Consider particular solution in the form $y_p = ax + b$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : [ ] [?] $\Longrightarrow$ $a =$ [ ] [?]

x^2 : [ ] [?] $b =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 9. Solve $y'' - 2y' + 2y = x^2 - 1$.

Consider particular solution in the form $y_p = ax^2 + bx + c$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^2 : [ ] ? $\qquad a =$ [ ] ?

x^1 : [ ] ? $\qquad \Longrightarrow \quad b =$ [ ] ?

x^0 : [ ] ? $\qquad c =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 10. Solve $y'' + y' - 2y = 2x + 1$.
Consider particular solution in the form $y_p = ax + b$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : [ ] [?] $\quad\Longrightarrow\quad a =$ [ ] [?]

x^2 : [ ] [?] $\qquad\qquad b =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 11. Solve $y'' - y' - 2y = 4x + 5$.
Consider particular solution in the form $y_p = ax + b$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : [ ] [?] $\Longrightarrow$ $a =$ [ ] [?]

x^0 : [ ] [?] $b =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 12. Solve $y'' + 2y' + y = 5x$.
Consider particular solution in the form $y_p = ax + b$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : [ ] [?] $\Longrightarrow$ $a =$ [ ] [?]

x^0 : [ ] [?] $b =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 13. Solve $y'' - y = xe^x$.

Consider particular solution in the form $y_p = e^x(ax^2 + bx)$.

1. Find the first two derivatives

$y_p' =$?

$y_p'' =$?

2. Substitute particular solution and its derivatives into the equation:

?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : ? $\quad \Longrightarrow \quad a =$?

x^0 : ? $\quad b =$?

4. Write the particular solution:

$y_p =$?

5. Write the general solution (use constants A and B):

$y =$?

Quiz 14. Solve $y'' - y = 3xe^x$.

Consider particular solution in the form $y_p = (ax^2 + bx)e^x$.

1. Find the first two derivatives

$y_p' =$?

$y_p'' =$?

2. Substitute particular solution and its derivatives into the equation:

?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : ? $\qquad \Longrightarrow \qquad a =$?

x^0 : ? $\qquad\qquad\quad\;\; b =$?

4. Write the particular solution:

$y_p =$?

5. Write the general solution (use constants A and B):

$y =$?

Quiz 15. Solve $y'' - y = (3x - 2)e^x$.

Consider particular solution in the form $y_p = (ax^2 + bx)e^x$.

1. Find the first two derivatives

$y_p' =$?

$y_p'' =$?

2. Substitute particular solution and its derivatives into the equation:

?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^1 : ? $\qquad \Longrightarrow \qquad a =$?

x^0 : ? $\qquad\qquad\quad\;\; b =$?

4. Write the particular solution:

$y_p =$?

5. Write the general solution (use constants A and B):

$y =$?

Quiz 16. Solve $y'' + 2y' + y = 2x^2 + 1$.

Consider particular solution in the form $y_p = ax^2 + bx + c$.

1. Find the first two derivatives

$y_p' =$ [ ] ?

$y_p'' =$ [ ] ?

2. Substitute particular solution and its derivatives into the equation:

[ ] ?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^2 : [ ] ? $\qquad a =$ [ ] ?

x^1 : [ ] ? $\qquad \Longrightarrow \quad b =$ [ ] ?

x^0 : [ ] ? $\qquad c =$ [ ] ?

4. Write the particular solution:

$y_p =$ [ ] ?

5. Write the general solution (use constants A and B):

$y =$ [ ] ?

Quiz 17. Solve $y'' + 4y = x^2$.

Consider particular solution in the form $y_p = ax^2 + bx + c$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^2 : [ ] [?] $\quad a =$ [ ] [?]

x^1 : [ ] [?] $\quad \Longrightarrow \quad b =$ [ ] [?]

x^0 : [ ] [?] $\quad c =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 18. Solve $y'' + 2y' - 3y = 6x^3 + 2x + 1$.
Consider particular solution in the form $y_p = ax^3 + bx^2 + cx + d$.

1. Find the first two derivatives

$y_p' =$?

$y_p'' =$?

2. Substitute particular solution and its derivatives into the equation:

?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^3 : ? $\qquad a =$?

x^2 : ? $\qquad b =$?

x^1 : ? $\implies \quad c =$?

x^0 : ? $\qquad d =$?

4. Write the particular solution:

$y_p =$?

5. Write the general solution (use constants A and B):

$y =$?

Quiz 19. Solve $y'' + 2y' + y = x^3$.

Consider particular solution in the form $y_p = ax^3 + bx^2 + cx + d$.

1. Find the first two derivatives

$y_p' =$?

$y_p'' =$?

2. Substitute particular solution and its derivatives into the equation:

?

3. If an exponential factor appears in the equation, divide by this factor. Then build linear system for undetermined constants by comparing like powers of x and solve this system.

x^3 : ?

x^2 : ?

x^1 : ?

x^0 : ?

$\Longrightarrow$

$a =$?

$b =$?

$c =$?

$d =$?

4. Write the particular solution:

$y_p =$?

5. Write the general solution (use constants A and B):

$y =$?

Quiz 20. Solve $y'' - 4y = \sin x$.
Consider particular solution in the form $y_p = b\sin(x) + c\cos(x)$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. Build the linear system for constants a and b by comparing coefficients at corresponding trigonometric functions and solve this system

$\sin(x)$: [ ] [?]

$\cos(x)$: [ ] [?]

$\Longrightarrow$ $b =$ [ ] [?]

$c =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]

Quiz 21. Solve $y'' - 4y' + 4y = \sin x$.
Consider particular solution in the form $y_p = b\sin(x) + c\cos(x)$.

1. Find the first two derivatives

$y_p' =$ [ ] [?]

$y_p'' =$ [ ] [?]

2. Substitute particular solution and its derivatives into the equation:

[ ] [?]

3. Build the linear system for constants a and b by comparing coefficients at corresponding trigonometric functions and solve this system

$\sin(x)$: [ ] [?]

$\cos(x)$: [ ] [?]

$\Longrightarrow$ $b =$ [ ] [?]

$c =$ [ ] [?]

4. Write the particular solution:

$y_p =$ [ ] [?]

5. Write the general solution (use constants A and B):

$y =$ [ ] [?]