Second order homogeneous linear differential equation Interactive tests

Robert Mařík

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

1. Theory

Theorem 1 Consider the homogeneous second order ODE with constant coefficients

$$\boxed{y'' + py' + qy = 0}, \qquad p, q \in \mathbb{R} \tag{1}$$

and its characteristic equation $\boxed{z^2 + pz + q = 0}$ with unknown z.

- If $z_1, z_2 \in \mathbb{R}$ are mutually different real zeros of the characteristic equation, we put

$$y_1(x) = e^{z_1 x} \qquad \text{and} \qquad y_2(x) = e^{z_2 x}.$$

- If $z_1 \in \mathbb{R}$ is a double real zero of the characteristic equation, we put

$$y_1(x) = e^{z_1 x} \qquad \text{and} \qquad y_2(x) = x e^{z_1 x}.$$

- If $z_{1,2} = \alpha \pm i\beta \notin \mathbb{R}$ are complex zeros of the characteristic equation, we put

$$y_1(x) = e^{\alpha x} \cos(\beta x) \qquad \text{and} \qquad y_2(x) = e^{\alpha x} \sin(\beta x).$$

The functions y_1 and y_2 form a fundamental system of solutions of equation (1). The general solution of equation (1) is

$$\boxed{y(x) = Ay_1(x) + By_2(x)}, \qquad A \in \mathbb{R},\ B \in \mathbb{R}.$$

2. Two terms homogeneous LDE

Quiz Second order linear differential equation with two terms on the left hand side.

- Find the characteristic equation in z variable, write e.g. " `z^2+3z-8=0` ".
- Find the fundamental system from Theorem 1 as comma separated unordered list, write e.g. " `exp(x)*cos(3x), exp(x)*sin(3x)` ".
- Find the general solution as a linear combination of functions from the fundamental system. Use constants A and B! Hence, write something like " `A*exp(x)*cos(3x)+B*exp(x)*sin(3x)` ".

1. $y'' + y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

2. $y'' - y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

3. $y'' + 4y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

4. $y'' - 4y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

5. $y'' + 2y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

6. $y'' - 2y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

7. $4y'' + y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

8. $4y'' - y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

9. $y'' + 9y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

10. $y'' - 9y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

3. General homogeneous LDE

Instructions are the same as in the first test.

Quiz

1. $4y'' + 4y' - 3y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

2. $y'' - 4y' + 4y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

3. $y'' - 3y' - 10y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

4. $y'' + 2y' + 2y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

5. $y'' - 2y' + 10y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

6. $y'' - 4y' + 13y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

7. $y'' + y' - 2y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

8. $y'' + 6y' + 9y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

9. $y'' + y' + y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

10. $y'' + y' - 6y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

11. $4y'' - 4y + y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

12. $y'' - 6y' + y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

13. $y'' + 2y' + 3y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

14. $2y'' - 5y' + 2y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?

15. $y'' - 4y' + 3y = 0$

Characteristic equation (in z): ?

Fundamental system: ?

General solution: $y(x) =$?