First order linear differential equation Interactive tests

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

1. Theory

Definition 1 (first order linear ODE) *Let a, b be continuous function on I. The equation*

$$y' + a(x)y = b(x) \tag{1}$$

is said to be the first order linear ordinary differential equation (shortly LDE). If $b(x) \equiv 0$ on I, then the equation is called homogeneous and nonhomogeneous otherwise.

Definition 2 (associated homogeneous equation) *Consider nonhomogeneous equation (1). Homogeneous equation*

$$y' + a(x)y = 0 \tag{2}$$

with the left-hand side identical to the left-hand side of (1) is said to be a homogeneous equation associated with the nonhomogeneous equation (1).

In this file, the form (1) is refferred as a *normal form* of the first order linear differential equation.

Theorem 1 (general solution of LDE)

- *If $y_p(x)$ is a particular solution of nonhomogeneous LDE and $y_0(x)$ is a general solution of the associated homogeneous LDE, then the function*

$$y(x) = y_p(x) + y_0(x)$$

is a general solution of nonhomogeneous LDE.

- *If $y_p(x)$ is a particular solution of nonhomogeneous LDE and $y_0(x)$ a nontrivial particular solution of the associated homogeneous LDE, then the function*

$$y(x) = y_p(x) + Cy_0(x)$$

is a general solution of nonhomogeneous LDE.

$$y' + a(x)y = b(x) \qquad \text{linear ODE} \tag{3}$$

$$y' + a(x)y = 0 \qquad \text{associated hom. ODE} \tag{4}$$

$$y_0(x) = C\,\underbrace{e^{-\int a(x)\,\mathrm{d}x}}_{\text{particular solution of (4)}}, \qquad C \in \mathbb{R} \qquad \text{General solution of (4)} \tag{5}$$

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right] \qquad \text{General solution of (3)} \tag{6}$$

$$= \underbrace{e^{-\int a(x)\,\mathrm{d}x}\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x}_{y_p(x)\text{ — a part. sol. of (3)}} + \underbrace{Ce^{-\int a(x)\,\mathrm{d}x}}_{y_0(x)} \qquad C \in \mathbb{R}$$

Here $\int \cdots \mathrm{d}x$ is *one* of primitive functions (no constant of integration). This expression is unique up to an aditive constant. Thus $e^{\pm\int a(x)\,\mathrm{d}x}$ is unique up to a nonzero constant multiple and the particular solution is unique up to an aditive factor which is a solution of homogeneous equation.

2. Tests

Test are on the following pages — one test (one equation) per page.

- The answer on the standard form of the linear differential equation and functions a and b is unique.
- The answer on the integral of a is unique up to an additive constant of integration. The test can proceed this anwers and grade correctly. You get hint which of the primitive functions should be used in the sequel.
- As usual, you can see the answer by pressing ? button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these quizzes. The problems on exam are harder[1]!
- As usual: If you have any comments or suggestions concerning this test, let me know, please!

[1] this means slightly longer computation of derivatives and integrals and so on

Quiz 1. Solve the 1-st order linear ODE $y' - y = 2$

1. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

2. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

3.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

4. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

5. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 2. Solve the 1-st order linear ODE $xy' + y = x^2$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 3. Solve the 1-st order linear ODE $xy' - y = 1$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 4. Solve the 1-st order linear ODE $(x+1)y' - 2y = (x+1)^4$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

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Theory

Tests

Quiz 5. Solve the 1-st order linear ODE $y' \cos(x) + y \sin(x) = 1$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 6. Solve the 1-st order linear ODE $y' - y = \frac{1+x^2}{x} e^x$

1. Comparing to the normal form of the linear ODE we see that

$a(x) =$ ___ ?

$b(x) =$ ___ ?

2. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ ___ ?

3.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ ___ ?

4. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ ___ ?

5. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ ___ ?

Quiz 7. Solve the 1-st order linear ODE $y' - y\tan x = \sin x$

1. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

2. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

3.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

4. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

5. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 8. Solve the 1-st order linear ODE $y' \cos x = (y + 2\cos x) \sin x$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 9. Solve the 1-st order linear ODE $(2x+1)y' + y = \sqrt{2x+1} + 3$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?

Quiz 10. Solve the 1-st order linear ODE $xy' + 2y = e^{-x^2}$

1. The standard form of the equation is [ ] ?

2. Comparing to the normal form of the linear ODE we see that

$a(x) =$ [ ] ?

$b(x) =$ [ ] ?

3. Integrating we get (use zero constant of integration)

$\int a(x)\,\mathrm{d}x =$ [ ] ?

4.

Find $b(x)e^{\int a(x)\,\mathrm{d}x} =$ [ ] ?

5. Integrate: $\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x =$ [ ] ?

6. Find general solution of the equation

$$y(x) = e^{-\int a(x)\,\mathrm{d}x}\left[\int b(x)e^{\int a(x)\,\mathrm{d}x}\,\mathrm{d}x + C\right]$$

$=$ [ ] ?