

Rational functions

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Look at three or four or twenty my quizzes
and then fill in my _____ please!

To integrate rational functions, we have to be able to recognize the type of this
function first and then we use the proper method.

Theory

Test1

Test2

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1. Theory

In this test we divide all rational functions into three groups.

- The function which is a *partial fraction* can be integrated using basic formulas.
- The *proper rational function* which is not a partial fraction can be written as a sum of partial fractions. Then we integrate partial fractions using basic formulas.
- The *improper rational function* can be written as sum of a polynomial and a proper rational function. Then we integrate both polynomial and proper rational fraction separately.

Definition 1 Let $P_n(x)$ be an n -degree polynomial, $Q_m(x)$ be an m -degree polynomial and $R(x) = \frac{P_n(x)}{Q_m(x)}$ be a *rational function*. The function $R(x)$ is said be *proper* if $n \geq m$ and *improper* if $n < m$.

Definition 2 *Partial fractions are functions which can be written in one of the following forms*

$$\frac{A_1}{x - a'} \quad \frac{A_n}{(x - a)^{n'}} \quad \frac{Ax + B}{x^2 + Mx + N}$$

where $n \geq 2$ is a positive integer, x a variable, a , A_i , M , N real constants and the polynomial $x^2 + Mx + N$ has no real zeros.

2. Test1

Quiz Distinguish the correct type of the function

1. $\frac{x}{x^2 + 4}$

2. $\frac{x}{x^2 - 4}$

3. $\frac{x + 1}{(x - 1)^2}$

4. $\frac{x}{(x - 1)^2}$

5. $\frac{3}{(x - 1)^2}$

6. $\frac{\sqrt{x}}{x^2 + x + 1}$

7. $\frac{x^3 - 1}{x + 2}$

8. $\frac{6x - 1}{x^2 + 8x + 100}$

9. $\frac{1}{x^3 + 1}$

10. $\frac{3}{x + 5}$

Partial fraction

Proper rational function, but not a partial fraction

Improper rational function

Not rational function



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Integrals - rational func.

file int-rf1.tex

Theory

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$$11. \frac{x+2}{x^2+4x+6}$$

$$12. \frac{x}{x^2+4x+6}$$

$$13. \frac{x}{x^3+4x}$$

$$14. \frac{x-1}{(x+2)^3}$$

$$15. \frac{6}{(x-\sqrt{3})^4}$$

$$16. \frac{x^2}{x+1}$$

$$17. \frac{x-1}{x(x-2)(x-3)}$$

$$18. \frac{x^3-1}{x(x-2)(x-3)}$$

$$19. \frac{x^2-1}{x(x-2)^2}$$

$$20. \frac{x}{x+1}$$

$$21. \frac{(x+1)(x-1)(x+2)^2}{x-1}$$

$$22. \frac{\sin(x)}{\cos(x)}$$

Partial fraction

Proper rational function, but not a partial fraction

Improper rational function

Not rational function



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$$23. \frac{\sqrt{x^2 - 1}}{x(x - 2)^2}$$

$$24. \frac{x^{2/3}}{(x + 1)(x + 2)^2}$$

$$25. \frac{6 - x}{x^2 + 3x + 9}$$

$$26. \frac{1}{x^2 + 1}$$

$$27. \frac{2x + 1}{(x + 1)^2}$$

Partial fraction

Proper rational function, but not a partial fraction

Improper rational function

Not rational function

3. Test2

Quiz

- Divide polynomials and write the improper rational function as sum of a polynomial and a proper rational function.
- Write the quotient (a polynomial) into the white field and the remainder (the numerator of the proper function) into the yellow field.

$$1. \frac{x^2 + 2x + 1}{x + 1} = \boxed{} + \frac{\boxed{}}{x + 1}$$

$$2. \frac{x^2}{x + 2} = \boxed{} + \frac{\boxed{}}{x + 2}$$

$$3. \frac{x^3 + x + 1}{x^2 + 2} = \boxed{} + \frac{\boxed{}}{x^2 + 2}$$

$$4. \frac{x^2 + 4x + 1}{x - 2} = \boxed{} + \frac{\boxed{}}{x - 2}$$

$$5. \frac{x^4 + 3x^2 + 4x + 5}{x^2 + 1} = \boxed{} + \frac{\boxed{}}{x^2 + 1}$$