# Integration by parts

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Look at three or four or twenty my quizzes
and then fill in my questionnaire please!

# 1. Theory

The formula for integration by parts reads

$$\int u(x)v'(x)\mathrm{d}x = u(x)v(x) - \int u'(x)v(x)\mathrm{d}x.$$

Let $P(x)$ be a polynomial. Typically we use the integration by parts in the evaluation of the integrals like

$$\int P(x)e^{\alpha x+\beta}\mathrm{d}x, \int P(x)\sin(\alpha x+\beta)\mathrm{d}x, \int P(x)\cos(\alpha x+\beta)\mathrm{d}x,$$

and

$$\int P(x)\operatorname{atan} x\mathrm{d}x, \int P(x)\ln^m x\mathrm{d}x,$$

where $\alpha, \beta \in \mathbb{R}$ and $m \in \mathbb{N}$. In the first set of the integrals we choose the functions $u$ and $v$ such that the polynomial function is differentiated and the exponential (or trigonometric) function is integrated. In the second set of the integrals we choose the functions $u$ and $v$ such that the functions $\operatorname{atan} x$ or $\ln x$ are differentiated and the polynomial is integrated.

## 2. Multiple choice test

Quiz 1. You have to answer whether the integral is suitable for integration by parts.

| | | Not suitable for integratino by parts. | By parts, the red part is $u$ and will be differentiated. | By parts, the red part is $v'$ and will be integrated. |
|---|---|---|---|---|
| **1.** | $\int (x+1)\, e^{x} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **2.** | $\int (x+1)\, \ln(x) \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **3.** | $\int (x+1)\, \sin(x) \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **4.** | $\int (x+1)\, e^{x^2} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **5.** | $\int x^2\, e^{2x} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **6.** | $\int (x^2-1)\, \operatorname{atan}(x+1) \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **7.** | $\int (x^3-2)\, e^{x^2+x} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **8.** | $\int e^{-x^2} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **9.** | $\int x\, e^{x^2} \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **10.** | $\int (3x+1)\, e^{-x+1} \mathrm{d}x$ | ☐ | ☐ | ☐ |

| | Not suitable for integratino by parts. | By parts, the red part is $u$ and will be differentiated. | By parts, the red part is $v'$ and will be integrated. |
|---|---|---|---|
| **11.** $\int x^2 e^x \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **12.** $\int (x+4)\ \operatorname{atan}\frac{x}{2}\mathrm{d}x$ | ☐ | ☐ | ☐ |
| **13.** $\int x\ \sin x^2 \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **14.** $\int x^2\ \ln x \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **15.** $\int \operatorname{atan} x \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **16.** $\int x \ln x\ \cos x \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **17.** $\int x\ \cos^3 x \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **18.** $\int (2+x)\ \cos(2x) \mathrm{d}x$ | ☐ | ☐ | ☐ |
| **19.** $\int (x^3-1)\ \sin\left(\frac{\pi}{2}-x\right) \mathrm{d}x$ | ☐ | ☐ | ☐ |

# 3. Test 1

- Integrate by parts. Use zero constant of integration when evaluating $v'(x)$ from $v(x)$.
- As usual, you can see the answer by pressing [?] button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these quizzes. The problems on exam are harder[1]!
- As usual: If you have any comments or suggestions concerning this test, let me know, please!

Quiz 2.

**1.** $\int \ln(x)\mathrm{d}x =$ $u =$ [ ] $u' =$ [ ] $v' =$ [ ] $v =$ [ ] [?]

$= $ [ ] $- \int$ [ ] $\mathrm{d}x$

$= $ [ ] [?]

[1] this means slightly longer computation of derivatives and integrals and so on

**2.** $\int xe^x \mathrm{d}x =$ $u =$ $u' =$ $v' =$ $v =$ ?

$=$ $-\int$ $\mathrm{d}x$

$=$ ?

**3.** $\int x \ln(x+1) \mathrm{d}x =$ $u =$ $u' =$ $v' =$ $v =$ ?

$=$ $-\int$ $\mathrm{d}x$

$=$ ?

**4.** $\int \operatorname{atan}(x)\,\mathrm{d}x =$ $u =$ [ ] $u' =$ [ ] $v' =$ [ ] $v =$ [ ] ?

$=$ [ ] $-\int$ [ ] $\mathrm{d}x$

$=$ [ ] ?

**5.** $\int (x+1)e^{-x}\,\mathrm{d}x =$ $u =$ [ ] $u' =$ [ ] $v' =$ [ ] $v =$ [ ] ?

$=$ [ ] $-\int$ [ ] $\mathrm{d}x$

$=$ [ ] ?

**6.** $\int (x-1)\sin(x)\mathrm{d}x =$ $u =$ $u' =$ $v' =$ $v =$ ?

$=$ $-\int$ $\mathrm{d}x$

$=$ ?

**7.** $\int (x-2)\cos(2x)\mathrm{d}x =$ $u =$ $u' =$ $v' =$ $v =$ ?

$=$ $-\int$ $\mathrm{d}x$

$=$ ?

**8.** $\int x^4 \ln x \, \mathrm{d}x =$

$u =$ $u' =$

$v' =$ $v =$ ?

$=$ $- \int$ $\mathrm{d}x$

$=$ ?

**9.** $\int x \frac{1}{\cos^2 x} \mathrm{d}x =$

$u =$ $u' =$

$v' =$ $v =$ ?

$=$ $- \int$ $\mathrm{d}x$

$=$ ?

# 4. Test 2

Quiz 3.

- Questions (integrals) in this quiz are on the following pages, one integral per page.
- In this test two computations by parts are required to evaluate the inetgral.
- The second pattern is disclosed after correct responses to the first integration by parts. As in the preceeding test, use zero constant of integration when evaluate $v$.

**1.** $I_1 = \int x^2 e^x \mathrm{d}x =$

$u =$ [ ] $u' =$ [ ]

$v' =$ [ ] $v =$ [ ] ?

$=$ [ ] ? $- \int$ [ ] ? $\mathrm{d}x$

??????????????????

**2.** $I_2 = \int (x^2 + x - 1)e^{-x}\,\mathrm{d}x =$

$u =$ [ ] $u' =$ [ ]

$v' =$ [ ] $v =$ [ ] ?

$=$ [ ] ? $- \int$ [ ] ? $\mathrm{d}x$

??????????????????

**3.** $I_3 = \int (x^2 - 1)\cos(x)\mathrm{d}x =$

$u =$ [ ] $u' =$ [ ]

$v' =$ [ ] $v =$ [ ] ?

$=$ [ ] ? $- \int$ [ ] ? $\mathrm{d}x$

??????????????????

**4.** $I_4 = \int x^2 e^{2x} \mathrm{d}x =$ $u =$ [ ] $u' =$ [ ] $v' =$ [ ] $v =$ [ ] ?

$=$ [ ] ? $- \int$ [ ] ? $\mathrm{d}x$

??????????????????

**5.** $I_5 = \int x \ln^2 x \mathrm{d}x =$

$u =$ $u' =$

$v' =$ $v =$ ?

$=$ ? $- \int$ ? $\mathrm{d}x$

??????????????????

**6.** $I_6 = \int (1 - x^2) \sin(x) \mathrm{d}x =$ $u =$ [ ] $u' =$ [ ] $v' =$ [ ] $v =$ [ ] ?

$=$ [ ] ? $- \int$ [ ] ? $\mathrm{d}x$

??????????????????