

Integration by parts

Robert Mařík

July 23, 2006

Look at three or four or twenty my quizzes
and then fill in my please!



ROBERT MAŘÍK
Integration by parts
file int-parts.tex

Theory

Multiple ...

Test 1

Test 2

Home Page

Print

Title Page

◀◀

▶▶

◀

▶

Page 1 of 16

Go Back

Full Screen

Close

Quit



1. Theory

The formula for integration by parts reads

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx.$$

Let $P(x)$ be a polynomial. Typically we use the integration by parts in the evaluation of the integrals like

$$\int P(x)e^{\alpha x + \beta}dx, \int P(x)\sin(\alpha x + \beta)dx, \int P(x)\cos(\alpha x + \beta)dx,$$

and

$$\int P(x)\operatorname{atan} x dx, \int P(x)\ln^m x dx,$$

where $\alpha, \beta \in \mathbb{R}$ and $m \in \mathbb{N}$. In the first set of the integrals we choose the functions u and v such that the polynomial function is differentiated and the exponential (or trigonometric) function is integrated. In the second set of the integrals we choose the functions u and v such that the functions $\operatorname{atan} x$ or $\ln x$ are differentiated and the polynomial is integrated.



2. Multiple choice test

Quiz 1. You have to answer whether the integral is suitable for integration by parts.

1. $\int (x + 1) e^x dx$

2. $\int (x + 1) \ln(x) dx$

3. $\int (x + 1) \sin(x) dx$

4. $\int (x + 1) e^{x^2} dx$

5. $\int x^2 e^{2x} dx$

6. $\int (x^2 - 1) \operatorname{atan}(x + 1) dx$

7. $\int (x^3 - 2) e^{x^2+x} dx$

8. $\int e^{-x^2} dx$

9. $\int x e^{x^2} dx$

10. $\int (3x + 1) e^{-x+1} dx$

Not suitable for integratio by parts.

By parts, the red part is u and will be differentiated.

By parts, the red part is v' and will be integrated.



ROBERT MAŘÍK

Integration by parts

file int-parts.tex

Theory

Multiple ...

Test 1

Test 2

Home Page

Print

Title Page



Page 4 of 16

Go Back

Full Screen

Close

Quit

11. $\int x^2 e^x dx$

12. $\int (x + 4) \operatorname{atan} \frac{x}{2} dx$

13. $\int x \sin x^2 dx$

14. $\int x^2 \ln x dx$

15. $\int \operatorname{atan} x dx$

16. $\int x \ln x \cos x dx$

17. $\int x \cos^3 x dx$

18. $\int (2 + x) \cos(2x) dx$

19. $\int (x^3 - 1) \sin \left(\frac{\pi}{2} - x \right) dx$

Not suitable for integratio by parts.

By parts, the red part is u and will be differentiated.

By parts, the red part is v' and will be integrated.



3. Test 1

- Integrate by parts. Use zero constant of integration when evaluating $v'(x)$ from $v(x)$.
- As usual, you can see the answer by pressing button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these quizzes. The problems on exam are harder¹!
- As usual: If you have any comments or suggestions concerning this test, let me know, please!

Quiz 2.

1. $\int \ln(x) dx =$

$u =$	$u' =$
$v' =$	$v =$

 dx

$=$ $-\int$

$=$

¹this means slightly longer computation of derivatives and integrals and so on

$$2. \int xe^x dx =$$

$$u =$$

$$u' =$$

$$v' =$$

$$v =$$

=

$-\int$

dx

=

$$3. \int x \ln(x+1) dx =$$

$$u =$$

$$u' =$$

$$v' =$$

$$v =$$

=

$-\int$

dx

=



ROBERT MAŘÍK

Integration by parts

file int-parts.tex

Theory

Multiple...

Test 1

Test 2

Home Page

Print

Title Page

◀▶

◀▶

Page 6 of 16

Go Back

Full Screen

Close

Quit

$$4. \int \arctan(x) dx =$$

$$u =$$

$$u' =$$

$$v' =$$

$$v =$$

=

$-\int$

dx

=

$$5. \int (x+1)e^{-x} dx =$$

$$u =$$

$$u' =$$

$$v' =$$

$$v =$$

=

$-\int$

dx

=



ROBERT MAŘÍK

Integration by parts

file int-parts.tex

Theory

Multiple ...

Test 1

Test 2

Home Page

Print

Title Page

◀▶

◀▶

Page 7 of 16

Go Back

Full Screen

Close

Quit

$$6. \int (x - 1) \sin(x) dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int \quad - \int \quad dx$$

=

$$7. \int (x - 2) \cos(2x) dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int \quad - \int \quad dx$$

=



ROBERT MAŘÍK
Integration by parts
file int-parts.tex

Theory

Multiple ...

Test 1

Test 2

Home Page

Print

Title Page

◀▶

◀▶

Page 8 of 16

Go Back

Full Screen

Close

Quit

$$8. \int x^4 \ln x dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \quad - \int \quad dx$$

=

$$9. \int x \frac{1}{\cos^2 x} dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \quad - \int \quad dx$$

=





4. Test 2

Quiz 3.

- Questions (integrals) in this quiz are on the following pages, one integral per page.
- In this test two computations by parts are required to evaluate the inetgral.
- The second pattern is disclosed after correct responses to the first integration by parts. As in the preceeding test, use zero constant of integration when evaluate v .

$$1. I_1 = \int x^2 e^x dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int (x^2 e^x - 2 \int x e^x dx) dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$\Rightarrow \begin{array}{|l} u = \\ v' = \end{array} \begin{array}{|l} u' = \\ v = \end{array}$$

$$I_1 = x^2 e^x - 2 \left(\int x e^x dx \right)$$

=



$$2. I_2 = \int (x^2 + x - 1)e^{-x} dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int (x^2 + x - 1)e^{-x} dx - \int (2x + 1)e^{-x} dx$$

$$= -e^{-x}(x^2 + x - 1) + \int (2x + 1)e^{-x} dx$$

$$\Rightarrow \begin{array}{|l} u = \quad \quad \quad u' = \\ v' = \quad \quad \quad v = \end{array}$$

$$I_2 = -e^{-x}(x^2 + x - 1) + \left(\int (2x + 1)e^{-x} dx - \int (2x + 1)e^{-x} dx \right)$$

=



ROBERT MAŘÍK

Integration by parts

file int-parts.tex

Theory

Multiple ...

Test 1

Test 2

Home Page	
Print	
Title Page	
◀◀	▶▶
◀	▶
Page 12 of 16	
Go Back	
Full Screen	
Close	
Quit	

$$3. I_3 = \int (x^2 - 1) \cos(x) dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int (x^2 - 1) \cos(x) dx - \int x \sin(x) dx$$

$$= (x^2 - 1) \sin x - 2 \int x \sin x dx$$

$$\Rightarrow \begin{array}{|l} u = \quad \quad \quad u' = \\ v' = \quad \quad \quad v = \end{array}$$

$$I_3 = (x^2 - 1) \sin x - 2 \left(\int x \sin x dx - \int \sin x dx \right)$$

=



$$4. I_4 = \int x^2 e^{2x} dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \quad - \int \quad dx$$

$$= \frac{x^2}{2} e^{2x} - \int x e^{2x} dx$$

$$\Rightarrow$$

$u =$	$u' =$
$v' =$	$v =$

$$I_4 = \frac{x^2}{2} e^{2x} - \left(\quad - \int \quad dx \right)$$

=



$$5. I_5 = \int x \ln^2 x dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= \int x \ln^2 x - \int x \ln(x) dx \quad dx$$

$$= \frac{x^2}{2} \ln^2 x - \int x \ln(x) dx$$

$$\Rightarrow$$

$u =$	$u' =$
$v' =$	$v =$

$$I_5 = \frac{x^2}{2} \ln^2 x - \left(\int x \ln(x) dx \right)$$

=



$$6. I_6 = \int (1 - x^2) \sin(x) dx =$$

$u =$	$u' =$
$v' =$	$v =$

$$= - \int \quad dx$$

$$= -(1 - x^2) \cos(x) - 2 \int x \cos x dx$$

$$\Rightarrow \begin{array}{|l} u = \quad \quad \quad u' = \\ v' = \quad \quad \quad v = \end{array}$$

$$I_6 = -(1 - x^2) \cos(x) - 2 \left(- \int \quad dx \right)$$

=

