# Integration by parts

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Look at three or four or twenty my quizzes and then fill in my please!





$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx.$$

Let P(x) be a polynomial. Typically we use the integration by parts in the evaluation of the integrals like

$$\int P(x)e^{\alpha x+\beta} dx, \int P(x)\sin(\alpha x+\beta) dx, \int P(x)\cos(\alpha x+\beta) dx,$$

and

$$\int P(x) \operatorname{atan} x dx$$
,  $\int P(x) \ln^m x dx$ ,

where  $\alpha, \beta \in \mathbb{R}$  and  $m \in \mathbb{N}$ . In the first set of the integrals we choose the functions u and v such that the polynomial function is differentiated and the exponential (or trigonometric) function is integrated. In the second set of the integrals we choose the functions u and v such that the functions u are differentiated and the polynomial is integrated.



Multiple . . .

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## **Multiple choice test**

Quiz 1. You have to answer whether the integral is suitable for integration by parts.

Not suitable for integratino by parts

- $1. \int (x+1) e^x dx$
- $2. \int (x+1) \ln(x) dx$
- $3. \int (x+1) \sin(x) dx$
- **4.**  $\int (x+1) e^{x^2} dx$
- **5.**  $\int x^2 e^{2x} dx$
- **6.**  $\int (x^2 1) \arctan(x + 1) dx$
- 7.  $\int (x^3 2) e^{x^2 + x} dx$
- **8.**  $\int e^{-x^2} dx$
- $9. \int x e^{x^2} dx$
- **10.**  $\int (3x+1)e^{-x+1}dx$

By parts, the red part is v' and will be integrated

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11. 
$$\int x^2 e^x dx$$
12. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
13. 
$$\int x \sin x^2 dx$$
14. 
$$\int x^2 \ln x dx$$
15. 
$$\int \arctan x \cos x dx$$
16. 
$$\int x \ln x \cos x dx$$
17. 
$$\int x \cos^3 x dx$$
18. 
$$\int (2+x) \cos(2x) dx$$
19. 
$$\int (x^3-1) \sin \left(\frac{\pi}{2}-x\right) dx$$
19. 
$$\int (x^3-1) \sin \left(\frac{\pi}{2}-x\right) dx$$
10. 
$$\int (x^3-1) \sin \left(\frac{\pi}{2}-x\right) dx$$
11. 
$$\int (x^3-1) \sin \left(\frac{\pi}{2}-x\right) dx$$
12. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
13. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
14. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
15. 
$$\int \arctan \frac{x}{2} dx$$
16. 
$$\int x \ln x \cos x dx$$
17. 
$$\int x \cos^3 x dx$$
18. 
$$\int (2+x) \cos(2x) dx$$
19. 
$$\int (x^3-1) \sin \left(\frac{\pi}{2}-x\right) dx$$
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14. 
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17. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
18. 
$$\int (x+4) \arctan \frac{x}{2} dx$$
19. 
$$\int (x+4) \arctan \frac{x}{2} dx$$

- Integrate by parts. Use zero constant of integration when evaluating v'(x) from v(x).
- As usual, you can see the answer by pressing button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these guizzes. The problems on exam are harder<sup>1</sup>!
- As usual: If you have any comments or suggestions concerning this test, let me know, please!

#### Quiz 2.

 $\mathbf{1.} \int \ln(x) dx = \begin{vmatrix} u = \\ v' = \end{vmatrix}$ v = Theory

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dx





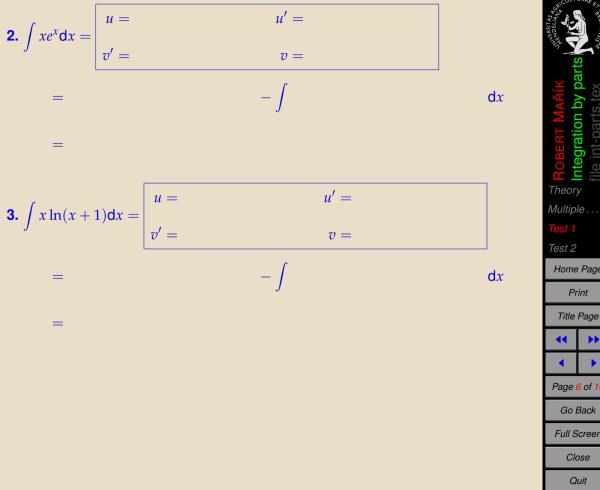


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<sup>&</sup>lt;sup>1</sup>this means slightly longer computation of derivatives and integrals and so on



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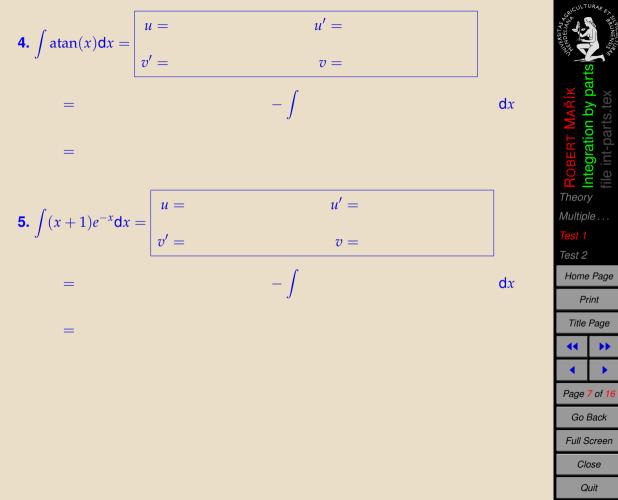


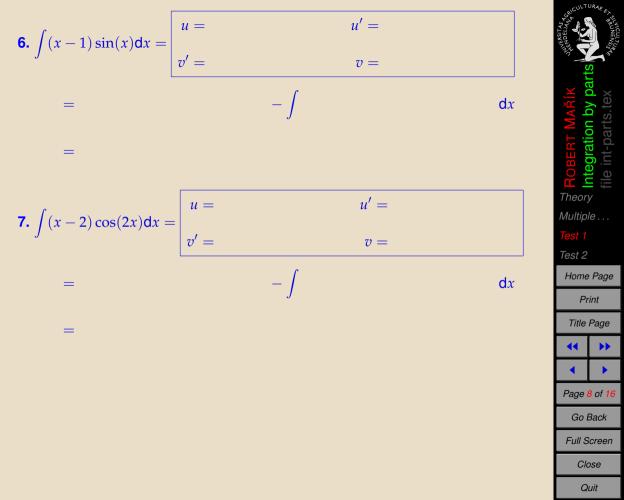


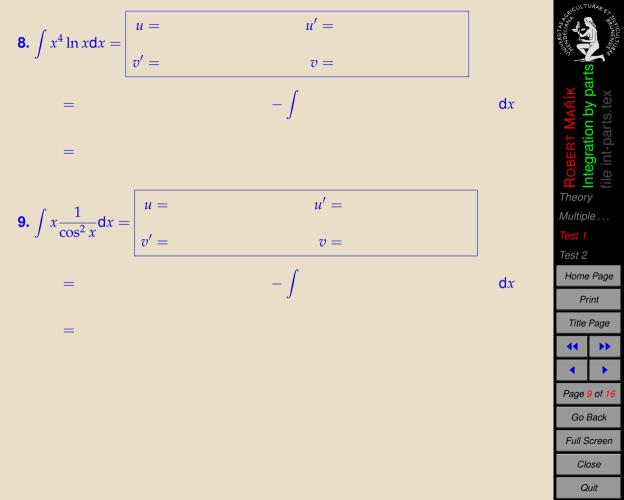
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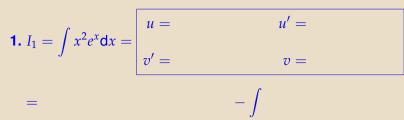


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#### Quiz 3.

- Questions (integrals) in this quiz are on the following pages, one integral per page.
- In this test two computations by parts are required to evaluate the inetgral.
- The second pattern is disclosed after correct responses to the first integration by parts. As in the preceding test, use zero constant of integration when evaluate v.





$$= x^2 e^x - 2 \int x e^x dx$$

$$I_1 = x^2 e^x - 2\left( -\int dx \right)$$



dx

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2. 
$$I_2 = \int (x^2 + x - 1)e^{-x} dx = \begin{bmatrix} u = & u' = \\ v' = & v = \end{bmatrix}$$

$$= -\int dx$$

$$= -e^{-x}(x^2 + x - 1) + \int (2x + 1)e^{-x} dx$$

$$I_2 = -e^{-x}(x^2 + x - 1) + \left( -\int_{-\infty}^{\infty} dx - \int_{-\infty}^{\infty} dx - \int_{-\infty}^{\infty}$$

dx

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3. 
$$I_3 = \int (x^2 - 1)\cos(x)dx = \begin{bmatrix} u = & u' = \\ v' = & v = \end{bmatrix}$$

$$= -\int dx$$

$$= (x^2 - 1)\sin x - 2 \int x \sin x dx$$

$$I_3 = (x^2 - 1)\sin x - 2\left( -\int dx \right)$$

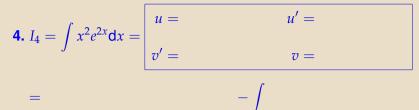


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$$=\frac{x^2}{2}e^{2x}-\int xe^{2x}\mathrm{d}x$$

$$=$$
  $u' =$ 

$$\Rightarrow \begin{array}{c} u = & u' = \\ v' = & v = \end{array}$$

$$I_4 = \frac{x^2}{2}e^{2x} - \left($$

$$-\int$$
 dx



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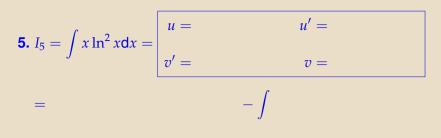
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$$= \frac{x^2}{2} \ln^2 x - \int x \ln(x) dx$$

$$\begin{array}{ccc}
u &= & u' &= \\
v' &= & v &= \\
\end{array}$$

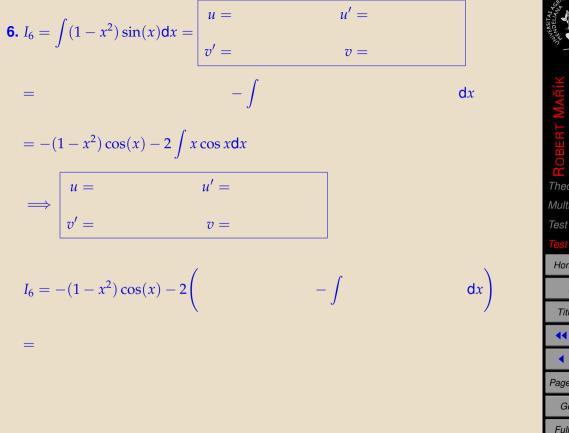
$$I_5 = \frac{x^2}{2} \ln^2 x - \left( -\int \right)$$



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dx

dx



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