Integral calculus
Decomposition into partial fractions
Interactive quizzes

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Look at three or four or twenty my quizzes and then fill in my questionnaire please!

1. Decomposition into partial fractions

Let $R(x) = \dfrac{P_n(x)}{Q_m(x)}$ be a proper rational function. Suppose that the polynomials $P_n(x)$ and $Q_m(x)$ have no common zeros and the polynomial $Q_m(x)$ has only real zeros or complex zeros of the multiplicity one.

- Let us assign to each simple real root c of the polynomial $Q_m(x)$ the fraction
$$\frac{A}{x-c},$$
where A is some (not yet determined) real constant.

- Let us assign to each real root c of the multiplicity k of the polynomial $Q_m(x)$ the k-tuple of the fractions
$$\frac{A_1}{x-c}, \frac{A_2}{(x-c)^2}, \dots, \frac{A_k}{(x-c)^k},$$
where A_i are some (not yet determined) real constants.

- For each pair of the mutually conjugate complex roots of the polynomial $Q_m(x)$ the factorization of the polynomial $Q_m(x)$ contains the factor of the type $(x^2 + Mx + N)$. Let us assign to this pair of complex roots the fraction
$$\frac{Bx + C}{x^2 + Mx + N},$$
where B and C are some (not yet determined) real numbers.

Then for a convenient particular choice of the not constants A, A_i, B, C, …the function $R(x)$ can be written as a sum of all of the fractions, considered above. This decomposition is unique up to the order in the sum.

Definition 1 (partial fractions) *The fractions presented in the preceding theorem are called partial fractions.*

2. Test

- Find formal decomposition (with undetermined constants) of proper rational functions.
- Write partial fractions as comma separated unordered list. Use consecutively constants A, B, C ....
- For the function $\dfrac{x}{(x-1)^2(x+1)}$ *correct answers* include
  - `A/(x+1) , B/(x-1) , C/(x-1)^2`
  - `B/(x+1) , A/(x-1) , C/(x-1)^2`
  - `A/(x-1) , B/(x+1) , C/(x-1)^2`

  and *incorrect answers* include
  - `A/(x+1) , B/(x-1) , D/(x-1)^2`
    (use C as the third constant – not D)
  - `B/(x+1) , A/(x-1) , C/(x-1)`
    (incorrect, of course)
  - `A/(x-1) , A/(x+1) , A/(x-1)^2`
    (you cannot use the same name for all constants)
- The green boundary indicates correct answer, the red boundary indicates wrong answer.
- As usual, you can see the answer by pressing ? button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these quizzes. The prob-

lems on exam are harder[1]! If there are more fields to be filled, click repeatedly.

- As usual: If you have any comments or suggestions concerning this test, let me know, please!

Quiz

1. $\dfrac{x}{(x-1)^2(x+1)} \Rightarrow$?

2. $\dfrac{1}{(x-1)^2(x+1)} \Rightarrow$?

3. $\dfrac{2x-7}{(x-1)^2(x+1)} \Rightarrow$?

4. $\dfrac{2}{x^2(x-1)} \Rightarrow$?

5. $\dfrac{2x+1}{x^2(x-1)^2} \Rightarrow$?

6. $\dfrac{5x^2-1}{(x^2+3)(x^2+1)} \Rightarrow$?

[1] this means slightly longer computation of derivatives and integrals and so on

7. $\dfrac{x^3-1}{x(x+2)^4} \Rightarrow$?

8. $\dfrac{5}{(x^2+3)(x-1)^2} \Rightarrow$?

9. $\dfrac{x}{x^2-1} \Rightarrow$?

10. $\dfrac{1}{x^3+x} \Rightarrow$?

11. $\dfrac{x^2+1}{(x-3)^2(x+1)} \Rightarrow$?

12. $\dfrac{3x^2-9}{x(x+6)^3} \Rightarrow$?