Integral calculus
Integration of partial fracions
Interactive quizzes

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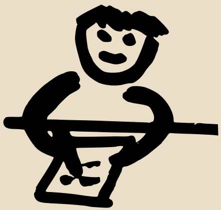
July 23, 2006

Look at three or four or twenty my quizzes and then fill in my questionnaire please!



- In this file we divide all partial fractions into several classes and explain a method how to integrate the functions from these classes.
- Fill in blank fields and press Enter.
- The green boundary indicates correct answer, the red boundary indicates wrong answer.
- As usual, you can see the answer by pressing ? button. But don't use this button too much, please. All (or at least almost all) computations are easy. We have to learn the technique in these quizzes. The problems on exam are harder[1]! If there are more fields to be filled, click repeatedly.
- As usual: If you have any comments or suggestions concerning this test, let me know, please!

[1] this means slightly longer computation of derivatives and integrals and so on

1. Type A

Quiz The partial fraction of the type $\frac{A}{x-c}$ can be simply integrated by the formula

$$\int \frac{A}{x-c}\,\mathrm{d}x = A\ln(|x-c|) + C.$$

1. $\int \frac{4}{x+3}\,\mathrm{d}x = \boxed{} + C$?

2. $\int \frac{3}{x-7}\,\mathrm{d}x = \boxed{} + C$?

3. $\int \frac{5}{x+9}\,\mathrm{d}x = \boxed{} + C$?

4. $\int \frac{10}{x+6}\,\mathrm{d}x = \boxed{} + C$?

5. $\int \frac{5}{x+1}\,\mathrm{d}x = \boxed{} + C$?

6. $\int \frac{-1}{x+3}\mathrm{d}x =$ ______ $+ C$?

7. $\int \frac{5}{x-9}\mathrm{d}x =$ ______ $+ C$?

8. $\int \frac{7}{x-4}\mathrm{d}x =$ ______ $+ C$?

9. $\int \frac{3}{x+2}\mathrm{d}x =$ ______ $+ C$?

10. $\int \frac{9}{x}\mathrm{d}x =$ ______ $+ C$?

11. $\int \frac{8}{x-2}\mathrm{d}x =$ ______ $+ C$?

12. $\int \frac{5}{x-4}\mathrm{d}x =$ ______ $+ C$?

13. $\int \frac{6}{x+\sqrt{2}}\mathrm{d}x =$ ______ $+ C$?

2. Type B

Quiz The partial fraction od the type $\frac{A}{(x-c)^n}$, $n > 1$ can be simply integrated by the formula

$$\int \frac{A}{(x-c)^n}\mathrm{d}x = \int A(x-c)^{-n}\mathrm{d}x = A\frac{(x-c)^{-n+1}}{-n+1} = \frac{A}{(1-n)(x-c)^{n-1}} + C.$$

1. $\int \frac{1}{(x+5)^2}\mathrm{d}x =$ ______ $+C$?
2. $\int \frac{5}{(x-2)^3}\mathrm{d}x =$ ______ $+C$?
3. $\int \frac{6}{(x-1)^7}\mathrm{d}x =$ ______ $+C$?
4. $\int \frac{6}{(x+5)^3}\mathrm{d}x =$ ______ $+C$?
5. $\int \frac{5}{x^2}\mathrm{d}x =$ ______ $+C$?

6. $\int \frac{12}{x^3}\mathrm{d}x =$ ____ $+C$?

7. $\int \frac{12}{x^2}\mathrm{d}x =$ ____ $+C$?

8. $\int \frac{12}{x^4}\mathrm{d}x =$ ____ $+C$?

9. $\int \frac{1}{(x+2)^2}\mathrm{d}x =$ ____ $+C$?

10. $\int \frac{1}{(x+1)^5}\mathrm{d}x =$ ____ $+C$?

11. $\int \frac{3}{(x-1)^3}\mathrm{d}x =$ ____ $+C$?

3. Type C

The partial fracion of the type $\dfrac{Ax+B}{x^2+\beta^2}$ can be integated as follows: We write the fraction as linear combination of two *special* fractions. The numerator of the first fraction is $2x$ (the derivative of denominator) and the numerator of the second fraction is the number 1. Then the first fraction can be integrated by the formula

$$\int \frac{f'(x)}{f(x)}\mathrm{d}x = \ln|x| + C$$

and the second one by the formula

$$\int \frac{1}{x^2+\beta^2}\mathrm{d}x = \frac{1}{\beta}\operatorname{atan}\frac{x}{\beta} + C$$

Quiz Write the function in the form suggested above and integate. Write the coeficients of linear combination (numbers) into blue fields and the antiderivative into the long white field.

1. $$\int \frac{3x+7}{x^2+9}\,\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+9} + \boxed{}\,\frac{1}{x^2+9}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

2. $$\int \frac{5x-2}{x^2+25}\,\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+25} + \boxed{}\,\frac{1}{x^2+25}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

3. $$\int \frac{x+1}{x^2+4}\,\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+4} + \boxed{}\,\frac{1}{x^2+4}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

4. $$\int \frac{4x-6}{x^2+3}\,\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+3} + \boxed{}\,\frac{1}{x^2+3}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

5. $$\int \frac{7x+1}{x^2+5}\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+5} + \boxed{}\,\frac{1}{x^2+5}\mathrm{d}x$$

$$= \boxed{} + C$$

?

6. $$\int \frac{4-3x}{x^2+9}\mathrm{d}x = \int \boxed{}\,\frac{2x}{x^2+9} + \boxed{}\,\frac{1}{x^2+9}\mathrm{d}x$$

$$= \boxed{} + C$$

?

4. Type D

The partial fracion of the type $\dfrac{Ax+B}{x^2+\beta x+\gamma}$ can be integated as follows:

We write the fraction as linear combination of two *special* fractions. The numerator of the first fraction is $2x+\beta$ (the derivative of denominator) and the numerator of the second fraction is the number 1. Then the first fraction can be integrated by the formula

$$\int \frac{f'(x)}{f(x)}\mathrm{d}x = \ln|x| + C$$

and the second one by the formula

$$\int \frac{1}{(x+m)^2+n^2}\mathrm{d}x = \frac{1}{n}\operatorname{atan}\frac{x+m}{n} + C,$$

where m and n have to be found by completing square in the denominator.

Quiz

- Complete the pattern for the integration and find the antiderivative.
- The answer in the blue field is a number.
- The answer in the red field is derivative of denominator.
- The answer in the white field is the antiderivative. Omit the constant of integration, please.

1. $$\int \frac{x}{x^2+2x+2}\,\mathrm{d}x = \int \boxed{}\,\frac{\boxed{}}{x^2+2x+2} + \boxed{}\,\frac{1}{(x+\boxed{})^2+\boxed{}}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

2. $$\int \frac{2x+1}{x^2+4x+9}\,\mathrm{d}x = \int \boxed{}\,\frac{\boxed{}}{x^2+4x+9} + \boxed{}\,\frac{1}{(x+\boxed{})^2+\boxed{}}\,\mathrm{d}x$$

$$= \boxed{} + C$$?

3. $\displaystyle\int \frac{3x+1}{x^2-2x+9}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2-2x+9} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$

$= \square + C$?

4. $\displaystyle\int \frac{-5x-7}{x^2+8x+20}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2+8x+20} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$

$= \square + C$?

5. $\displaystyle\int \frac{x-1}{x^2-6x+10}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2-6x+10} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$

$= \square + C$?

6. $\displaystyle\int \frac{x-1}{x^2+x+1}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2+x+1} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$

$= \square + C$?

7. $$\int \frac{3x+7}{x^2+10x+29}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2+10x+29} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$$

$$= \square + C$$?

8. $$\int \frac{x-1}{x^2-4x+6}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2-4x+6} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$$

$$= \square + C$$?

9. $$\int \frac{x+7}{x^2-4x+8}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2-4x+8} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$$

$$= \square + C$$?

10. $$\int \frac{x}{x^2-x+1}\,\mathrm{d}x = \int \square\,\frac{\square}{x^2-x+1} + \square\,\frac{1}{(x+\square)^2+\square}\,\mathrm{d}x$$

$$= \square + C$$?

11. $\displaystyle\int \frac{5x-6}{x^2+2x+10}\,\mathrm{d}x = \int \boxed{}\,\frac{\boxed{}}{x^2+2x+10} + \boxed{}\,\frac{1}{(x+\boxed{})^2+\boxed{}}\,\mathrm{d}x$

$\displaystyle = \boxed{} + C$?