Differential Equations and Social Diffusion

Robert Mařík

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Social diffusion is the spreading of information in a population of constant size M. The number y of people who have this information at time x is supposed to be a continuous differentiable function.

The population is divided into two classes, those that have the information and those that do not. It is reasonable to assume that the spread of information, i.e. the rate of change y', is proportional to the number of people x who have this information and to the number of people who do not.

This gives the **mathematical formulation** for our problem: given numbers M and k, find the function $y = y(x)$ defined on $\mathbb{R}$ with values in the compact interval $[0, M]$ which satisfies the equation

$$y' = k \cdot y \cdot (M - y).$$

We can solve the equation

$$y' = k \cdot y \cdot (M - y).$$

by separating variables.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = ky(M-y)$$

$$\frac{1}{y(M-y)}\,\mathrm{d}y = k\,\mathrm{d}x$$

$$\frac{1}{M}\left(\frac{1}{y} + \frac{1}{M-y}\right)\mathrm{d}y = k\,\mathrm{d}x$$

$$\frac{1}{M}\int \frac{1}{y} + \frac{1}{M-y}\,\mathrm{d}y = \int k\,\mathrm{d}x,$$

$$\frac{1}{M}\Big(\ln y - \ln(M-y)\Big) = kx + c$$

$$\ln\frac{y}{M-y} = M(kx+c)$$

$$\frac{y}{M-y} = e^{M(kx+c)}$$

$$\frac{y}{M-y} = e^{Mc}e^{Mkx}$$

$$\frac{y}{M-y} = Ce^{Mkx}$$

$$y = MCe^{Mkx} - yCe^{Mkx}$$

$$y(1+Ce^{Mkx}) = MCe^{Mkx}$$

$$y = \frac{Ce^{Mkx}}{1+Ce^{Mkx}}M$$

where $C = e^{Mc}$ is a positive constant.

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