Definition (expanded set of the real numbers)**.** Under *an expanded set of the real numbers* $\mathbb{R}^*$ we understand the set $\mathbb{R}$ of the real numbers enriched by the numbers $\pm\infty$ in the following way: We set $\mathbb{R}^* = \mathbb{R} \cup \{\infty, -\infty\}$ and for $a \in \mathbb{R}$ we set:

$$a+\infty=\infty, \quad a-\infty=-\infty, \quad \infty+\infty=\infty, \quad -\infty-\infty=-\infty$$

$$\infty.\infty=-\infty.(-\infty)=\infty, \qquad \infty.(-\infty)=-\infty, \qquad \frac{a}{\infty}=\frac{a}{-\infty}=0$$

$$-\infty<a<\infty, \qquad |\pm\infty|=\infty.$$

Further, for $a>0$ we set $\quad a.\infty=\infty \quad a.(-\infty)=-\infty,$

and for $a<0$ we set $\quad a.\infty=-\infty \quad a.(-\infty)=\infty.$

Another operations we define with the commutativity of the operation "$+$" and "$.$".

Remark 1 (indeterminate forms)**.** The operations "$\infty-\infty$", "$\pm\infty.0$" and "$\frac{\pm\infty}{\pm\infty}$" remain undefined. Of course, the division by a zero remains undefined as well.

Theorem 1 (uniqueness of the limit)**.** *The function f possesses at the point a at most one limit (or one-sided limit).*

Theorem 2 (the relationship between the limit and the one-sided limits)**.** *The limit of the function f at the point $a \in \mathbb{R}$ exists if and only if both one-sided limits at the point a exist and are equal. More precisely: If the limits $f(a-)$ and $f(a+)$ exist and $f(a-)=f(a+)$, then the limit $f(a\pm)$ exists as well and $f(a\pm)=f(a-)=f(a+)$. If one of the one-sided limits does not exist or if $f(a-)\neq f(a+)$, then the limit $f(a\pm)$ does not exist.*

Theorem 3 (algebra of limits)**.** *Let $a \in \mathbb{R}^*$, $f,g:\mathbb{R}\to\mathbb{R}$. The following relations hold whenever the limits on the right exist and the formula on the right is well-defined.*

$$\lim_{x\to a}(f(x)\pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) \tag{1}$$

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) \tag{2}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \tag{3}$$

The same holds for one sided limits as well.

Theorem 4 (limit of the composite function with continuous component)**.** *Let $\lim\limits_{x\to a} f(x)=b$ and $g(x)$ be a function continuous at b. Then $\lim\limits_{x\to a} g(f(x))=g(b)$, i.e.*

$$\lim_{x\to a} g(f(x)) = g(\lim_{x\to a} f(x)).$$

The same holds for one-sided limits as well.

Theorem 5 (limit of the composite function)**.** *Let $\lim\limits_{x\to a} f(x)=b$, $\lim\limits_{y\to b} g(y)=L$ and let there exist a ring neighborhood of the point $x=a$ such that $f(x)\neq b$ for all x in this neighborhood. Then $\lim\limits_{x\to a} g(f(x))=L$.*

Theorem 6 (limit of the type "$\frac{L}{0}$")**.** *Let $a\in\mathbb{R}^*$, $\lim\limits_{x\to a} g(x)=0$ and $\lim\limits_{x\to a} f(x)=L\in\mathbb{R}^*\setminus\{0\}$. Suppose that there exists ring neighborhood of the point a such that the function $g(x)$ does not change its sign in this neighborhood. Then*

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \begin{cases} +\infty & \text{if } g(x) \text{ and } L \text{ have common sign,} \\ -\infty & \text{if } g(x) \text{ and } L \text{ have an opposite sign,} \end{cases}$$

in the neighborhood under consideration. The same holds for one sided limits as well.

Theorem 7 (limit of the polynomial or of the rational functions at $\pm\infty$)**.** *It holds*

$$\lim_{x\to\pm\infty}(a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n) = \lim_{x\to\pm\infty} a_0x^n,$$

$$\lim_{x\to\pm\infty}\frac{a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n}{b_0x^m+b_1x^{m-1}+\cdots+b_{m-1}x+b_m} = \lim_{x\to\pm\infty}\frac{a_0}{b_0}x^{n-m}.$$