The partial derivatives

Robert Mařík

February 20, 2006

Contents

Remark. In all the following exercises the mixed partial derivatives $(z'_x)'_y$ and $(z'_y)'_x$ are identical and we make no difference between them.

Find derivatives of $z(x, y) = x^2 + xy + y^3$ up to the 2nd order.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0$$

We differentiate the sum $(x^2 + xy + y^3)$ with respect to x.

- x^2 is differentiated as the function of one variable.
- The variable y in the expression xy is considered to be a constant factor and we use the constant multiple rule

  $$(xy)'_x = y(x)'_x.$$

  The derivative of x with respect to x is the usual derivative.
- Term y^3 does not involve the variable x. Hence this term is treated to be constant and the derivative is zero.

Find derivatives of $z(x, y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$

We simplify.

Find derivatives of $z(x,y)=x^2+xy+y^3$ up to the 2nd order.

$$z'_x = 2x + 1\cdot y + 0 = 2x + y$$
$$z'_y = 0 + x\cdot 1 + 3y^2$$

We differentiate the sum (x^2+xy+y^3) with respect to y.

- x^2 is differentiated as a constant, since it does not involve the variable y.
- The variable x in the expression xy is considered to be a constant factor and we use the constant multiple rule $(xy)'_y = x(y)'_y$. The derivative of y with respect to y is the usual derivative.
- Term y^3 is differentiated as one-variable function.

Find derivatives of $z(x, y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$
$$z'_y = 0 + x \cdot 1 + 3y^2 = x + 3y^2$$

We simplify.

Find derivatives of $z(x,y)=x^2+xy+y^3$ up to the 2nd order.

$$z'_x = 2x + 1\cdot y + 0 = 2x + y$$
$$z'_y = 0 + x\cdot 1 + 3y^2 = x + 3y^2$$
$$z''_{xx} = (2x+y)'_x$$

We differentiate z'_x with respect to x

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$

$$z'_y = 0 + x \cdot 1 + 3y^2 = x + 3y^2$$

$$z''_{xx} = (2x + y)'_x = 2 \cdot 1 + 0$$

We use the sum rule, the constant multiple rule and the rule for the derivative of constant function.

Find derivatives of $z(x,y)=x^2+xy+y^3$ up to the 2nd order.

$$z'_x = 2x + 1\cdot y + 0 = 2x + y$$
$$z'_y = 0 + x\cdot 1 + 3y^2 = x + 3y^2$$
$$z''_{xx} = (2x+y)'_x = 2\cdot 1 + 0 = 2$$

We simplify.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$
$$z'_y = 0 + x \cdot 1 + 3y^2 = x + 3y^2$$
$$z''_{xx} = (2x + y)'_x = 2 \cdot 1 + 0 = 2$$
$$z''_{xy} = (2x + y)'_y$$

To find z''_{xy} we differentiate z'_x with respect to y.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$\begin{aligned}
z'_x &= 2x + 1 \cdot y + 0 = 2x + y \\
z'_y &= 0 + x \cdot 1 + 3y^2 = x + 3y^2 \\
z''_{xx} &= (2x + y)'_x = 2 \cdot 1 + 0 = 2 \\
z''_{xy} &= (2x + y)'_y = 0 + 1
\end{aligned}$$

We use the sum rule. Since x is treated to be a constant, $(2x)$ is constant as well.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$\begin{aligned}
z'_x &= 2x + 1 \cdot y + 0 = 2x + y \\
z'_y &= 0 + x \cdot 1 + 3y^2 = x + 3y^2 \\
z''_{xx} &= (2x + y)'_x = 2 \cdot 1 + 0 = 2 \\
z''_{xy} &= (2x + y)'_y = 0 + 1 = 1
\end{aligned}$$

We simplify.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$
$$z'_y = 0 + x \cdot 1 + 3y^2 = x + 3y^2$$
$$z''_{xx} = (2x + y)'_x = 2 \cdot 1 + 0 = 2$$
$$z''_{xy} = (2x + y)'_y = 0 + 1 = 1$$
$$z''_{yy} = (x + 3y^2)'_y$$

To find z''_{yy} we differentiate z'_y with respect to y.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$
\begin{aligned}
z'_x &= 2x + 1 \cdot y + 0 = 2x + y \\
z'_y &= 0 + x \cdot 1 + 3y^2 = x + 3y^2 \\
z''_{xx} &= (2x + y)'_x = 2 \cdot 1 + 0 = 2 \\
z''_{xy} &= (2x + y)'_y = 0 + 1 = 1 \\
z''_{yy} &= (x + 3y^2)'_y = 0 + 3 \cdot 2y^1
\end{aligned}
$$

We use the sum rule, the derivative of constant function, the constant multiple rule and the power rule.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$
\begin{aligned}
z'_x &= 2x + 1 \cdot y + 0 = 2x + y \\
z'_y &= 0 + x \cdot 1 + 3y^2 = x + 3y^2 \\
z''_{xx} &= (2x + y)'_x = 2 \cdot 1 + 0 = 2 \\
z''_{xy} &= (2x + y)'_y = 0 + 1 = 1 \\
z''_{yy} &= (x + 3y^2)'_y = 0 + 3 \cdot 2y^1 = 6y
\end{aligned}
$$

We simplify.

Find derivatives of $z(x,y) = x^2 + xy + y^3$ up to the 2nd order.

$$z'_x = 2x + 1 \cdot y + 0 = 2x + y$$

$$z'_y = 0 + x \cdot 1 + 3y^2 = x + 3y^2$$

$$z''_{xx} = (2x + y)'_x = 2 \cdot 1 + 0 = 2$$

$$z''_{xy} = (2x + y)'_y = 0 + 1 = 1$$

$$z''_{yy} = (x + 3y^2)'_y = 0 + 3 \cdot 2y^1 = 6y$$

All derivatives up to the second order have been found.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y) \cdot (e^{-x})'_x$$

- The function consists from the product of two factors $\boxed{z = (x+y) \cdot e^{-x}}$.
- Both factors involve the variable x and hence we differentiate by the product rule.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot (-1)$$

The usual rules are employed and the variable y is treated as a constant.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y)$$

We take out the repeating factor e^{-x}.

Find derivatives of $z(x,y) = (x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y) \cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y) \cdot e^{-x} \cdot (-1) = e^{-x} \cdot (1-x-y)$$
$$z'_y = (x+y)'_y \cdot e^{-x}$$

- We differentiate with respect to y. The function is a product of two factors $\boxed{z = (x+y) \cdot e^{-x}}$.
- The green expression does not involve the variable and it is considered to be constant. Hence we have a constant multiple of the function $(x+y)$ and work with the constant multiple rule.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$
\begin{aligned}
z'_x &= (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x \\
&= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y) \\
z'_y &= (x+y)'_y \cdot e^{-x} = (0+1)e^{-x}
\end{aligned}
$$

We use the sum rule, the variable x is considered to be a constant parameter.

Find derivatives of $z(x,y) = (x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y) \cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y) \cdot e^{-x} \cdot (-1) = e^{-x} \cdot (1-x-y)$$
$$z'_y = (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x}$$

We simplify.

Find derivatives of $z(x,y) = (x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y) \cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y) \cdot e^{-x} \cdot (-1) = e^{-x} \cdot (1-x-y)$$
$$z'_y = (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x}$$
$$z''_{xx} = e^{-x} \cdot (-1) \cdot (1-x-y) + e^{-x}(0-1-0)$$

- To find z''_{xx} we differentiate the first derivative z'_x with respect to x.
- The variable x is involved in both factors and we have to use the product rule.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$\begin{aligned}
z'_x &= (x+y)'_x\cdot e^{-x}+(x+y)\cdot(e^{-x})'_x\\
&= (1+0)e^{-x}+(x+y)\cdot e^{-x}\cdot(-1)=e^{-x}\cdot(1-x-y)\\
z'_y &= (x+y)'_y\cdot e^{-x}=(0+1)e^{-x}=e^{-x}\\
z''_{xx} &= e^{-x}\cdot(-1)\cdot(1-x-y)+e^{-x}(0-1-0)\\
&= e^{-x}(-1+x+y-1)
\end{aligned}$$

We take out the common factor.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$\begin{aligned} z'_x &= (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x \\ &= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y) \\ z'_y &= (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x} \\ z''_{xx} &= e^{-x}\cdot(-1)\cdot(1-x-y) + e^{-x}(0-1-0) \\ &= e^{-x}(-1+x+y-1) = e^{-x}(x+y-2) \end{aligned}$$

We simplify.

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$\begin{aligned}
z'_x &= (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x \\
&= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y) \\
z'_y &= (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x} \\
z''_{xx} &= e^{-x}\cdot(-1)\cdot(1-x-y) + e^{-x}(0-1-0) \\
&= e^{-x}(-1+x+y-1) = e^{-x}(x+y-2) \\
z''_{xy} &= (e^{-x})'_x
\end{aligned}$$

To find the mixed derivative we find either the derivative $(z'_x)'_y$ or $(z'_y)'_x$. The second possibility seems to be easier.

Find derivatives of $z(x,y) = (x+y)e^{-x}$ up to the 2nd order.

$$\begin{aligned} z'_x &= (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x \\ &= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y) \\ z'_y &= (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x} \\ z''_{xx} &= e^{-x}\cdot(-1)\cdot(1-x-y) + e^{-x}(0-1-0) \\ &= e^{-x}(-1+x+y-1) = e^{-x}(x+y-2) \\ z''_{xy} &= (e^{-x})'_x = -e^{-x} \end{aligned}$$

Since the function is a function of one variable, the partial derivative becomes to be the usual derivative. We use the chain rule as follows.

$$(e^{-x})' = e^{-x}(-x)' = e^{-x}(-1)$$

Find derivatives of $z(x,y)=(x+y)e^{-x}$ up to the 2nd order.

$$
\begin{aligned}
z'_x &= (x+y)'_x \cdot e^{-x} + (x+y)\cdot (e^{-x})'_x \\
&= (1+0)e^{-x} + (x+y)\cdot e^{-x}\cdot(-1) = e^{-x}\cdot(1-x-y) \\
z'_y &= (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x} \\
z''_{xx} &= e^{-x}\cdot(-1)\cdot(1-x-y) + e^{-x}(0-1-0) \\
&= e^{-x}(-1+x+y-1) = e^{-x}(x+y-2) \\
z''_{xy} &= (e^{-x})'_x = -e^{-x} \\
z''_{yy} &= 0
\end{aligned}
$$

To find the derivative z''_{yy} we differentiate z'_y with respect to y. However, the variable y is missing in the expression for z'_y. Hence z'_y is constant and its derivative is zero.

Find derivatives of $z(x,y) = (x+y)e^{-x}$ up to the 2nd order.

$$z'_x = (x+y)'_x \cdot e^{-x} + (x+y) \cdot (e^{-x})'_x$$
$$= (1+0)e^{-x} + (x+y) \cdot e^{-x} \cdot (-1) = e^{-x} \cdot (1-x-y)$$
$$z'_y = (x+y)'_y \cdot e^{-x} = (0+1)e^{-x} = e^{-x}$$
$$z''_{xx} = e^{-x} \cdot (-1) \cdot (1-x-y) + e^{-x}(0-1-0)$$
$$= e^{-x}(-1+x+y-1) = e^{-x}(x+y-2)$$
$$z''_{xy} = (e^{-x})'_x = -e^{-x}$$
$$z''_{yy} = 0$$

Find derivatives of $z(x, y) = \dfrac{x + y^2}{y - 1}$ up to the 2nd order.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1}\cdot(1+0)$$

- In order to differentiate the function with respect to x we write the function as the product of two factors: $\boxed{\dfrac{1}{y-1}\cdot(x+y^2)}$.
- The factor $\dfrac{1}{y-1}$ does not involve the variable x and it is a constant multiple. We use the constant multiple rule.
- It remains to differentiate the sum $(x+y^2)$ by the sum rule.

Find derivatives of $z(x,y) = \dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x = \frac{1}{y-1} \cdot (1+0) = \frac{1}{y-1}$$

We simplify.

Find derivatives of $z(x,y)=\frac{x+y^2}{y-1}$ up to the 2nd order.

$z'_x=\frac{1}{y-1}$,

$$z'_y=\frac{(x+y^2)'_y(y-1)-(x+y^2)(y-1)'_y}{(y-1)^2}$$

To find z'_y we have to use the quotient rule, since the variable y is in both numerator and denominator. Hence we differentiate

$$\frac{x+y^2}{y-1}$$

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},$$

$$z'_y=\frac{(x+y^2)'_y(y-1)-(x+y^2)(y-1)'_y}{(y-1)^2}$$
$$=\frac{(0+2y)(y-1)-(x+y^2)(1-0)}{(y-1)^2}$$

We evaluate the derivative of the numerator and denominator. To do this we use the sum rule, constant rule and power rule.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},$$

$$\begin{aligned} z'_y &= \frac{(x+y^2)'_y(y-1)-(x+y^2)(y-1)'_y}{(y-1)^2} \\ &= \frac{(0+2y)(y-1)-(x+y^2)(1-0)}{(y-1)^2} \\ &= \frac{2y^2-2y-(x+y^2)}{(y-1)^2} \end{aligned}$$

We simplify.

Find derivatives of $z(x,y) = \dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x = \frac{1}{y-1},$$

$$\begin{aligned} z'_y &= \frac{(x+y^2)'_y(y-1) - (x+y^2)(y-1)'_y}{(y-1)^2} \\ &= \frac{(0+2y)(y-1) - (x+y^2)(1-0)}{(y-1)^2} \\ &= \frac{2y^2-2y-(x+y^2)}{(y-1)^2} \\ &= \frac{y^2-2y-x}{(y-1)^2} \end{aligned}$$

We simplify even more.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},$$

The first derivatives have been found.

Find derivatives of $z(x,y) = \dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x = \frac{1}{y-1}, \quad z'_y = \frac{y^2-2y-x}{(y-1)^2}, \quad z''_{xx} = 0,$$

$$z''_{xx} = 0$$

- To find z''_{xx} we differentiate z'_x with respect to x.
- Since z'_x does not involve the variable x, it is treated as a constant and the derivative is zero by the constant rule.

Find derivatives of $z(x,y) = \dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x = \frac{1}{y-1}, \quad z'_y = \frac{y^2-2y-x}{(y-1)^2}, \quad z''_{xx} = 0,$$

$$z''_{xx} = 0$$

$$z''_{xy} = -1 \cdot (y-1)^{-2} \cdot (1-0)$$

- To find z''_{xy} we differentiate z'_x with respect to y.
- Since the expression for z'_x does not involve the variable x, it is a one-variable function and the partial derivative is the usual derivative.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},\quad z''_{xx}=0,\quad z''_{xy}=-\frac{1}{(y-1)^2}$$

$$z''_{xx}=0$$

$$z''_{xy}=-1\cdot(y-1)^{-2}\cdot(1-0)=-\frac{1}{(y-1)^2}$$

We simplify.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},\quad z''_{xx}=0,\quad z''_{xy}=-\frac{1}{(y-1)^2}$$

$$z''_{yy}=\frac{(2y-2)(y-1)^2-(y^2-2y-x)\cdot 2\cdot (y-1)\cdot (1-0)}{(y-1)^4}$$

- To find z''_{yy} we differentiate $z'_y=\dfrac{y^2-2y-x}{(y-1)^2}$ with respect to y. Since y is in both numerator and denominator, we use the quotient rule.
- The expression $(y-1)^2$ is differentiated by the chain rule.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},\quad z''_{xx}=0,\quad z''_{xy}=-\frac{1}{(y-1)^2}$$

$$z''_{yy}=\frac{(2y-2)(y-1)^2-(y^2-2y-x)\cdot 2\cdot (y-1)\cdot (1-0)}{(y-1)^4}$$

$$=2(y-1)\frac{(y-1)^2-(y^2-2y-x)}{(y-1)^4}$$

We take out the common factor $2(y-1)$

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},\quad z''_{xx}=0,\quad z''_{xy}=-\frac{1}{(y-1)^2}$$

$$z''_{yy}=\frac{(2y-2)(y-1)^2-(y^2-2y-x)\cdot 2\cdot (y-1)\cdot(1-0)}{(y-1)^4}$$

$$=2(y-1)\frac{(y-1)^2-(y^2-2y-x)}{(y-1)^4}$$

$$=2\frac{x+1}{(y-1)^3}$$

We simplify the numerator by expanding the power of the sum and adding the corresponding terms.

Find derivatives of $z(x,y)=\dfrac{x+y^2}{y-1}$ up to the 2nd order.

$$z'_x=\frac{1}{y-1},\quad z'_y=\frac{y^2-2y-x}{(y-1)^2},\quad z''_{xx}=0,\quad z''_{xy}=-\frac{1}{(y-1)^2}$$

$$\begin{aligned}z''_{yy}&=\frac{(2y-2)(y-1)^2-(y^2-2y-x)\cdot 2\cdot (y-1)\cdot (1-0)}{(y-1)^4}\\&=2(y-1)\frac{(y-1)^2-(y^2-2y-x)}{(y-1)^4}\\&=2\frac{x+1}{(y-1)^3}\end{aligned}$$

All derivatives up to the second order have been found.

Find derivatives of $z(x, y) = \operatorname{arctg} \frac{y}{x}$ up to the 2nd order.

Find derivatives of $z(x,y)=\operatorname{arctg}\dfrac{y}{x}$ up to the 2nd order.

$$z'_x=\frac{1}{1+\frac{y^2}{x^2}}\cdot y\cdot(-1)x^{-2}$$

- We differentiate the $\operatorname{arctg}(\cdot)$ function by the rule $\left(\operatorname{arctg} f(x)\right)'=\dfrac{1}{1+f^2(x)}\cdot f'(x)$ (formula for arctangent and the chain rule).
- The expression $\dfrac{y}{x}$ is differentiated as the product or the constant factor and the power function, i.e. $\dfrac{y}{x}=y\cdot x^{-1}$.

Find derivatives of $z(x,y) = \operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$$z'_x = \frac{1}{1+\frac{y^2}{x^2}} \cdot y \cdot (-1)x^{-2} = -\frac{x^2}{x^2+y^2} \cdot \frac{y}{x^2}$$

We simplify. Among others, we use

$$\frac{1}{1+\frac{y^2}{x^2}} = \frac{x^2}{x^2\left(1+\frac{y^2}{x^2}\right)} = \frac{x^2}{x^2+y^2}$$

and

$$x^{-2} = \frac{1}{x^2}.$$

Find derivatives of $z(x,y)=\operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$$z'_x=\frac{1}{1+\frac{y^2}{x^2}}\cdot y\cdot(-1)x^{-2}=-\frac{x^2}{x^2+y^2}\cdot\frac{y}{x^2}=-\frac{y}{x^2+y^2}$$

We multiply the fractions. The term x^2 cancels.

Find derivatives of $z(x,y) = \operatorname{arctg} \frac{y}{x}$ up to the 2nd order.

$$z'_x = \frac{1}{1+\frac{y^2}{x^2}} \cdot y \cdot (-1)x^{-2} = -\frac{x^2}{x^2+y^2} \cdot \frac{y}{x^2} = -\frac{y}{x^2+y^2}$$

$$z'_y = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} \cdot 1$$

Formula $(\operatorname{arctg}(f(x)))' = \frac{1}{1+f^2(x)} f'(x)$ is used and the expression $\frac{y}{x}$ is treated as a product or the constant factor and the power function, i.e. $\frac{y}{x} = \frac{1}{x} \cdot y$.

Find derivatives of $z(x,y)=\operatorname{arctg}\dfrac{y}{x}$ up to the 2nd order.

$$z'_x=\frac{1}{1+\frac{y^2}{x^2}}\cdot y\cdot(-1)x^{-2}=-\frac{x^2}{x^2+y^2}\cdot\frac{y}{x^2}=-\frac{y}{x^2+y^2}$$

$$z'_y=\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}\cdot 1=\frac{x^2}{x^2+y^2}\cdot\frac{1}{x}$$

We simplify.

Find derivatives of $z(x,y)=\operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$$z'_x=\frac{1}{1+\frac{y^2}{x^2}}\cdot y\cdot(-1)x^{-2}=-\frac{x^2}{x^2+y^2}\cdot\frac{y}{x^2}=-\frac{y}{x^2+y^2}$$

$$z'_y=\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}\cdot 1=\frac{x^2}{x^2+y^2}\cdot\frac{1}{x}=\frac{x}{x^2+y^2}$$

We multiply and cancel x.

Find derivatives of $z(x,y) = \operatorname{arctg} \frac{y}{x}$ up to the 2nd order.

$$z'_x = -\frac{y}{x^2+y^2}, \qquad z'_y = \frac{x}{x^2+y^2},$$

The first derivatives are known.

Find derivatives of $z(x,y) = \operatorname{arctg}\dfrac{y}{x}$ up to the 2nd order.

$$\boxed{z'_x = -\frac{y}{x^2+y^2}}, \quad \boxed{z'_y = \frac{x}{x^2+y^2}},$$

$$z''_{xx} = -y \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot (2x+0)$$

We differentiate $z'_x = -y \cdot (x^2+y^2)^{-1}$ with respect to x. The factor $(-y)$ is a constant multiple and the constant multiple rule is folowed by the chain rule for $(x^2+y^2)^{-1}$.

Find derivatives of $z(x,y)=\operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$$z'_x=-\frac{y}{x^2+y^2},\qquad z'_y=\frac{x}{x^2+y^2},$$

$$z''_{xx}=-y\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(2x+0)=\frac{2xy}{(x^2+y^2)^2}$$

We simplify.

Find derivatives of $z(x,y)=\operatorname{arctg}\dfrac{y}{x}$ up to the 2nd order.

$$z'_x=-\frac{y}{x^2+y^2},\qquad z'_y=\frac{x}{x^2+y^2},$$

$$z''_{xx}=-y\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(2x+0)=\frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy}=-\frac{1\cdot(x^2+y^2)-y\cdot(0+2y)}{(x^2+y^2)^2}$$

We differentiate $z'_x=-\dfrac{y}{x^2+y^2}$ with respect to y by the quotient rule.

Find derivatives of $z(x,y)=\operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$z'_x = -\frac{y}{x^2+y^2}$, $z'_y = \frac{x}{x^2+y^2}$,

$$z''_{xx} = -y\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(2x+0) = \frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy} = -\frac{1\cdot(x^2+y^2)-y\cdot(0+2y)}{(x^2+y^2)^2} = -\frac{x^2-y^2}{(x^2+y^2)^2}$$

We simplify the numerator.

Find derivatives of $z(x,y) = \operatorname{arctg} \frac{y}{x}$ up to the 2nd order.

$\boxed{z'_x = -\frac{y}{x^2+y^2}}$, $\boxed{z'_y = \frac{x}{x^2+y^2}}$,

$$z''_{xx} = -y \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot (2x+0) = \frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy} = -\frac{1 \cdot (x^2+y^2) - y \cdot (0+2y)}{(x^2+y^2)^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

We multiply the fraction by -1 which stays in the front of the fraction.

Find derivatives of $z(x,y) = \operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$z'_x = -\frac{y}{x^2+y^2}$, $z'_y = \frac{x}{x^2+y^2}$,

$$z''_{xx} = -y\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(2x+0) = \frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy} = -\frac{1\cdot(x^2+y^2) - y\cdot(0+2y)}{(x^2+y^2)^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$z''_{yy} = x\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(0+2y)$$

We differentiate $z'_y = x\cdot(x^2+y^2)^{-1}$ with respect to y, treating x as a constant and $(x^2+y^2)^{-1}$ as a power function with inside function (x^2+y^2).

Find derivatives of $z(x,y) = \operatorname{arctg}\dfrac{y}{x}$ up to the 2nd order.

$$\boxed{z'_x = -\frac{y}{x^2+y^2}}, \quad \boxed{z'_y = \frac{x}{x^2+y^2}},$$

$$z''_{xx} = -y\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(2x+0) = \frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy} = -\frac{1\cdot(x^2+y^2) - y\cdot(0+2y)}{(x^2+y^2)^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$z''_{yy} = x\cdot(-1)\cdot(x^2+y^2)^{-2}\cdot(0+2y) = -\frac{2xy}{(x^2+y^2)^2}$$

We simplify.

Find derivatives of $z(x,y) = \operatorname{arctg}\frac{y}{x}$ up to the 2nd order.

$$\boxed{z'_x = -\frac{y}{x^2+y^2}}, \quad \boxed{z'_y = \frac{x}{x^2+y^2}},$$

$$z''_{xx} = -y \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot (2x+0) = \frac{2xy}{(x^2+y^2)^2}$$

$$z''_{xy} = -\frac{1 \cdot (x^2+y^2) - y \cdot (0+2y)}{(x^2+y^2)^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$z''_{yy} = x \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot (0+2y) = -\frac{2xy}{(x^2+y^2)^2}$$

All derivatives have been found.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=\frac{1}{2}(1-x^2-y^2)^{-1/2}(-2x)$$

We use the power rule with power $\frac{1}{2}$ and the chain rule

$$\Big(f(g(x))\Big)'=f'(g(x))\cdot g'(x)$$

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=\frac{1}{2}(1-x^2-y^2)^{-1/2}(-2x)=-\frac{x}{\sqrt{1-x^2-y^2}}$$

We simplify

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=\frac{1}{2}(1-x^2-y^2)^{-1/2}(-2x)=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_x=\frac{1}{2}(1-x^2-y^2)^{-1/2}(-2y)$$

We use the power rule with power $\frac{1}{2}$ and the chain rule

$$\Big(f(g(x))\Big)'=f'(g(x))\cdot g'(x)$$

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x = \frac{1}{2}(1-x^2-y^2)^{-1/2}(-2x) = -\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_x = \frac{1}{2}(1-x^2-y^2)^{-1/2}(-2y) = -\frac{y}{\sqrt{1-x^2-y^2}}$$

We simplify.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$z''_{xx}=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}$$

We use the quotient rule

$$\left(\frac{u}{v}\right)'=\frac{u'\cdot v-u\cdot v'}{v^2}$$

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$z''_{xx}=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}$$

$$=-\frac{(1-x^2-y^2)+x^2}{(1-x^2-y^2)^{3/2}}$$

We multiply both numerator and denominator by the expression $\sqrt{1-x^2-y^2}$. This removes the composite fraction.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$\begin{aligned} z''_{xx}&=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}\\ &=-\frac{(1-x^2-y^2)+x^2}{(1-x^2-y^2)^{3/2}}=\frac{y^2-1}{(1-x^2-y^2)^{3/2}}\end{aligned}$$

We simplify.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$z''_{xx}=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}$$

$$=-\frac{(1-x^2-y^2)+x^2}{(1-x^2-y^2)^{3/2}}=\frac{y^2-1}{(1-x^2-y^2)^{3/2}}$$

$$z''_{xy}=-x\left(-\frac{1}{2}\right)(1-x^2-y^2)^{-3/2}(-2y)$$

We write the derivative with respect to x in the form of $z'_x=-x\cdot(1-x^2-y^2)^{-1/2}$, treat x as a constant (we differentiate with respect to y) and use the constant multiple rule and the chain rule.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$z''_{xx}=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}$$

$$=-\frac{(1-x^2-y^2)+x^2}{(1-x^2-y^2)^{3/2}}=\frac{y^2-1}{(1-x^2-y^2)^{3/2}}$$

$$z''_{xy}=-x\left(-\frac{1}{2}\right)(1-x^2-y^2)^{-3/2}(-2y)=-\frac{xy}{(1-x^2-y^2)^{3/2}}$$

We simplify.

Find derivatives of $z(x,y)=\sqrt{1-x^2-y^2}$ up to the 2nd order.

$$z'_x=-\frac{x}{\sqrt{1-x^2-y^2}}$$

$$z'_y=-\frac{y}{\sqrt{1-x^2-y^2}}$$

$$z''_{xx}=-\frac{1\cdot\sqrt{1-x^2-y^2}-x\cdot\frac{1}{2}\cdot(1-x^2-y^2)^{-1/2}(-2x)}{1-x^2-y^2}$$

$$=-\frac{(1-x^2-y^2)+x^2}{(1-x^2-y^2)^{3/2}}=\frac{y^2-1}{(1-x^2-y^2)^{3/2}}$$

$$z''_{xy}=-x\left(-\frac{1}{2}\right)(1-x^2-y^2)^{-3/2}(-2y)=-\frac{xy}{(1-x^2-y^2)^{3/2}}$$

$$z''_{yy}=\frac{x^2-1}{(1-x^2-y^2)^{3/2}}$$

An evaluation of z''_{yy} is similar to z''_{xx}.

Find derivatives of $z(x,y) = (x^2 + y)e^{x^2 - y}$ up to the 2nd order.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x \cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x)$$

Product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

Find derivatives of $z(x,y) = (x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x \cdot e^{x^2-y} + (x^2+y) \cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$

We simplify by taking out the factor $2xe^{x^2-y}$.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$

$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1)$$

We use the product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$

$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

We simplify by taking out the factor e^{x^2-y}.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{xx} = e^{x^2-y}(2x)\cdot(2x^3+2xy+2x) + e^{x^2-y}(6x^2+2y+2)$$

We write the derivative z'_x in the form

$$z'_x = e^{x^2-y}\cdot\left(2x^3+2xy+2x\right)$$

and use the product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

Find derivatives of $z(x,y) = (x^2 + y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x \cdot e^{x^2-y} + (x^2 + y) \cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2 + y + 1)$$

$$z'_y = 1 \cdot e^{x^2-y} + (x^2 + y) \cdot e^{x^2-y}(-1) = e^{x^2-y}(1 - x^2 - y^2)$$

$$z''_{xx} = e^{x^2-y}(2x) \cdot (2x^3 + 2xy + 2x) + e^{x^2-y}(6x^2 + 2y + 2)$$

$$= 2e^{x^2-y}(2x^4 + 2x^2y + 2x^2 + 3x^2 + y + 1)$$

We factorize, the repeating factor is $2e^{x^2-y}$.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{xx} = e^{x^2-y}(2x)\cdot(2x^3+2xy+2x) + e^{x^2-y}(6x^2+2y+2)$$
$$= 2e^{x^2-y}(2x^4+2x^2y+2x^2+3x^2+y+1)$$
$$= 2e^{x^2-y}(2x^4+2x^2y+5x^2+y+1)$$

We simplify in the parenthesis.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{xy} = e^{x^2-1}(2x)\cdot(1-x^2-y) + e^{x^2-y}\cdot(-2x)$$

We start with

$$z'_y = e^{x^2-y}(1-x^2-y^2)$$

and differentiate with respect to x by the product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{xy} = e^{x^2-1}(2x)\cdot(1-x^2-y) + e^{x^2-y}\cdot(-2x)$$
$$= 2xe^{x^2-y}(1-x^2-y-1)$$

We factorize, the repeating factor is $2xe^{x^2-y}$.

Find derivatives of $z(x,y) = (x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x \cdot e^{x^2-y} + (x^2+y) \cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1 \cdot e^{x^2-y} + (x^2+y) \cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{xy} = e^{x^2-1}(2x) \cdot (1-x^2-y) + e^{x^2-y} \cdot (-2x)$$
$$= 2xe^{x^2-y}(1-x^2-y-1)$$
$$= -2xe^{x^2-y}(x^2+y)$$

We simplify.

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{yy} = e^{x^2-y}(-1)\cdot(1-x^2-y) + e^{x^2-y}\cdot(-1)$$

To find z''_{yy} we start with

$$z'_y = e^{x^2-y}(1-x^2-y^2)$$

and use the product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'.$$

Find derivatives of $z(x,y)=(x^2+y)e^{x^2-y}$ up to the 2nd order.

$$z'_x = 2x\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(2x) = 2xe^{x^2-y}(x^2+y+1)$$
$$z'_y = 1\cdot e^{x^2-y} + (x^2+y)\cdot e^{x^2-y}(-1) = e^{x^2-y}(1-x^2-y^2)$$

$$z''_{yy} = e^{x^2-y}(-1)\cdot(1-x^2-y) + e^{x^2-y}\cdot(-1)$$
$$= (-1)e^{x^2-y}(2-x^2-y)$$

The repeating factor is $(-1)e^{x^2-y}$

Find derivatives of $z(x,y) = e^{x^2+y^2}$ up to the 2nd order.

Find derivatives of $z(x,y)=e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2} \cdot 2x$$

The chain rule

$$\Big(f(g(x))\Big)' = f'(g(x)) \cdot g'(x).$$

Find derivatives of $z(x,y) = e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2} \cdot 2x$$

$$z'_y = e^{x^2+y^2} \cdot 2y$$

The chain rule

$$\Big(f(g(x))\Big)' = f'(g(x)) \cdot g'(x).$$

Find derivatives of $z(x,y)=e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2}\cdot 2x$$

$$z'_y = e^{x^2+y^2}\cdot 2y$$

$$z''_{xx} = e^{x^2+y^2}2x\cdot 2x + e^{x^2+y^2}\cdot 2$$

The product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'.$$

Find derivatives of $z(x,y) = e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2} \cdot 2x$$

$$z'_y = e^{x^2+y^2} \cdot 2y$$

$$z''_{xx} = e^{x^2+y^2} 2x \cdot 2x + e^{x^2+y^2} \cdot 2 = 2e^{x^2+y^2}(1+2x^2)$$

Simplifying.

Find derivatives of $z(x,y) = e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2} \cdot 2x$$

$$z'_y = e^{x^2+y^2} \cdot 2y$$

$$z''_{xx} = e^{x^2+y^2} 2x \cdot 2x + e^{x^2+y^2} \cdot 2 = 2e^{x^2+y^2}(1+2x^2)$$

$$z''_{xy} = 2x \cdot e^{x^2+y^2} 2y$$

The constant multiple rule.

Find derivatives of $z(x,y)=e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2}\cdot 2x$$

$$z'_y = e^{x^2+y^2}\cdot 2y$$

$$z''_{xx} = e^{x^2+y^2}2x\cdot 2x + e^{x^2+y^2}\cdot 2 = 2e^{x^2+y^2}(1+2x^2)$$

$$z''_{xy} = 2x\cdot e^{x^2+y^2}2y = 4xye^{x^2+y^2}$$

Simplifying.

Find derivatives of $z(x,y)=e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2}\cdot 2x$$

$$z'_y = e^{x^2+y^2}\cdot 2y$$

$$z''_{xx} = e^{x^2+y^2}2x\cdot 2x + e^{x^2+y^2}\cdot 2 = 2e^{x^2+y^2}(1+2x^2)$$

$$z''_{xy} = 2x\cdot e^{x^2+y^2}2y = 4xye^{x^2+y^2}$$

$$z''_{yy} = e^{x^2+y^2}2y\cdot 2y + e^{x^2+y^2}\cdot 2$$

The product rule

$$(u\cdot v)' = u'\cdot v + u\cdot v'.$$

Find derivatives of $z(x,y)=e^{x^2+y^2}$ up to the 2nd order.

$$z'_x = e^{x^2+y^2} \cdot 2x$$

$$z'_y = e^{x^2+y^2} \cdot 2y$$

$$z''_{xx} = e^{x^2+y^2} 2x \cdot 2x + e^{x^2+y^2} \cdot 2 = 2e^{x^2+y^2}(1+2x^2)$$

$$z''_{xy} = 2x \cdot e^{x^2+y^2} 2y = 4xye^{x^2+y^2}$$

$$z''_{yy} = e^{x^2+y^2} 2y \cdot 2y + e^{x^2+y^2} \cdot 2 = 2e^{x^2+y^2}(1+2y^2)$$

Simplifying.

That's all . . .