Limits

Robert Mařík

May 9, 2006

Contents

Find $\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1}$$

Find $\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg} 1}{1+1}$$

- We substitute $x = 1$.
- The expression is well-defined. Hence the function is continuous at $x = 1$ and the value of the function is the same as the value of the limit.

Find $\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg} 1}{1+1}$$
$$= \frac{\frac{\pi}{4}}{2}$$

We evaluate $\operatorname{arctg} 1$. We have to complete the pattern

$$\operatorname{tg}(\cdot) = 1.$$

The solution is

$$\operatorname{tg} \frac{\pi}{4} = 1$$

and hence $\operatorname{arctg} 1 = \frac{\pi}{4}$.

Find $\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to 1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg} 1}{1+1} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$$

We simplify the fraction. The problem is solved.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$$

Find $\lim_{x \to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x \to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1}$$

We substitute ...

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

...and simplify.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1}$$

The limit is of the type $\frac{\text{nonzero}}{\text{zero}}$. Hence we have to investigate the one-sided limits first. We start with the limit from the right.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{0}$$

We know, what we obtain after the substitution $x = -1$.

Find $\lim\limits_{x\to -1} \dfrac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0}$$

- We have to establish the sign of the function in the denominator.
- If x is on the right of the number -1, then $x > -1$ and the relation $x+1 > 0$ holds.
- Hence the denominator is positive.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0} = -\infty$$

The value of the limit from the right is $-\infty$.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0} = -\infty$$

$$\lim_{x\to -1^-} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{0}$$

We investigate the limit from the left.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0} = -\infty$$

$$\lim_{x\to -1^-} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{-0}$$

- If x is on the left from -1, then $x < -1$.
- Hence $x + 1 < 0$ and the denominator is negative.

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0} = -\infty$$

$$\lim_{x\to -1^-} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{-0} = +\infty$$

The value of the limit from the left is $+\infty$

Find $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1} = \frac{\operatorname{arctg}(-1)}{-1+1} = \frac{-\frac{\pi}{4}}{0}$$

$$\lim_{x\to -1^+} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{+0} = -\infty$$

$$\lim_{x\to -1^-} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{4}}{-0} = +\infty$$

The two-sided limit $\lim_{x\to -1} \frac{\operatorname{arctg} x}{x+1}$ does not exist.

Both one-sided limits are different. Hence the two-sided limit does not exist.

Find $\lim\limits_{x\to-\infty} \dfrac{\operatorname{arctg} x}{x+1}$

Find $\lim\limits_{x\to-\infty} \dfrac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to-\infty} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{2}}{}$$

- We evaluate the limit of the numerator and the denominator separately.
- $\lim\limits_{x\to-\infty} \operatorname{arctg} x$ can be established by investigating the graph of the function $y = \operatorname{arctg} x$.
- The function $y = \operatorname{arctg} x$ has a horizontal asymptote $y = -\dfrac{\pi}{2}$ at $-\infty$. Hence the limit of the numerator is $-\dfrac{\pi}{2}$

Find $\lim_{x\to-\infty} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to-\infty} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{2}}{-\infty}$$

The limit of the denominator is: $-\infty + 1 = -\infty$.

Find $\lim_{x\to-\infty} \frac{\operatorname{arctg} x}{x+1}$

$$\lim_{x\to-\infty} \frac{\operatorname{arctg} x}{x+1} = \frac{-\frac{\pi}{2}}{-\infty} = 0$$

A finite quantity divided by infinity equals zero. The problem is solved!

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x$$

We start with the limit at $+\infty$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty}$$

- We try to establish the limits of the functions in the product.
- If we obtain something different from the indeterminate form 0∞, the problem becomes easy.
- We substitute. Under $e^{-\infty}$ we understand the limit $\lim_{x\to-\infty} e^{x}$.

Find $\lim\limits_{x\to\pm\infty} e^{-x}\operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x}\operatorname{arctg} x = e^{-\infty}\operatorname{arctg}\infty$$

We substitute into the second function. Under $\operatorname{arctg}\infty$ we understand the limit $\lim\limits_{x\to\infty} \operatorname{arctg} x$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2}$$

Investigating the graphs of $y = e^x$ and $y = \operatorname{arctg} x$ we can see that

$$\lim_{x\to-\infty} e^x = 0$$

and

$$\lim_{x\to\infty} \operatorname{arctg} x = \frac{\pi}{2}.$$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

The product equals to zero.

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

$$\lim_{x\to-\infty} e^{-x} \operatorname{arctg} x =$$

We continue with the limit at $-\infty$.

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

$$\lim_{x\to-\infty} e^{-x} \operatorname{arctg} x = e^{\infty}$$

- We try to establish the limits of the functions in the product.
- If we obtain something different from the indeterminate form 0∞, the problem becomes easy.
- We substitute. Since $-(-\infty) = \infty$, we have e^{∞} in the first factor. Under this expression we understand the limit $\lim_{x\to\infty} e^{x}$.

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

$$\lim_{x\to-\infty} e^{-x} \operatorname{arctg} x = e^{\infty} \operatorname{arctg}(-\infty)$$

We substitute into the second function. Under $\operatorname{arctg}(-\infty)$ we understand the limit $\lim_{x\to-\infty} \operatorname{arctg} x$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

$$\lim_{x\to-\infty} e^{-x} \operatorname{arctg} x = e^{\infty} \operatorname{arctg}(-\infty) = \infty(-\frac{\pi}{2})$$

Investigating the graphs of $y = e^x$ and $y = \operatorname{arctg} x$ we can see that

$$\lim_{x\to\infty} e^x = \infty$$

and

$$\lim_{x\to-\infty} \operatorname{arctg} x = -\frac{\pi}{2}.$$

Find $\lim_{x\to\pm\infty} e^{-x} \operatorname{arctg} x$

$$\lim_{x\to\infty} e^{-x} \operatorname{arctg} x = e^{-\infty} \operatorname{arctg} \infty = 0\frac{\pi}{2} = 0$$

$$\lim_{x\to-\infty} e^{-x} \operatorname{arctg} x = e^{\infty} \operatorname{arctg}(-\infty) = \infty(-\frac{\pi}{2}) = -\infty$$

The product equals $-\infty$. Both problems are solved.

Find $\lim_{x \to \pm\infty} x^3 + 2x^2 - 4$

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4$$

- We start with the limit at $+\infty$. We substitute.
- Under ∞^3 we understand either $\lim_{x\to\infty} x^3$, or equivalently the product $\infty \times \infty \times \infty$.
- Similarly we treat the expression ∞^2.

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4$$

$$\infty^3 = \infty, \qquad \infty^2 = \infty$$

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\infty + \infty - 4 = \infty$$

by the rules for working with infinity.

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x^2 - 4$$

We continue with the limit at $-\infty$.

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x^2 - 4 = (-\infty)^3 + 2(-\infty)^2 - 4$$

We substitute.

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x^2 - 4 = (-\infty)^3 + 2(-\infty)^2 - 4$$

$$= -\infty + \infty - 4$$

$$(-\infty)\times(-\infty)\times(-\infty) = -\infty \qquad 2(-\infty)(-\infty) = \infty$$

by the rules for working with infinity.
Problem! We obtained an indeterminate form $-\infty + \infty$.

Find $\lim_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x^2 - 4 = (-\infty)^3$$

$$= -\infty$$

$$= -\infty$$

- We know from theory how to resolve this problem.
- It can be shown that the leading coefficients are the only "important" terms at $\pm\infty$. Hence we can omitt the terms with smaller powers.
- The limit of the leading term equals $-\infty$.

Find $\lim\limits_{x\to\pm\infty} x^3 + 2x^2 - 4$

$$\lim_{x\to\infty} x^3 + 2x^2 - 4 = \infty^3 + 2\infty^2 - 4 = \infty + \infty - 4 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x^2 - 4 = (-\infty)^3$$

$$= -\infty$$

$$= -\infty$$

Both problems are solved.

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3}$$

We start with the limit at $+\infty$.

Find $\lim\limits_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

- The limit of both numerator and denominator are $+\infty$.
- We have an indeterminate form.

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2}$$

From theory we know that the limit can be established as the limit of the fraction from leading terms in numerator and denominator.

Find $\lim_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2}$$

We simplify.

$$\frac{x^3}{2x^2} = \frac{x}{2}.$$

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2}$$

We substitute $x = \infty$.

Find $\lim\limits_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

We use the rules for working with infinity.

Find $\lim\limits_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

- We continue with the limit at $-\infty$.
- Sunstituting $x = -\infty$ we obtain again an indeterminate form.

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^2}$$

As in the case of the limit at $+\infty$, we consider only leading terms of numerator and denominator.

Find $\lim_{x\to\pm\infty} \frac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^2} = \lim_{x\to-\infty} \frac{x}{2}$$

We simplify the function in the limit.

Find $\lim_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^2} = \lim_{x\to-\infty} \frac{x}{2} = \frac{-\infty}{2}$$

We substitute.

Find $\lim_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^2} = \lim_{x\to-\infty} \frac{x}{2} = \frac{-\infty}{2} = -\infty$$

We use the rules for working with infinity.

Find $\lim_{x\to\pm\infty} \dfrac{x^3+3x^2+1}{2x^2-3}$

$$\lim_{x\to\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{\infty}{\infty}}$$

$$= \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to-\infty} \frac{x^3+3x^2+1}{2x^2-3} = \boxed{\frac{-\infty}{\infty}}$$

$$= \lim_{x\to-\infty} \frac{x^3}{2x^2} = \lim_{x\to-\infty} \frac{x}{2} = \frac{-\infty}{2} = -\infty$$

Finished!

Find $\lim_{x\to\pm\infty} \dfrac{2x^4 + 4x + 5}{3x^4 - x^3 + 4x + 1}$

Find $\lim_{x\to\pm\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

We start with the limit at $+\infty$.

Find $\lim_{x\to\pm\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

We substitute $x = \infty$.

Find $\lim_{x\to\pm\infty} \dfrac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

- We obtained an indeterminate form.
- From theory we know that it is sufficient to consider the leading coefficients only.
- We omitt the coefficients with smaller degrees.

Find $\lim_{x\to\pm\infty} \dfrac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

We simplify

$$\frac{2x^4}{3x^4} = \frac{2}{3}$$

Find $\lim\limits_{x\to\pm\infty} \dfrac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$$

The limit of a constant function is the constant.

Find $\lim_{x\to\pm\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}}$$

We continue with the limit at $-\infty$. We substitute $x = -\infty$.

Find $\lim_{x\to\pm\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to-\infty} \frac{2x^4}{3x^4}$$

- We obtained an indeterminate form.
- From theory we know that it is sufficient to consider the leading coefficients only.
- We omitt the coefficients with smaller degrees.

Find $\lim_{x\to\pm\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to-\infty} \frac{2x^4}{3x^4} = \lim_{x\to-\infty} \frac{2}{3}$$

We simplify

$$\frac{2x^4}{3x^4} = \frac{2}{3}$$

Find $\lim\limits_{x\to\pm\infty} \dfrac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to-\infty} \frac{2x^4}{3x^4} = \lim_{x\to-\infty} \frac{2}{3} = \frac{2}{3}$$

The limit of a constant function is the constant.

Find $\lim\limits_{x\to\pm\infty} \dfrac{2x^4+4x+5}{3x^4-x^3+4x+1}$

$$\lim_{x\to\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to\infty} \frac{2x^4}{3x^4} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{2x^4+4x+5}{3x^4-x^3+4x+1} = \boxed{\frac{\infty}{\infty}} = \lim_{x\to-\infty} \frac{2x^4}{3x^4} = \lim_{x\to-\infty} \frac{2}{3} = \frac{2}{3}$$

Finished!

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

Find $\lim_{x\to\infty} \left[2\ln x - \ln(x^2 + x + 1)\right]$.

$$\lim_{x\to\infty} \left[2\ln x - \ln(x^2 + x + 1)\right]$$

We start with the limit.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty - \infty}$$

Since $\lim_{x\to\infty}\ln x = \infty$, we have an indeterminate form $\boxed{\infty - \infty}$.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty - \infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

- In general, the limits written as fractions are easier (remember the l'Hospital's rule). We write the function inside the limit as a fraction.
- We write both terms as logarithms first.
- We use the rule $\boxed{r\ln a = \ln a^r}$.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty-\infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty} \ln \frac{x^2}{x^2+x+1}$$

We subtract logarithms. We use the rule $\boxed{\ln a - \ln b = \ln \frac{a}{b}}$.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty-\infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty} \ln \frac{x^2}{x^2+x+1} = \ln\left(\lim_{x\to\infty}\frac{x^2}{x^2+x+1}\right)$$

We evaluate the limit of the composite function. We look for the limit of the inside function first.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty - \infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty} \ln \frac{x^2}{x^2+x+1} = \ln\left(\lim_{x\to\infty}\frac{x^2}{x^2+x+1}\right) = \boxed{\ln\frac{\infty}{\infty}}$$

The limit of the inside function is an indeterminate form.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty-\infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty}\ln\frac{x^2}{x^2+x+1} = \ln\left(\lim_{x\to\infty}\frac{x^2}{x^2+x+1}\right) = \boxed{\ln\frac{\infty}{\infty}}$$

$$= \ln\left(\lim_{x\to\infty}\frac{x^2}{x^2}\right)$$

- We have the limit of rational function at infinity.
- Only the leading terms are important in the limit.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty - \infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty} \ln \frac{x^2}{x^2+x+1} = \ln\left(\lim_{x\to\infty} \frac{x^2}{x^2+x+1}\right) = \boxed{\ln \frac{\infty}{\infty}}$$

$$= \ln\left(\lim_{x\to\infty} \frac{x^2}{x^2}\right) = \ln 1$$

The numerator and the denominator in $\frac{x^2}{x^2}$ cancel and $\lim_{x\to\infty} 1 = 1$.

Find $\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right]$.

$$\lim_{x\to\infty}\left[2\ln x - \ln(x^2+x+1)\right] = \boxed{\infty - \infty}$$

$$= \lim_{x\to\infty}\left[\ln x^2 - \ln(x^2+x+1)\right]$$

$$= \lim_{x\to\infty} \ln \frac{x^2}{x^2+x+1} = \ln\left(\lim_{x\to\infty} \frac{x^2}{x^2+x+1}\right) = \boxed{\ln\frac{\infty}{\infty}}$$

$$= \ln\left(\lim_{x\to\infty}\frac{x^2}{x^2}\right) = \ln 1 = 0$$

$\boxed{\ln 1 = 0}$. The problem is solved.