Main ides of differential calculus

Robert Mak

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Isaac Newton

Gottfried Wilhelm Leibniz

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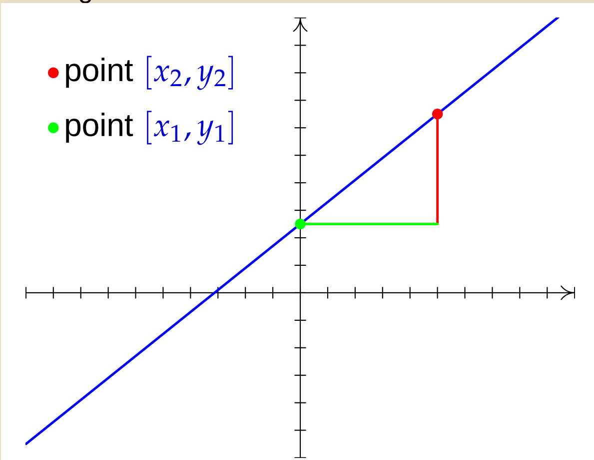
1. Rate of change – linear growth

Slope of the line (denoted by k) and evaluated as a the quotient of the vertical and corresponding horizontal change is a convenient tool which measures the rate of growth.



$$k = \frac{\text{red part}}{\text{green part}} = \frac{y_2 - y_1}{x_2 - x_1} = 0.8$$

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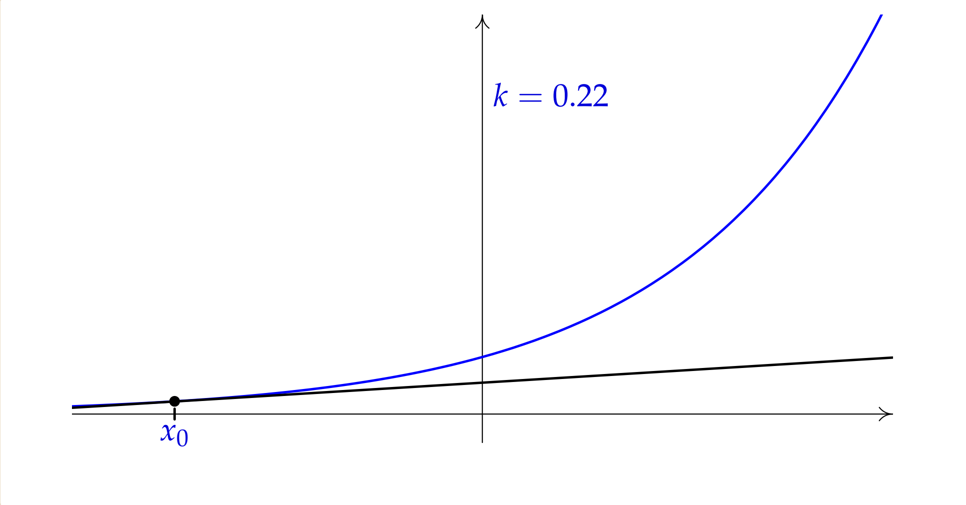
2. Rate of change – nonlinear growth

The rate of change of nonlinear function at a point is defined as the rate of change of the best linear approximation (tangent) at this point. This rate may change along the curve.



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We have to find the slope of the tangent.

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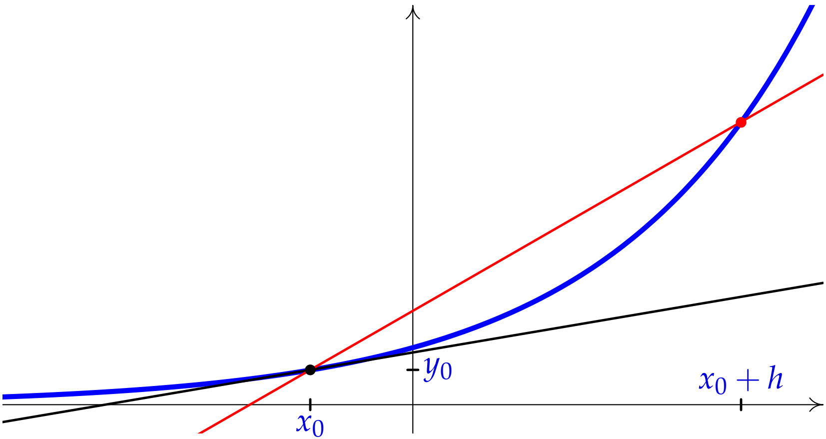
3. Tangent

It is easy to find the slope of the secant (we have two points of the line). Tangent can be considered as a limit position of secants.



Secant slope is $k = \dfrac{f(x_0+h) - f(x_0)}{h} = 2.07$.

Tangent slope is $y = 0.61x + 0.91$.

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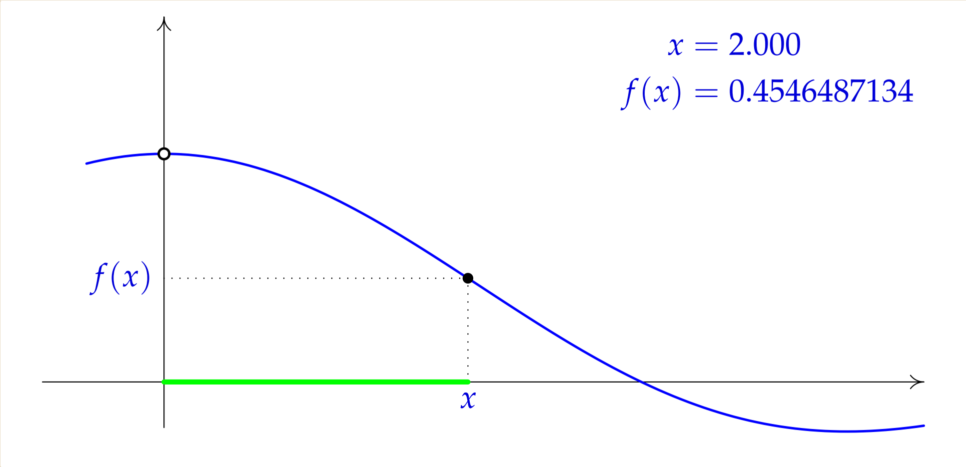
4. “Approaching” 1

Limit process $\lim\limits_{x\to 0^+} \dfrac{\sin x}{x}$



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It holds $\lim\limits_{x\to 0^+} \dfrac{\sin x}{x} = 1$

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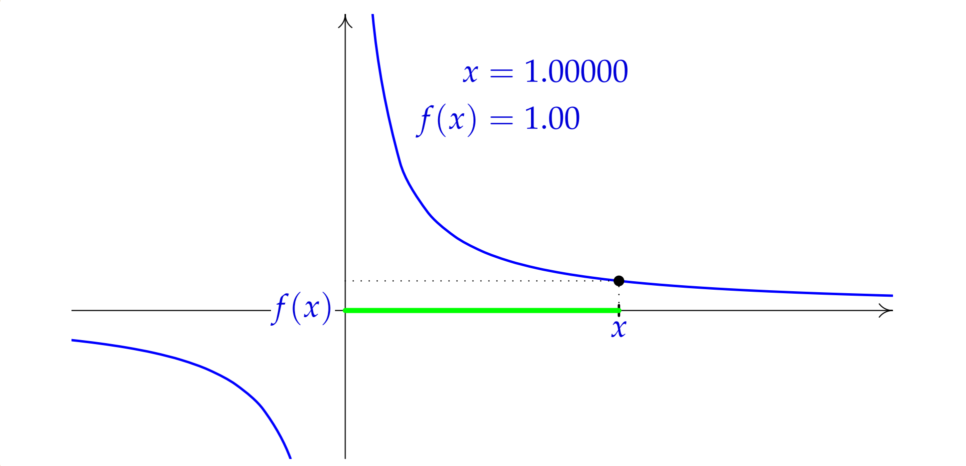
5. “Approaching” 2

Limit process $\lim\limits_{x\to 0^+} \dfrac{1}{x}$



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It holds $\lim\limits_{x\to 0^+} \dfrac{1}{x} = \infty$

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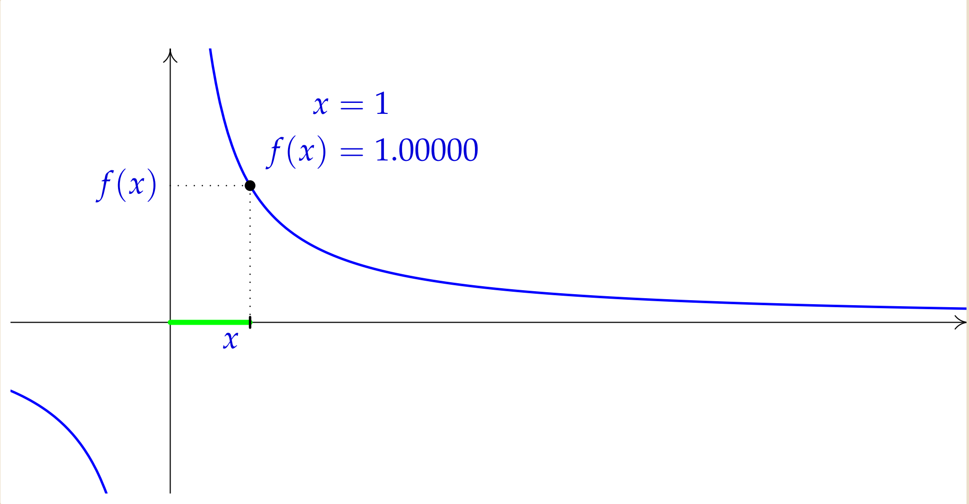
6. "Approaching" 3

Limit process $\lim\limits_{x\to\infty}\dfrac{1}{x}$



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Plat $\lim\limits_{x\to\infty}\dfrac{1}{x} = 0$

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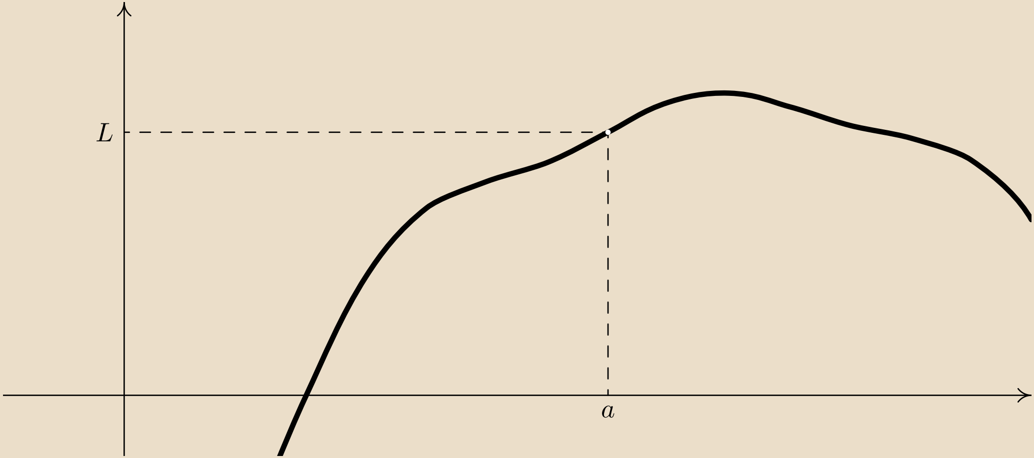
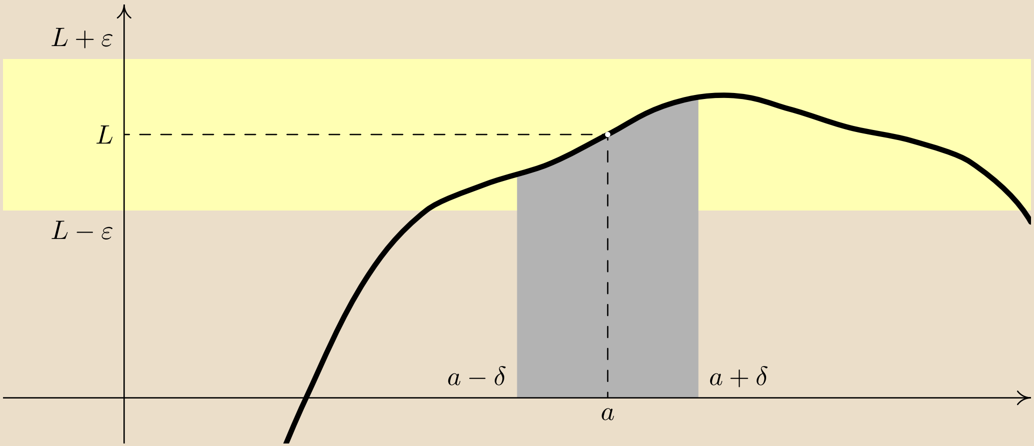
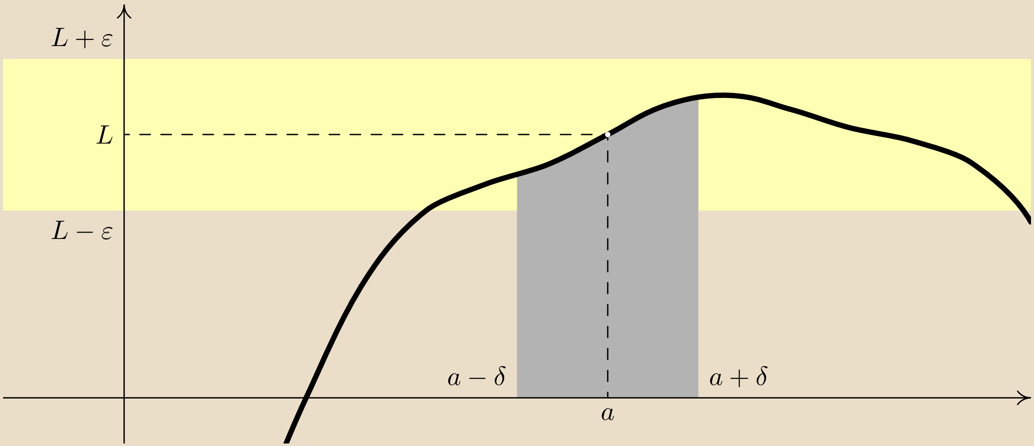
7. Definition of the limit



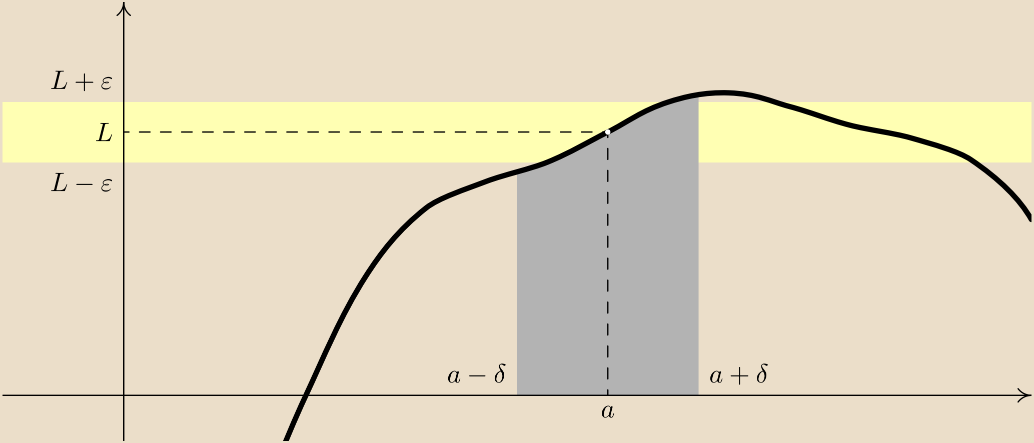
- From the picture it seems that $f(x)$ approaches L if x approaches a
- Now we will define exactly the limit – a mathematical description of the fact that “something approaches something”.



- The neighborhood of the point b is every open interval which contains this point.
- The ring neighborhood of the point b is the neighborhood of b, with exception of the point b.



- For every neighborhood of the point L we can find a ring neighborhood of the point a such that the image of all points from this ring neighborhood belongs to the neighborhood of the point L.
- The value $f(a)$ plays no role in our investigations. It may be arbitrary or remain undefined.



- We consider smaller neighborhood of the point L (yellow strip).
- Values at some of the points in the neighborhood of the point a (the top of gray set) is outside the yellow strip.

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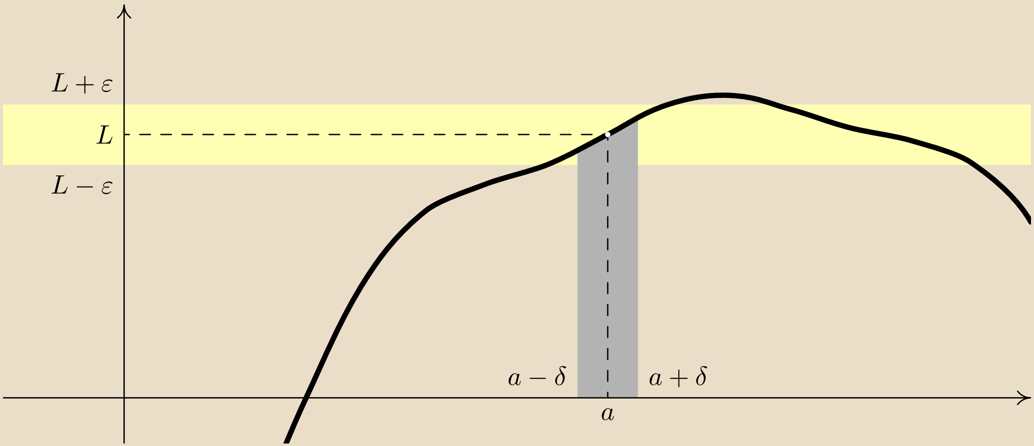

- If we consider smaller neighborhood of the point a, the top of the gray set is again in the yellow strip.

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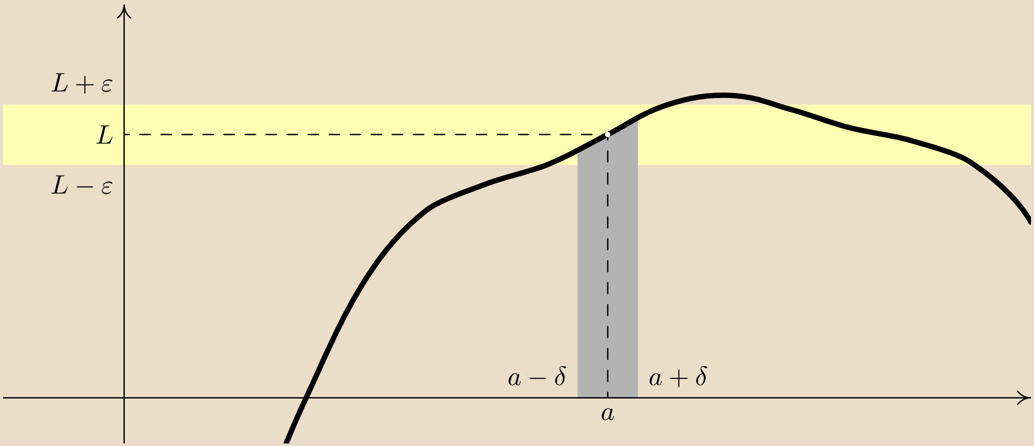
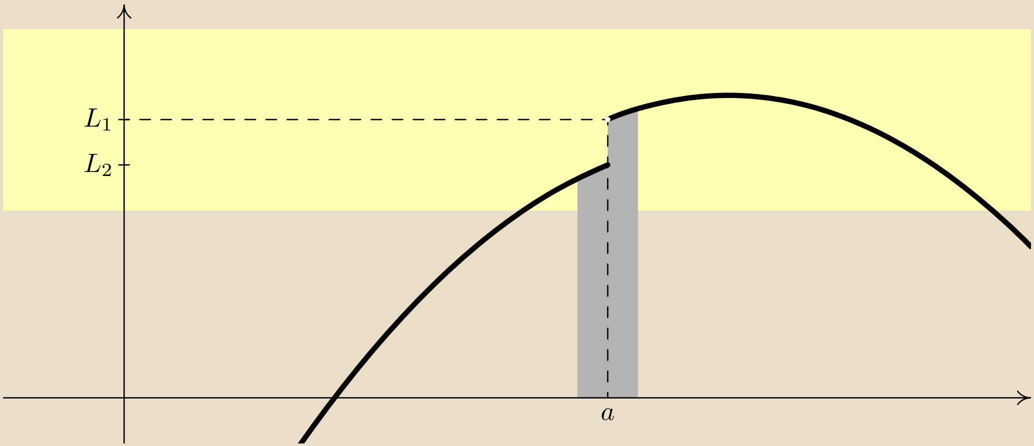
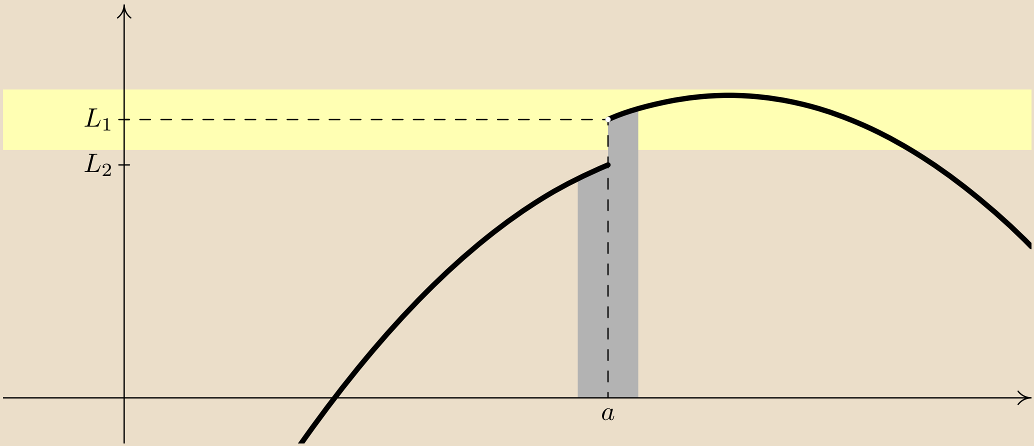


- For every neighborhood of the point L we can find a ring neighborhood of the point a such that the values of all points in this neighborhood belong to the neighborhood of the point L.

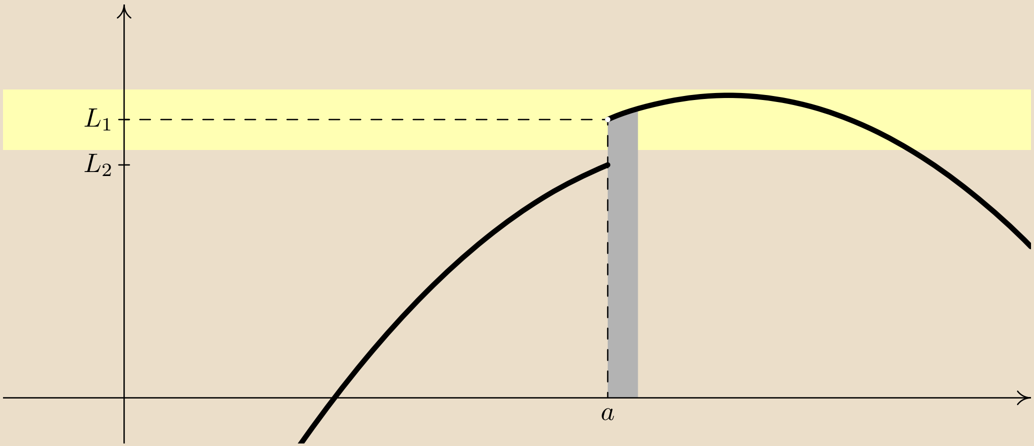
Definice 1 (definition of limit) If the above condition is fulfilled, then we say that the limit of the function f at the point a is L. We write $\boxed{\lim_{x\to a} f(x) = L}$.



- Consider another function.



- For this neighborhood of the point L there is no neighborhood of the point a with the property from the definition of the limit.
- The point L_1 is not the limit of the function at a.
- Similarly, neither L_2 nor any other number is the limit at a. The function possesses no limit at the point a.



- **BUT:** For every neighborhood of L_1 we can find a right ring neighborhood of the point a such that all points from this neighborhood have images in the neighborhood of the point L_1.

Definice 2 (definition of one-sided limit) If the last condition holds, we say that the limit of the function f at the point a equals L_1. We write $\boxed{\lim_{x\to a^+} f(x) = L_1}$.

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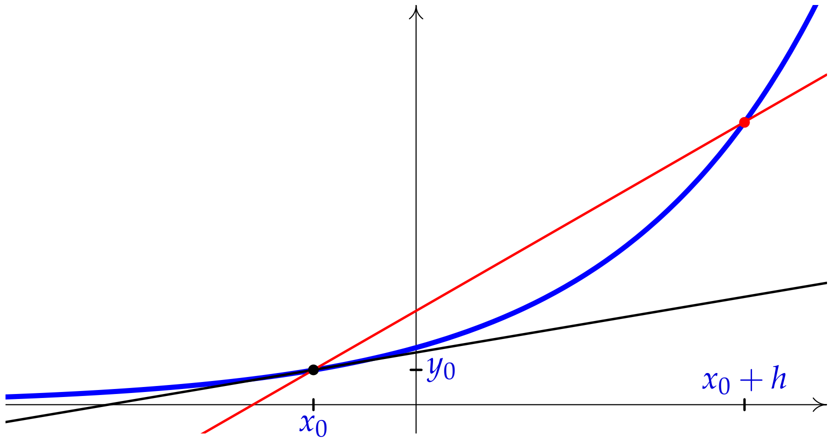
8. Tangent

It is easy to find the slope of the secant (we have two points of the line). Tangent can be considered as a limit position of secants.



Secant slope is $k = \dfrac{f(x_0+h) - f(x_0)}{h} = 2.07$.

Tangent slope is $y = 0.61x + 0.91$.

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9. Derivative

Definice 3 (derivative of the function) Let $x \in \mathrm{D}(f)$. The function f has *derivative at the point* x if the finite limit

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}. \tag{1}$$

exists.