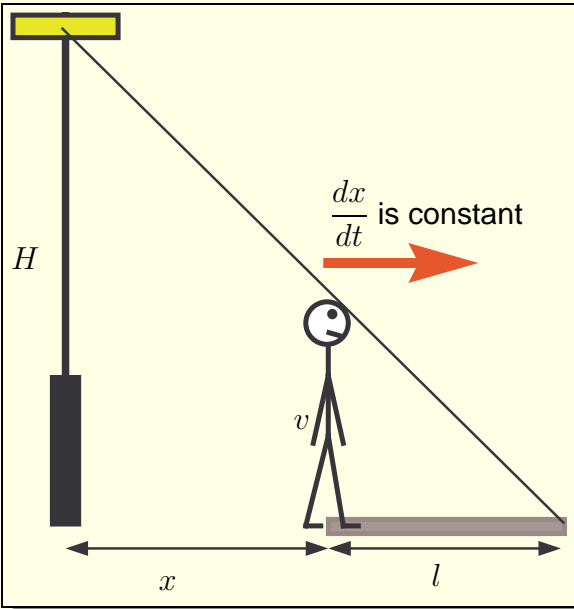


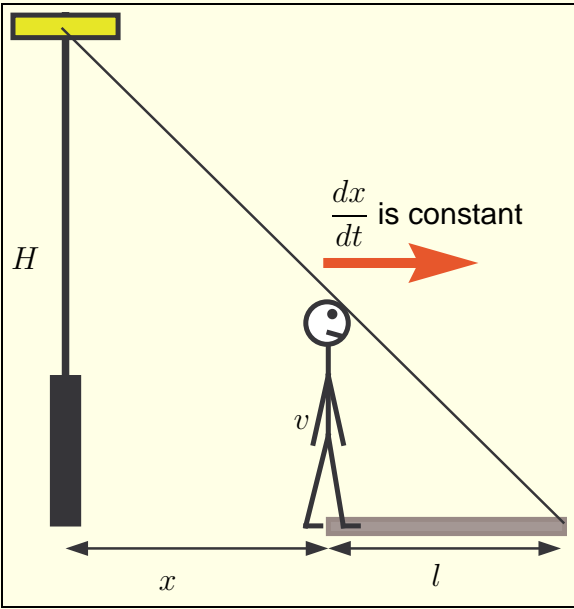
The Derivative as the Rate of Change

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January 15, 2006

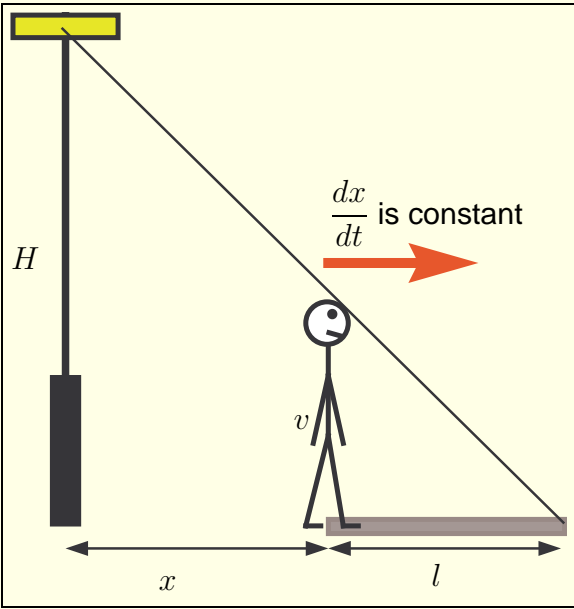


- A man is walking at a constant speed, as on the picture.
- What is the rate of change of his shadow?
- Is the rate smaller when the shadow is small or when the shadow is large?



$$\frac{H}{x+l} = \frac{v}{l}$$

We find relation between x , v , H and l from similar triangles.

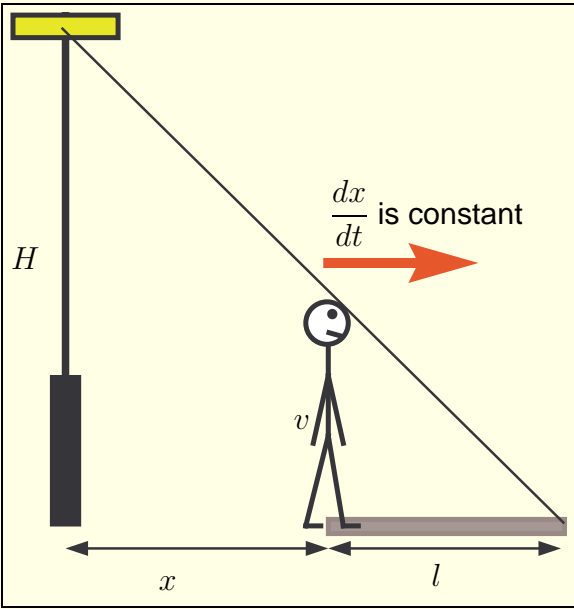


$$\frac{H}{x+l} = \frac{v}{l}$$

$$lH = vx + vl$$

$$l = \frac{v}{H-v} \cdot x$$

We find the length l of the shadow as a function of the distance x between the man and the lamp.



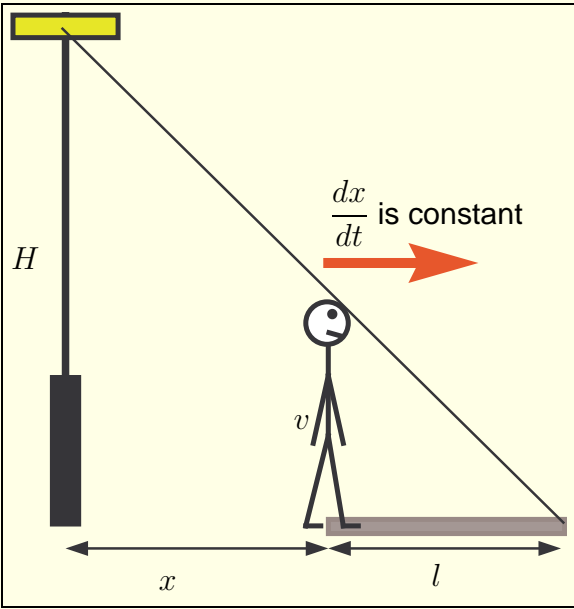
$$\frac{H}{x+l} = \frac{v}{l}$$

$$lH = vx + vl$$

$$l = \frac{v}{H-v} \cdot x$$

$$\frac{dl}{dt} = \frac{\overbrace{v}^{\text{const}}}{H-v} \cdot \frac{\overbrace{dx}^{\text{const}}}{dt}$$

We find the rate of the change $\frac{dl}{dt}$ of the shadow as a function of the known speed of walk $\frac{dx}{dt}$.



$$\frac{H}{x+l} = \frac{v}{l}$$

$$lH = vx + vl$$

$$l = \frac{v}{H-v} \cdot x$$

$$\frac{dl}{dt} = \frac{\overbrace{v}^{\text{const}}}{H-v} \cdot \overbrace{\frac{dx}{dt}}^{\text{const}}$$

The rate of the growth of the shadow is constant neglecting its length.