Improper integral

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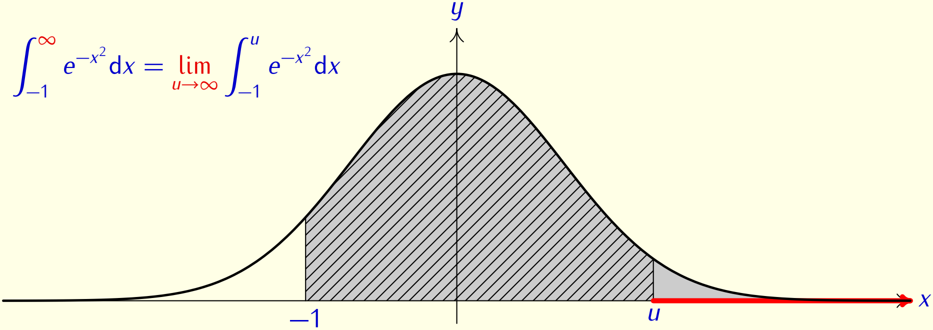
1 Improper integral

In the following we extend the concept of Riemann integral for integration on the unbounded intervals like $[1,\infty)$, $(-\infty,\infty)$ and so on. This is necessary especially because of applications in statistics.

If one of the limits a, b in the integral $\int_a^b f(x)\,\mathrm{d}x$ is $\pm\infty$, then the integral is called *improper* and the corresponding unbounded limit of integration is called *singular point*.

$$\int_{-1}^{\infty} e^{-x^2}\,\mathrm{d}x = \lim_{u\to\infty}\int_{-1}^{u} e^{-x^2}\,\mathrm{d}x$$



Definition (improper integral). Let a be a real number and f be a function integrable in the sense of Riemann on the interval $[a, t]$ for every $t > a$. Under an *improper* integral

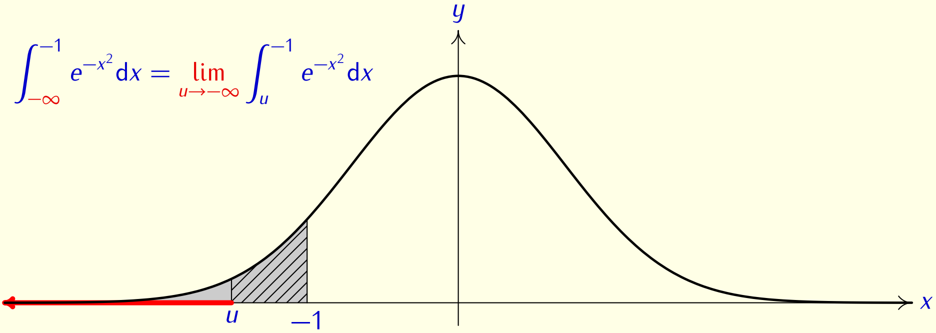
$$I = \int_a^{\infty} f(x)\,\mathrm{d}x \qquad (1)$$

we understand the limit

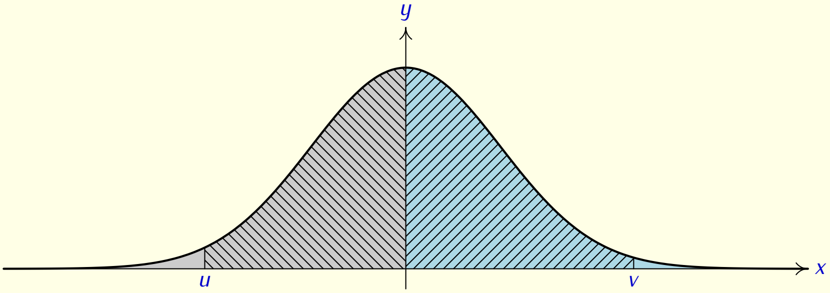
$$I = \lim_{t\to\infty} \int_a^t f(x)\,\mathrm{d}x,$$

if this limit exist as a finite number. In this case the integral is said be *convergent*. If the limit does not exists or equals $\pm\infty$, then the integral is said to be *divergent*.

$$\int_{-\infty}^{-1} e^{-x^2} \mathrm{d}x = \lim_{u\to-\infty} \int_{u}^{-1} e^{-x^2} \mathrm{d}x$$



Definition (improper integral). The integral $\int_{-\infty}^{a} f(x)\,\mathrm{d}x$ is defined in a similar way as the limit $\lim_{t\to-\infty} \int_{t}^{a} f(x)\,\mathrm{d}x$.



$$\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x = \int_{-\infty}^{0} e^{-x^2}\,\mathrm{d}x + \int_{0}^{\infty} e^{-x^2}\,\mathrm{d}x$$
$$= \lim_{u\to-\infty}\int_{u}^{0} e^{-x^2}\,\mathrm{d}x + \lim_{v\to\infty}\int_{0}^{v} e^{-x^2}\,\mathrm{d}x$$

The integral $\int_{-\infty}^{+\infty} f(x)\,\mathrm{d}x$ is defined as the sum of two integrals $\int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$ where $c \in \mathbb{R}$ is any real number, provided both integrals are convergent. It can be shown that the particular value of c has no influence to the value of the resulting integral.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x.$

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)} \, dx.$

$$I = \lim_{u \to \infty} \int_1^{u} \frac{1}{x(x^2+1)} \, dx$$

According to the definition, we substitute the upper limit by u.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x =$$

We decompose into partial fractions.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

We integrate using basic rules and formulas.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

We evaluate the Riemann integral by Newton–Leibniz formula.

Find $I = \int_1^\infty \frac{1}{x(x^2+1)}\,dx$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,dx$$

$$\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,dx = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty}\left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right]$$

We use the limit process $u \to \infty$.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty}\left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right] =$$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\ln\left(\lim_{u\to\infty}\frac{u^2}{u^2+1}\right) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2.$$

- The expression is $\infty - \infty$.
- We add the terms with logarithms and evaluate the limit as a limit of continuous function with continuous “outside” component.

Find $I = \int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty}\left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right] =$$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\ln\left(\lim_{u\to\infty}\frac{u^2}{u^2+1}\right) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2.$$

The integral is convergent and the value is $\frac{1}{2}\ln 2$.

Find $I = \int_2^\infty \frac{1}{x \ln x}\,\mathrm{d}x$.

We write

$$I = \lim_{u\to\infty} \int_2^u \frac{1}{x \ln x}\,\mathrm{d}x.$$

The indefinite integral satisfies

$$\int \frac{1}{x \ln x}\,\mathrm{d}x = \int \frac{\frac{1}{x}}{\ln x}\,\mathrm{d}x = \ln|\ln x|$$

and hence

$$I = \int_2^\infty \frac{1}{x \ln x}\,\mathrm{d}x = \lim_{u\to\infty} \int_2^u \frac{1}{x \ln x}\,\mathrm{d}x = \lim_{u\to\infty} \Big[\ln|\ln u| - \ln|\ln 2|\Big] = \infty$$

and the integral diverges.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}} \, dx.$

Find $I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

We start with the definition of this integral.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

We look for the antiderivative.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$x + 1 = t^2$$

We use the substitution which removes the radical.

Find $I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}} \, dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}} \, dx$$

$$\int \frac{1}{x\sqrt{x+1}} \, dx$$

$$x + 1 = t^2$$
$$x = t^2 - 1$$

We solve the substitution for x...

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\begin{aligned} x+1 &= t^2 \\ x &= t^2-1 \\ \mathrm{d}x &= 2t\,\mathrm{d}t \end{aligned}$$

...and find the relation between differentials.

Find $\quad I = \displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,\mathrm{d}t$$

We substitute. . .

Find $I = \int_1^\infty \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,dx$$

$$\int \frac{1}{x\sqrt{x+1}}\,dx \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ dx = 2t\,dt \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,dt = \int \frac{2}{t^2-1}\,dt$$

... and simplify.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,\mathrm{d}t = \int \frac{2}{t^2-1}\,\mathrm{d}t = \ln \frac{t-1}{t+1}$$

We expand into partial fractions and integrate.

$$\int \frac{2}{t^2-1}\,\mathrm{d}t = \int \frac{1}{t-1} - \frac{1}{t+1}\,\mathrm{d}t = \ln|t-1| - \ln|t+1|$$

$$= \ln\frac{|t-1|}{|t+1|} = \ln\frac{t-1}{t+1}$$

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,\mathrm{d}t = \int \frac{2}{t^2-1}\,\mathrm{d}t = \ln \frac{t-1}{t+1}$$

$$= \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

We use back substitution $t = \sqrt{x+1}$.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

The antiderivative is known. We continue with the definite integral.

Find $\quad I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^{u}$$

We use Newton–Leibniz formula.

Find $I = \int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

The application of Newton–Leibniz formula gives this value.

Find $I = \int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty}\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

The improper integral is a limit of the definite integral.

Find $\quad I = \displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty}\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty}\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right)$$

We use theorem concerning the limit of composite function.

Find $I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,dx \qquad \int \frac{1}{x\sqrt{x+1}}\,dx = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,dx = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

We have the indeterminate form $\frac{\infty}{\infty}$ and l'Hospital rule can be used.

Find $I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1$$

The numerator and denominator cancel.

Find $I = \int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^{u} = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1 = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$\ln 1 = 0$

Find $\quad I = \int_1^\infty \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,dx \qquad \int \frac{1}{x\sqrt{x+1}}\,dx = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,dx = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln \left(\lim_{u\to\infty} \frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1 = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

The problem is resolved.

Find $I = \int_0^{\infty} x e^{-x^2}\, \mathrm{d}x$

Find $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} xe^{-x^2}\,\mathrm{d}x$$

We start with the definition of the improper integral.

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x$$

We evaluate the indefinite integral first.

Find $I = \int_0^{\infty} x e^{-x^2} \, dx$

$$I = \lim_{u \to \infty} \int_0^u x e^{-x^2} \, dx$$

$$\int x e^{-x^2} \, dx$$

$$-x^2 = t$$

The composite function suggest the substitution for the inside function $(-x^2)$.

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x$$

$$-x^2 = t$$
$$-2x\,\mathrm{d}x = \mathrm{d}t$$
$$x\,\mathrm{d}x = -\frac{1}{2}\,\mathrm{d}t$$

We find the relationship between differentials. The expression $x\,\mathrm{d}x$ is present in the integral and the integral is ready for substitution.

Find $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} xe^{-x^2}\,\mathrm{d}x$$

$$\int xe^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t$$

We substitute,...

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t$$

evaluate the integral,...

Find $I = \int_0^{\infty} x e^{-x^2} \, dx$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2} \, dx$$

$$\int x e^{-x^2} \, dx \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x \, dx &= dt \\ x \, dx &= -\frac{1}{2} \, dt \end{aligned}} = -\frac{1}{2} \int e^t \, dt = -\frac{1}{2} e^t = \frac{1}{2} e^{-x^2}$$

and go back to the variable x.

Find $I = \int_0^{\infty} x e^{-x^2}\, \mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^u x e^{-x^2}\, \mathrm{d}x$$

$$\int x e^{-x^2}\, \mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\, \mathrm{d}x &= \mathrm{d}t \\ x\, \mathrm{d}x &= -\frac{1}{2}\, \mathrm{d}t \end{aligned}} = -\frac{1}{2} \int e^t\, \mathrm{d}t = -\frac{1}{2} e^t = \frac{1}{2} e^{-x^2}$$

$$\int_0^u x e^{-x^2}\, \mathrm{d}x$$

We continue with definite integral.

Find $I = \int_0^{\infty} x e^{-x^2}\, dx$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\, dx$$

$$\int x e^{-x^2}\, dx \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\, dx &= dt \\ x\, dx &= -\frac{1}{2}\, dt \end{aligned}} = -\frac{1}{2}\int e^t\, dt = -\frac{1}{2} e^t = \frac{1}{2} e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\, dx = \left[-\frac{1}{2} e^{-x^2}\right]_0^u$$

The antiderivative is known and we can use Newton–Leibniz formula.

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right)$$

An application of Newton–Leibniz formula gives this value...

Find $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^u xe^{-x^2}\,\mathrm{d}x$$

$$\int xe^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u xe^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

... which can be simplified.

Find $I = \int_0^\infty xe^{-x^2}\,dx$

$$I = \lim_{u\to\infty} \int_0^u xe^{-x^2}\,dx$$

$$\int xe^{-x^2}\,dx \quad \boxed{\begin{array}{c} -x^2 = t \\ -2x\,dx = dt \\ x\,dx = -\frac{1}{2}\,dt \end{array}} = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u xe^{-x^2}\,dx = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty}\frac{1}{2}e^{-u^2}$$

The improper integral is by definition limit of the definite integral.

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty}\frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty}$$

$\infty^2 = \infty$ (in the sense of limits)

Find $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty} \frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty} = \frac{1}{2}$$

$e^{-\infty} = 0$ (in the sense of limits)

Find $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^u xe^{-x^2}\,\mathrm{d}x$$

$$\int xe^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u xe^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty}\frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty} = \frac{1}{2}$$

The problem is solved.

Find $\quad I = \int_0^{\infty} x^2 e^{-x} \, dx.$

Find $I = \int_0^{\infty} x^2 e^{-x}\, dx.$

$$I = \lim_{u\to\infty} \int_0^{u} x^2 e^{-x}\, dx$$

We start with the definition of the improper integral.

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\, \mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\, \mathrm{d}x$$

$$\int x^2 e^{-x}\, \mathrm{d}x$$

We evaluate the antiderivative first.

Find $\quad I = \int_0^{\infty} x^2 e^{-x} \,\mathrm{d}x.$

$$I = \lim_{u \to \infty} \int_0^u x^2 e^{-x} \,\mathrm{d}x$$

$$\int x^2 e^{-x} \,\mathrm{d}x = -x^2 e^{-x} + 2 \int x e^{-x} \,\mathrm{d}x$$

We integrate by parts with $\begin{array}{ll} u = x^2 & u' = 2x \\ v' = e^{-x} & v = -e^{-x} \end{array}$.

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

We integrate by parts with $\boxed{\begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array}}$.

Find $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right)$$

We evaluate the integral ...

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

... and take out the repeating term $-e^{-x}$.

Find $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

We continue with the definite integral. The antiderivative is known and Newton–Leibniz formula can be used.

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\,(-x e^{-x} - e^{-x}) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - [-e^0(0 + 0 + 2)]$$

The application of the formula gives this value. This can be simplified.

Find $\quad I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

This is the definite integral. It remains to evaluate the limit.

Find $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^{0}(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2)$$

$\lim_{u\to\infty} e^{-u} = 0$ and $\boxed{0 \times \infty}$ is an indeterminate form.

Find $\quad I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

We convert the indeterminate form into quotient and use l'Hospital rule.

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

After application of l'Hospital rule we have still $\boxed{\frac{\infty}{\infty}}$. We use l'Hospital rule again.

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u + 2}{e^u}$$

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\,(-xe^{-x} - e^{-x}) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

Now we have $\lim\limits_{u\to\infty} \frac{2}{e^u} = \frac{2}{e^{\infty}} = \frac{2}{\infty} = 0.$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2+2u+2) = 2 - \lim_{u\to\infty} \frac{u^2+2u+2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u+2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u}$$

Find $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^{u} x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\,(-xe^{-x} - e^{-x}) = -e^{-x}(x^2+2x+2)$$

$$\int_0^{u} x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

Now we have $\lim_{u\to\infty} \frac{2}{e^u} = \frac{2}{e^{\infty}} = \frac{2}{\infty} = 0.$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2+2u+2) = 2 - \lim_{u\to\infty} \frac{u^2+2u+2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u+2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u} = 2 - 0$$

Find $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

$$= -e^{-u}(u^2+2u+2) - \left[-e^0(0+0+2)\right] = -e^{-u}(u^2+2u+2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2+2u+2) = 2 - \lim_{u\to\infty} \frac{u^2+2u+2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u+2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u} = 2 - 0 = 2$$

Find $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2+2x+2)$$

$$\int_0^u x^2e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

The problem is solved

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2+2u+2) = 2 - \lim_{u\to\infty}\frac{u^2+2u+2}{e^u}$$

$$= 2 - \lim_{u\to\infty}\frac{2u+2}{e^u} = 2 - \lim_{u\to\infty}\frac{2}{e^u} = 2 - 0 = 2$$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

We start with the definition of the improper integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

We evaluate the indefinite integral first.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$\operatorname{arctg} x = t$

We use the substitution $\operatorname{arctg} x = t$.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \qquad \begin{array}{|c|}\hline \operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t \\ \hline \end{array}$$

With this substitution we have $\frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t$ and the term $\frac{1}{x^2+1}\,\mathrm{d}x$ is present in the integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c} \operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t \end{array}} = \int t\,\mathrm{d}t$$

We substitute,...

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2}$$

... evaluate the integral ...

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

... and return to the variable x.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

We continue with the definite integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{gathered}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{gathered}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u$$

The antiderivative is known.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1} \,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1} \,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1} \,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t \,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1} \,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2}$$

Newton–Leibniz formula yields the value of the integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^{u} = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

Simplifications can be made.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{aligned} \operatorname{arctg} x &= t \\ \frac{1}{x^2+1}\,\mathrm{d}x &= \mathrm{d}t \end{aligned}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^{u} = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$
$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

We continue with the improper integral. It is a limit of the definite integral.

Find $I = \int_1^\infty \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$
$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32}$$

The function $y = \operatorname{arctg} x$ has an horizontal asymptote $y = \frac{\pi}{2}$ in $+\infty$. This is the value of the limit $\lim_{u\to\infty} \operatorname{arctg} u$.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{aligned} \operatorname{arctg} x &= t \\ \frac{1}{x^2+1}\,dx &= dt \end{aligned}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^{u} = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32}$$

We simplify.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c} \operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,dx = dt \end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^{u} = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$
$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

The integral is evaluated.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

We start with the integral.

Find $I = \displaystyle\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

There are two singularities: $\pm\infty$.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

We divide into two integrals on half-lines.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

We evaluate the indefinite integral.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2} \, \mathrm{d}x$$

We simplify the integrand. . .

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2} \,\mathrm{d}x$$

$$e^{x} = t$$
$$e^{x} \,\mathrm{d}x = \mathrm{d}t$$

...and substitute.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \int \frac{e^{x}}{1 + (e^{x})^2} \, dx \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x} \, dx = dt \end{array}} \quad = \int \frac{1}{1 + t^2} \, dt$$

The substitution gives this integral...

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}} \,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^{2}} \,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^{2}} \,\mathrm{d}t = \operatorname{arctg} t$$

. . . which can be integrated by direct formula.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x}=t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

Finally we return to the original variable.

$$\boxed{\text{Find } I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\boxed{\text{Find } I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^{2}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^{2}}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1 + t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \int \frac{e^x}{1 + (e^x)^2} \, dx \quad \boxed{\begin{array}{c} e^x = t \\ e^x \, dx = \, dt \end{array}} \quad = \int \frac{1}{1 + t^2} \, dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^x$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx = \left[\operatorname{arctg} e^x\right]_u^0 = \operatorname{arctg} e^0 - \operatorname{arctg} e^u = \operatorname{arctg} 1 - \operatorname{arctg} e^u$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^u$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^{2}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x}=t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^{2}}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to-\infty} \left(\frac{\pi}{4} - \operatorname{arctg} e^{u}\right)$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx \quad \boxed{\begin{array}{c} e^x = t \\ e^x\,dx = dt \end{array}} \quad = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^x$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^x]_u^0 = \operatorname{arctg} e^0 - \operatorname{arctg} e^u = \operatorname{arctg} 1 - \operatorname{arctg} e^u$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^u$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to-\infty} \left(\frac{\pi}{4} - \operatorname{arctg} e^u\right) = \frac{\pi}{4} - \operatorname{arctg} e^{-\infty}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x}=t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to-\infty}\left(\frac{\pi}{4} - \operatorname{arctg} e^{u}\right) = \frac{\pi}{4} - \operatorname{arctg} e^{-\infty} = \frac{\pi}{4} - \operatorname{arctg} 0$$

$$\boxed{\text{Find } I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty}\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty}\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx \quad \boxed{\begin{gathered} e^x = t \\ e^x\,dx = dt \end{gathered}} \quad = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^x$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \left[\operatorname{arctg} e^x\right]_u^0 = \operatorname{arctg} e^0 - \operatorname{arctg} e^u = \operatorname{arctg} 1 - \operatorname{arctg} e^u$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^u$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to-\infty}\left(\frac{\pi}{4} - \operatorname{arctg} e^u\right) = \frac{\pi}{4} - \operatorname{arctg} e^{-\infty} = \frac{\pi}{4} - \operatorname{arctg} 0$$

$$= \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_0^u$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}} \, dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}} \, dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}} \, dx$$

$$\int \frac{1}{e^{-x}+e^{x}} \, dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}} \, dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}} \, dx = \left[\operatorname{arctg} e^{x}\right]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right)$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \lim_{u \to \infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4} \right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \lim_{u \to \infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4} \right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x} + e^{-x}} \, dx$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x}+e^{-x}}\,dx = \frac{\pi}{4} + \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x}+e^{-x}}\,dx = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x} + e^{-x}} \,\mathrm{d}x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Finished.