# Matching Game – Integral by substitution

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**Instructions:** Assign an indefinite integral and the most convenient substitution. Select an integral clicking its checkbox and find the substitution. No guessing! A maximum of $3$ tries on any problem before you get $3$ penalty points! Passing is to complete the puzzle with only $4$ incorrect answers.

**To the picture:** There are several different types of integral in mathematics. Now we are familiar with the indefinite integral. Today's **Mr. X** is one of few mathematitians which have their own integral – integral which bears their name[1].

**Mr. X.**, a prominent Czech scientist, was born on May 7, 1926 in Prague. He is a specialist in ordinary differential equations and theory of integral. The simplicity of definition of his integral made some educators advocate that this integral should replace the usual Riemann integral in introductory calculus courses, but this idea never quite popularized.

**Mr. X** has been awarded the highest possible scientific prize of the Czech Republic, **Česká hlava** (the "Czech Mind") of the year 2006, as an acknowledgement of his life achievements.

Who is Mr. X? – Solve problems on next page.

[1] The same integral has been defined independently by British mathematitian Ralph Henstock and the integral has now the name **Mr. X**–Hensctock integral.

## Questions

1. $\int \frac{\arcsin^2 x}{\sqrt{1-x^2}}\,\mathrm{d}x$

2. $\int x^2 e^{x^3-1}\,\mathrm{d}x$

3. $\int x\sqrt{1+x^2}\,\mathrm{d}x$

4. $\int \sin^3 x\,\mathrm{d}x$

5. $\int 3xe^{-x^2}\,\mathrm{d}x$

6. $\int \frac{x+\sqrt[3]{x+2}}{x+3}\,\mathrm{d}x$

7. $\int \frac{\sqrt{x-3}}{x+1}\,\mathrm{d}x$

8. $\int \frac{1}{x}\sin(\ln x)\,\mathrm{d}x$

9. $\int e^x \cos(e^x)\,\mathrm{d}x$

10. $\int \frac{\sin^2 x \cos x}{\sin^2 x+1}\,\mathrm{d}x$

11. $\int \frac{1}{x+\sqrt{x+1}}\,\mathrm{d}x$

12. $\int \frac{\sin^2 x}{\cos^4 x}\,\mathrm{d}x$

## Answers

a. $e^x = t$

b. $x = \sin^2 t$

c. $x+1 = t^2$

d. $1-x^2 = t$

e. $x = t^2 - 3$

f. $\cos x = t$

g. $x^3 - 1 = t$

h. $\sin x = t$

i. $x - 3 = t^2$

j. $-x^2 = t$

k. $x + 2 = t^3$

l. $\operatorname{tg} x = t$

m. $\sqrt{1-x^2} = t$

n. $x = t^3$

o. $x = \cos^2 t$

p. $1 + x^2 = t^2$

q. $1 + x^2 = \sqrt{t}$

r. $\ln x = t$

s. $x + 2 = t^2$

t. $\arcsin x = t$

u. $x = t^2$