Derivatives

Robert Mařík

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Differentiate $y = \dfrac{x}{x^2+1}$.

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Differentiate $y = \dfrac{x}{x^2+1}$.

$$y' = \left(\frac{x}{x^2+1}\right)'$$

The function is a quotient.

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Differentiate $y = \dfrac{x}{x^2+1}$.

$$y' = \left(\frac{x}{x^2+1}\right)'$$
$$= \frac{(x)' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2}$$

We use the quotient rule $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

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Differentiate $y = \dfrac{x}{x^2+1}$.

$$y' = \left(\frac{x}{x^2+1}\right)'$$

$$= \frac{(x)' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2}$$

$$= \frac{1 \cdot (x^2+1) - x \cdot (2x+0)}{(x^2+1)^2}$$

$x' = 1$ by the power rule. $(x^2+1)' = (x^2)' + (1)' = 2x + 0 = 2x$ by the sum rule and the power rule.

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Differentiate $y = \dfrac{x}{x^2+1}$.

$$\begin{aligned} y' &= \left(\frac{x}{x^2+1}\right)' \\ &= \frac{(x)' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2} \\ &= \frac{1 \cdot (x^2+1) - x \cdot (2x+0)}{(x^2+1)^2} \\ &= \frac{1-x^2}{(1+x^2)^2} \end{aligned}$$

We multiply the parentheses and simplify the numerator.

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Differentiate $y = \dfrac{x}{x^2+1}$.

$$\begin{aligned} y' &= \left(\frac{x}{x^2+1}\right)' \\ &= \frac{(x)'\cdot(x^2+1) - x\cdot(x^2+1)'}{(x^2+1)^2} \\ &= \frac{1\cdot(x^2+1) - x\cdot(2x+0)}{(x^2+1)^2} \\ &= \frac{1-x^2}{(1+x^2)^2} \end{aligned}$$

The problem is finished.

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

$$y' = \left(\frac{1 - x^3}{x^2}\right)'$$

The function is a quotient.

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

$$y' = \left(\frac{1 - x^3}{x^2}\right)'$$
$$= \frac{(1 - x^3)' \cdot x^2 - (1 - x^3) \cdot (x^2)'}{(x^2)^2}$$

We use the quotient rule $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

$$y' = \left(\frac{1 - x^3}{x^2}\right)'$$
$$= \frac{(1 - x^3)' \cdot x^2 - (1 - x^3) \cdot (x^2)'}{(x^2)^2}$$
$$= \frac{(0 - 3x^2) \cdot x^2 - (1 - x^3) \cdot 2x}{(x^2)^2}$$

We differentiate $(1 - x^3)'$ by the sum rule and the power rule. The expression x^2 can be differentiated using the power rule.

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

$$y' = \left(\frac{1 - x^3}{x^2}\right)'$$
$$= \frac{(1 - x^3)' \cdot x^2 - (1 - x^3) \cdot (x^2)'}{(x^2)^2}$$
$$= \frac{(0 - 3x^2) \cdot x^2 - (1 - x^3) \cdot 2x}{(x^2)^2}$$
$$= \frac{-3x^4 - 2x + 2x^4}{x^4}$$

We multiply the parentheses.

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Differentiate $y = \dfrac{1 - x^3}{x^2}$

$$y' = \left(\frac{1-x^3}{x^2}\right)'$$

$$= \frac{(1-x^3)' \cdot x^2 - (1-x^3) \cdot (x^2)'}{(x^2)^2}$$

$$= \frac{(0-3x^2) \cdot x^2 - (1-x^3) \cdot 2x}{(x^2)^2}$$

$$= \frac{-3x^4 - 2x + 2x^4}{x^4} = -\frac{2+x^3}{x^3}$$

We add the common powers of x.

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Differentiate $y = \frac{1 - x^3}{x^2}$

$$y' = \left(\frac{1 - x^3}{x^2}\right)'$$
$$= \frac{(1 - x^3)' \cdot x^2 - (1 - x^3) \cdot (x^2)'}{(x^2)^2}$$
$$= \frac{(0 - 3x^2) \cdot x^2 - (1 - x^3) \cdot 2x}{(x^2)^2}$$
$$= \frac{-3x^4 - 2x + 2x^4}{x^4} = -\frac{2 + x^3}{x^3}$$

We finished.

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Differentiate $y = x\ln^2 x$.

$$y' = (\ x\ \ \ln^2 x\ )'$$

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Differentiate $y = x \ln^2 x$.

$$y' = (\ x\ \ \ln^2 x\ )' = (x)'\ \ln^2 x + x\ (\ln^2 x)'$$

We differentiate the product $(uv)'$ with $u = x$ and $v' = \ln^2 x$.

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Differentiate $y = x \ln^2 x$.

$$y' = (\ x\ \ \ln^2 x\ )' = (x)'\ \ \ln^2 x + x\ (\ln^2 x)'$$
$$= 1\ \ \ln^2 x$$

Derivative of x is formula.

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Differentiate $y = x \ln^2 x$.

$$y' = (x \ \ln^2 x)' = (x)' \ \ln^2 x + x \ (\ln^2 x)'$$
$$= 1 \ \ln^2 x + x \ 2 \ln x \ (\ln x)'$$

The function $\ln^2 x$ is a composite function $(\ln x)^2$. The outside function is the quadratic function and the inside function is the logarithmic function. We use the chain rule.

Differentiate $y = x \ln^2 x$.

$$y' = (\,x\ \ln^2 x\,)' = (x)'\ \ln^2 x + x\ (\ln^2 x)'$$
$$= 1\ \ln^2 x + x\ 2\ln x\ (\ln x)'$$

$$= \ln^2 x + x\ 2\ln x\ \frac{1}{x}$$

The derivative of logarithm is a formula.

Differentiate $y = x \ln^2 x$.

$$y' = (\,x\ \ln^2 x\,)' = (x)'\ \ln^2 x + x\ (\ln^2 x)'$$
$$= 1\ \ln^2 x + x\ 2\ln x\ (\ln x)'$$
$$= \ln^2 x + x\ 2\ln x\ \frac{1}{x}$$
$$= (2 + \ln x)\ln x$$

$x\frac{1}{x} = 1$ and the common factor $\ln x$ can be taken out.

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Differentiate $y = x \ln^2 x$.

$$\begin{aligned} y' &= (\ x\ \ \ln^2 x\ )' = (x)'\ \ \ln^2 x + x\ \ (\ln^2 x)' \\ &= 1\ \ \ln^2 x + x\ 2\ln x\ (\ln x)' \\ &= \ln^2 x + x\ 2\ln x\ \frac{1}{x} \\ &= (2+\ln x)\ln x \end{aligned}$$

Finished!

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Differentiate $y = (x^2 + 3x)e^{-2x}$

Differentiate $y = (x^2 + 3x)e^{-2x}$

$$y' = \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)'$$

We differentiate product of the function $u = x^2 + 3x$ and $v = e^{-2x}$. We use the product rule.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$y' = \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)'$$
$$= \left((x^2)' + 3(x)'\right) e^{-2x}$$

We differentiate the sum. We use the sum rule and the constant multiple rule.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$y' = \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)'$$
$$= \left((x^2)' + 3(x)'\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)'$$

We differentiate the composite function e^{-2x}. The outside function is an exponential function which does not change by differentiation. The inside function is the linear function $-2x$.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$
\begin{aligned}
y' &= \Big(x^2 + 3x\Big)' e^{-2x} + (x^2 + 3x)\Big(e^{-2x}\Big)' \\
&= \Big((x^2)' + 3(x)'\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)' \\
&= \Big(2x + 3 \cdot 1\Big) e^{-2x}
\end{aligned}
$$

Derivatives of x^2 and x can be evaluated by power rule.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$y' = \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)'$$
$$= \left((x^2)' + 3(x)'\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)'$$
$$= \left(2x + 3 \cdot 1\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)(x)'$$

Derivative of $(-2x)$ can be evaluated by a constant multiple rule ...

Differentiate $y = (x^2 + 3x)e^{-2x}$

$$y' = \Big(x^2 + 3x\Big)' e^{-2x} + (x^2 + 3x)\Big(e^{-2x}\Big)'$$
$$= \Big((x^2)' + 3(x)'\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)'$$
$$= \Big(2x + 3 \cdot 1\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)(x)'$$
$$= \Big(2x + 3\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)1$$

...and power rule ($x = x^1$ and hence $x' = 1x^0 = 1$).

Differentiate $y = (x^2 + 3x)e^{-2x}$

$$\begin{aligned} y' &= \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)' \\ &= \left((x^2)' + 3(x)'\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)' \\ &= \left(2x + 3 \cdot 1\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)(x)' \\ &= \left(2x + 3\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)1 \\ &= \left(2x + 3 + (-2)(x^2 + 3x)\right) e^{-2x} \end{aligned}$$

The common factor e^{-2x} can be taken out.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$\begin{aligned}
y' &= \Big(x^2 + 3x\Big)' e^{-2x} + (x^2 + 3x)\Big(e^{-2x}\Big)' \\
&= \Big((x^2)' + 3(x)'\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)' \\
&= \Big(2x + 3 \cdot 1\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)(x)' \\
&= \Big(2x + 3\Big) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)1 \\
&= \Big(2x + 3 + (-2)(x^2 + 3x)\Big) e^{-2x} \\
&= \Big(-2x^2 - 4x + 3\Big) e^{-2x}
\end{aligned}$$

We simplify inside the parentheses.

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Differentiate $y = (x^2 + 3x)e^{-2x}$

$$\begin{aligned}
y' &= \left(x^2 + 3x\right)' e^{-2x} + (x^2 + 3x)\left(e^{-2x}\right)' \\
&= \left((x^2)' + 3(x)'\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2x)' \\
&= \left(2x + 3 \cdot 1\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)(x)' \\
&= \left(2x + 3\right) e^{-2x} + (x^2 + 3x)e^{-2x}(-2)1 \\
&= \left(2x + 3 + (-2)(x^2 + 3x)\right) e^{-2x} \\
&= \left(-2x^2 - 4x + 3\right) e^{-2x} = -\left(2x^2 + 4x - 3\right) e^{-2x}
\end{aligned}$$

We take out the minus sign. Finished!

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Differentiate $y = \sqrt[3]{\frac{1+x^3}{1-x^3}}$.

$$=$$

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Differentiate $y = \sqrt[3]{\frac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-2/3}$$

We consider the third root to be the power with exponent $\frac{1}{3}$. We use the power rule.

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Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-2/3}\left(\frac{1+x^3}{1-x^3}\right)'$$

The expression inside the root is a function. We use the chain rule and multiply by the derivative of the inside function.

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Differentiate $y = \sqrt[3]{\frac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-2/3}\left(\frac{1+x^3}{1-x^3}\right)'$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3}\frac{(1+x^3)'(1-x^3)-(1+x^3)(1-x^3)'}{(1-x^3)^2}$$

The inside function is a fraction. We use the quotient rule $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$.

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Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-2/3}\left(\frac{1+x^3}{1-x^3}\right)'$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3}\frac{(1+x^3)'(1-x^3)-(1+x^3)(1-x^3)'}{(1-x^3)^2}$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3}\frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{(1-x^3)^2}$$

Derivative of the numerator and denominator is simply the sum rule and the power rule.

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Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{(1-x^3)^2}$$

Up to now we have this.

Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{(1-x^3)^2}$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{6x^2}{(1-x^3)^2}$$

We simplify the numerator . . .

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Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{(1-x^3)^2}$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{6x^2}{(1-x^3)^2}$$

$$= \sqrt[3]{\frac{1+x^3}{1-x^3}}\,\frac{1-x^3}{1+x^3}\,\frac{2x^2}{(1-x^3)^2}$$

. . . and do some other simplifications.

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Differentiate $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

$$y' = \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{(1-x^3)^2}$$

$$= \frac{1}{3}\left(\frac{1-x^3}{1+x^3}\right)^{2/3} \frac{6x^2}{(1-x^3)^2}$$

$$= \sqrt[3]{\frac{1+x^3}{1-x^3}}\,\frac{1-x^3}{1+x^3}\,\frac{2x^2}{(1-x^3)^2} = \sqrt[3]{\frac{1+x^3}{1-x^3}}\,\frac{2x^2}{1-x^6}$$

The problem is solved.

Differentiate $y = \left(\dfrac{x-1}{x+1}\right)^2$.

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Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$

The function is a second power of fraction. The power function is the outside function and is differentiated as the first, using the rule $(x^2)' = 2x$.
The derivative of the inside function follows.

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Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$
$$= 2\frac{x-1}{x+1}\cdot\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$

The fraction is differentiated by the rule

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}.$$

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Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$
$$= 2\frac{x-1}{x+1} \cdot \frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$
$$= 2\frac{x-1}{x+1} \cdot \frac{1.(x+1)-(x-1).1}{(x+1)^2}$$

It is easy to differentiate the expressions in the numerator. We use the sum rule and the rules $(x)' = 1$ and $(1)' = 0$.

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Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$

$$= 2\frac{x-1}{x+1} \cdot \frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1} \cdot \frac{1.(x+1)-(x-1).1}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1} \cdot \frac{2}{(x+1)^2}$$

We simplify the numerator and multiply.

Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$

$$= 2\frac{x-1}{x+1}\cdot\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1}\cdot\frac{1.(x+1)-(x-1).1}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1}\cdot\frac{2}{(x+1)^2} = 4\frac{x-1}{(x+1)^3}$$

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Differentiate $y = \left(\frac{x-1}{x+1}\right)^2$.

$$y' = 2\frac{x-1}{x+1}\left(\frac{x-1}{x+1}\right)'$$

$$= 2\frac{x-1}{x+1} \cdot \frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1} \cdot \frac{1.(x+1)-(x-1).1}{(x+1)^2}$$

$$= 2\frac{x-1}{x+1} \cdot \frac{2}{(x+1)^2} = 4\frac{x-1}{(x+1)^3}$$

Finished!

Differentiate $y' = x\ln(x^2 - 1)$.

y'

Differentiate $y' = x\ln(x^2-1)$.

$$y' = x'\ln(x^2-1) + x\Big(\ln(x^2-1)\Big)'$$

The function is a product of two functions. We use the product rule

$$(uv)' = u'v + uv'$$

with $u = x$ and $v = \ln(x^2-1)$.

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Differentiate $y' = x \ln(x^2 - 1)$.

$$y' = x' \ln(x^2 - 1) + x\Big(\ln(x^2 - 1)\Big)'$$
$$= 1 \ln(x^2 - 1) + x \frac{1}{x^2 - 1}(x^2 - 1)'$$

Derivative of $u = x$ is easy. The function $\ln(x^2 - 1)$ is a composite function with the outside function $\ln(\cdot)$ and the inside function $x^2 - 1$.

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Differentiate $y' = x\ln(x^2-1)$.

$$y' = x'\ln(x^2-1) + x\Big(\ln(x^2-1)\Big)'$$
$$= 1\ln(x^2-1) + x\frac{1}{x^2-1}(x^2-1)'$$
$$= \ln(x^2-1) + x\frac{1}{x^2-1}2x$$

$$(x^2-1)' = 2x - 0 = 2x$$

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Differentiate $y' = x\ln(x^2-1)$.

$$\begin{aligned} y' &= x'\ln(x^2-1) + x\Big(\ln(x^2-1)\Big)' \\ &= 1\ln(x^2-1) + x\frac{1}{x^2-1}(x^2-1)' \\ &= \ln(x^2-1) + x\frac{1}{x^2-1}2x \\ &= \ln(x^2-1) + \frac{2x^2}{x^2-1} \end{aligned}$$

We simplify.

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Differentiate $y' = x\ln(x^2-1)$.

$$
\begin{aligned}
y' &= x'\ln(x^2-1) + x\Big(\ln(x^2-1)\Big)' \\
&= 1\ln(x^2-1) + x\frac{1}{x^2-1}(x^2-1)' \\
&= \ln(x^2-1) + x\frac{1}{x^2-1}2x \\
&= \ln(x^2-1) + \frac{2x^2}{x^2-1}
\end{aligned}
$$

Finished!

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Differentiate $y = \dfrac{1}{4} \ln \dfrac{x^2-1}{x^2+1}$.

y'

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Differentiate $y = \frac{1}{4} \ln \frac{x^2 - 1}{x^2 + 1}$.

$$y' = \frac{1}{4}$$

The function is a constant multiple of a logarithm. We use the multiple rule.

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Differentiate $y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$.

$$y' = \frac{1}{4} \frac{x^2+1}{x^2-1}$$

The logarithm contains the fraction as its inside function. We use the rule $(\ln(x))' = \frac{1}{x}$ and the chain rule.

Remember that $\frac{1}{\frac{x^2-1}{x^2+1}} = \frac{x^2+1}{x^2-1}$.

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Differentiate $y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$.

$$y' = \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2}$$

We continue in using the chain rule. We evaluate the derivative of the inside function by the quotient rule.

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Differentiate $y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$.

$$y' = \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2}$$
$$= \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{4x}{(x^2+1)^2}$$

We simplify the numerator of the last fraction. Terms with x^3 cancel and $4x$ remains.

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Differentiate $y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$.

$$y' = \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2}$$
$$= \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{4x}{(x^2+1)^2}$$
$$= \frac{x}{(x^2-1)(x^2+1)}$$

We multiply the fraction.

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Differentiate $y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$.

$$y' = \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{2x(x^2+1) - (x^2-1)2x}{(x^2+1)^2}$$

$$= \frac{1}{4} \frac{x^2+1}{x^2-1} \frac{4x}{(x^2+1)^2}$$

$$= \frac{x}{(x^2-1)(x^2+1)}$$

Finished!

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Differentiate $y = \sqrt{x+1} - \ln(1 + \sqrt{x+1})$.

$$y'$$

Differentiate $y = \sqrt{x+1} - \ln(1 + \sqrt{x+1})$.

$$y' = \frac{1}{2\sqrt{x+1}} - \frac{1}{1+\sqrt{x+1}} \left(0 + \frac{1}{2\sqrt{x+1}}\right)$$

$$(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

by the power rule. We combine this rule by the chain rule and hence

$$(\sqrt{x+1})' = \frac{1}{2\sqrt{x+1}} \cdot 1 = \frac{1}{2\sqrt{x+1}}$$

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Differentiate $y = \sqrt{x+1} - \ln(1+\sqrt{x+1})$.

$$y' = \frac{1}{2\sqrt{x+1}} - \frac{1}{1+\sqrt{x+1}}\left(0 + \frac{1}{2\sqrt{x+1}}\right)$$
$$= \frac{1}{2\sqrt{x+1}}\left(1 - \frac{1}{1+\sqrt{x+1}}\right)$$

We take out the common factor $\frac{1}{2\sqrt{x+1}}$.

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Differentiate $y = \sqrt{x+1} - \ln(1+\sqrt{x+1})$.

$$y' = \frac{1}{2\sqrt{x+1}} - \frac{1}{1+\sqrt{x+1}}\left(0 + \frac{1}{2\sqrt{x+1}}\right)$$

$$= \frac{1}{2\sqrt{x+1}}\left(1 - \frac{1}{1+\sqrt{x+1}}\right)$$

$$= \frac{1}{2\sqrt{x+1}}\frac{\sqrt{x+1}}{1+\sqrt{x+1}}$$

We convert into common denominator in the parentheses and add.

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Differentiate $y = \sqrt{x+1} - \ln(1+\sqrt{x+1})$.

$$y' = \frac{1}{2\sqrt{x+1}} - \frac{1}{1+\sqrt{x+1}}\left(0 + \frac{1}{2\sqrt{x+1}}\right)$$

$$= \frac{1}{2\sqrt{x+1}}\left(1 - \frac{1}{1+\sqrt{x+1}}\right)$$

$$= \frac{1}{2\sqrt{x+1}}\frac{\sqrt{x+1}}{1+\sqrt{x+1}}$$

$$= \frac{1}{2(1+\sqrt{x+1})}$$

We cancel the factor $\sqrt{x+1}$. Finished.

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Differentiate $y = \sqrt{1-x} \arcsin \sqrt{x}$

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Differentiate $y = \sqrt{1-x} \arcsin \sqrt{x}$

$$y' = (\sqrt{1-x})'. \arcsin \sqrt{x} + \sqrt{1-x}.(\arcsin \sqrt{x})'$$

Product rule.

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Differentiate $y = \sqrt{1-x}\arcsin\sqrt{x}$

$$y' = (\sqrt{1-x})' . \arcsin\sqrt{x} + \sqrt{1-x} . (\arcsin\sqrt{x})'$$
$$= \frac{1}{2\sqrt{1-x}} . (1-x)' . \arcsin\sqrt{x}$$
$$+ \sqrt{1-x} . \frac{1}{\sqrt{1-(\sqrt{x})^2}} (\sqrt{x})'$$

Chain rules for $\sqrt{1-x}$ and for $\arcsin(\sqrt{x})$

Differentiate $y = \sqrt{1-x}\arcsin\sqrt{x}$

$$y' = (\sqrt{1-x})'.\arcsin\sqrt{x} + \sqrt{1-x}.(\arcsin\sqrt{x})'$$

$$= \frac{1}{2\sqrt{1-x}}.(1-x)'.\arcsin\sqrt{x}$$

$$+ \sqrt{1-x}.\frac{1}{\sqrt{1-(\sqrt{x})^2}}(\sqrt{x})'$$

$$= -\frac{1}{2\sqrt{1-x}}\arcsin\sqrt{x} + \sqrt{1-x}\frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}$$

Evalauted derivatives of the inside functions

Differentiate $y = \sqrt{1-x} \arcsin \sqrt{x}$

$$y' = (\sqrt{1-x})'. \arcsin \sqrt{x} + \sqrt{1-x}.(\arcsin \sqrt{x})'$$

$$= \frac{1}{2\sqrt{1-x}}.(1-x)'. \arcsin \sqrt{x}$$

$$+ \sqrt{1-x}. \frac{1}{\sqrt{1-(\sqrt{x})^2}}(\sqrt{x})'$$

$$= -\frac{1}{2\sqrt{1-x}} \arcsin \sqrt{x} + \sqrt{1-x}\frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}$$

$$= -\frac{\arcsin \sqrt{x}}{2\sqrt{1-x}} + \frac{1}{2\sqrt{x}}$$

$\sqrt{1-x}$ cancels. Finished!

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Differentiate $y = (x^2 + 1)\sin x + x\cos x$

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Differentiate $y = (x^2+1)\sin x + x\cos x$

$$y' = \Big((x^2+1)\sin x\Big)' + (x\cos x)'$$

Sum rule.

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Differentiate $y = (x^2+1)\sin x + x\cos x$

$$y' = \Big((x^2+1)\sin x\Big)' + (x\cos x)'$$
$$= (x^2+1)'\sin x + (x^2+1)(\sin x)' + x'\cos x + x(\cos x)'$$

Two times product rule.

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Differentiate $y = (x^2 + 1)\sin x + x\cos x$

$$y' = \Big((x^2+1)\sin x\Big)' + (x\cos x)'$$
$$= (x^2+1)'\sin x + (x^2+1)(\sin x)' + x'\cos x + x(\cos x)'$$
$$= 2x\sin x + (x^2+1)\cos x + 1\cdot\cos x + x(-\sin x)$$

Formulas.

$(x^2)' = 2x$ $\quad$ $(\sin x)' = \cos x$ $\quad$ $(\cos x)' = -\sin x$

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Differentiate $y = (x^2 + 1)\sin x + x\cos x$

$$y' = \Big((x^2+1)\sin x\Big)' + (x\cos x)'$$
$$= (x^2+1)'\sin x + (x^2+1)(\sin x)' + x'\cos x + x(\cos x)'$$
$$= 2x\sin x + (x^2+1)\cos x + 1\cdot\cos x + x(-\sin x)$$
$$= (2x - x)\sin(x) + (x^2+1+1)\cos x$$

We take out the trigonometric functions.

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Differentiate $y = (x^2 + 1)\sin x + x\cos x$

$$\begin{aligned} y' &= \Big((x^2+1)\sin x\Big)' + (x\cos x)' \\ &= (x^2+1)'\sin x + (x^2+1)(\sin x)' + x'\cos x + x(\cos x)' \\ &= 2x\sin x + (x^2+1)\cos x + 1\cdot\cos x + x(-\sin x) \\ &= (2x-x)\sin(x) + (x^2+1+1)\cos x \\ &= x\sin x + (x^2+2)\cos x \end{aligned}$$

We simplify.

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Differentiate $y = (x^2 + 1)\cos(2x)$

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Differentiate $y = (x^2 + 1)\cos(2x)$

$$y' = (x^2 + 1)'\cos(2x) + (x^2 + 1)\big(\cos(2x)\big)'$$

Product rule.

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Differentiate $y = (x^2+1)\cos(2x)$

$$y' = (x^2+1)'\cos(2x) + (x^2+1)\big(\cos(2x)\big)'$$
$$= 2x\cos(2x) + (x^2+1)\big(-\sin(2x)\big)(2x)'$$

We differentiate composite function.

$$(\cos x)' = -\sin x \qquad [\cos(f(x))]' = -\sin(f(x)) \cdot f'(x)$$

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Differentiate $y = (x^2+1)\cos(2x)$

$$y' = (x^2+1)'\cos(2x) + (x^2+1)\big(\cos(2x)\big)'$$
$$= 2x\cos(2x) + (x^2+1)\big(-\sin(2x)\big)(2x)'$$
$$= 2x\cos(2x) - (x^2+1)\sin(2x)2$$

We differentiate.

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Differentiate $y = (x^2+1)\cos(2x)$

$$\begin{aligned} y' &= (x^2+1)'\cos(2x) + (x^2+1)\big(\cos(2x)\big)' \\ &= 2x\cos(2x) + (x^2+1)\big(-\sin(2x)\big)(2x)' \\ &= 2x\cos(2x) - (x^2+1)\sin(2x)2 \\ &= 2x\cos(2x) - 2(x^2+1)\sin(2x) \end{aligned}$$

We simplify.

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$

$$y' = \frac{\left[(x^2+1)^3\right]' x^4 - (x^2+1)^3 (x^4)'}{(x^4)^2}$$

Quotient rule.

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Differentiate $y = \frac{(x^2+1)^3}{x^4}$

$$y' = \frac{\left[(x^2+1)^3\right]' x^4 - (x^2+1)^3 (x^4)'}{(x^4)^2}$$
$$= \frac{3(x^2+1)^2 (x^2+1)' x^4 - (x^2+1)^3 4x^3}{x^{2\cdot 4}}$$

Chain rule.

$(x^3)' = 3x^2$

$\left[\left(f(x)\right)^3\right]' = 3\left(f(x)\right)^2 f'(x)$

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Differentiate $y = \frac{(x^2+1)^3}{x^4}$

$$y' = \frac{\left[(x^2+1)^3\right]' x^4 - (x^2+1)^3 (x^4)'}{(x^4)^2}$$

$$= \frac{3(x^2+1)^2 (x^2+1)' x^4 - (x^2+1)^3 4x^3}{x^{2\cdot 4}}$$

$$= \frac{3(x^2+1)^2 (2x) x^4 - (x^2+1)^3 4x^3}{x^8}$$

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Differentiate $y = \frac{(x^2+1)^3}{x^4}$

$$y' = \frac{\left[(x^2+1)^3\right]' x^4 - (x^2+1)^3 (x^4)'}{(x^4)^2}$$

$$= \frac{3(x^2+1)^2 (x^2+1)' x^4 - (x^2+1)^3 4x^3}{x^{2\cdot 4}}$$

$$= \frac{3(x^2+1)^2 (2x) x^4 - (x^2+1)^3 4x^3}{x^8}$$

$$= \frac{2(x^2+1)^2 x^3 [3x^2 - 2(x^2+1)]}{x^8}$$

We take out the common factor in the numerator.

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$

$$y' = \frac{\left[(x^2+1)^3\right]' x^4 - (x^2+1)^3 (x^4)'}{(x^4)^2}$$

$$= \frac{3(x^2+1)^2 (x^2+1)' x^4 - (x^2+1)^3 4x^3}{x^{2\cdot 4}}$$

$$= \frac{3(x^2+1)^2 (2x) x^4 - (x^2+1)^3 4x^3}{x^8}$$

$$= \frac{2(x^2+1)^2 x^3 [3x^2 - 2(x^2+1)]}{x^8}$$

$$= 2\frac{(x^2+1)^2 (x^2-2)}{x^5}$$

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$. Perform simplification first.

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$. Perform simplification first.

$$y' = \left[\frac{x^6 + 3x^4 + 3x^2 + 1}{x^4}\right]'$$

We evaluate the power by the formula

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

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Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$. Perform simplification first.

$$y' = \left[\frac{x^6 + 3x^4 + 3x^2 + 1}{x^4}\right]'$$
$$= \left[x^2 + 3 + 3x^{-2} + x^{-4}\right]'$$

We divide ezch term of the numerator.

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Differentiate $y = \frac{(x^2+1)^3}{x^4}$. Perform simplification first.

$$y' = \left[\frac{x^6+3x^4+3x^2+1}{x^4}\right]'$$
$$= \left[x^2+3+3x^{-2}+x^{-4}\right]'$$
$$= 2x+0+3(-2)x^{-3}+(-4)x^{-5}$$

We differentiate the sum (linear combination) of four power functions.

Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$. Perform simplification first.

$$y' = \left[\frac{x^6+3x^4+3x^2+1}{x^4}\right]'$$

$$= \left[x^2+3+3x^{-2}+x^{-4}\right]'$$

$$= 2x+0+3(-2)x^{-3}+(-4)x^{-5}$$

$$= 2x - \frac{6}{x^3} - \frac{4}{x^5}$$

We convert negative powers into fractions.

Differentiate $y = \dfrac{(x^2+1)^3}{x^4}$. Perform simplification first.

$$\begin{aligned} y' &= \left[\frac{x^6+3x^4+3x^2+1}{x^4}\right]' \\ &= \left[x^2+3+3x^{-2}+x^{-4}\right]' \\ &= 2x+0+3(-2)x^{-3}+(-4)x^{-5} \\ &= 2x-\frac{6}{x^3}-\frac{4}{x^5} = \frac{2x^6-6x^2-4}{x^5} \end{aligned}$$

We simplify. The differentiation was easier then in the preceeding method, but solving equation $y'=0$ is more difficult.

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Differentiate $y = \ln\left(x + \arcsin(2\sqrt{x})\right)$

Differentiate $y = \ln\left(x + \arcsin(2\sqrt{x})\right)$

$$y' = \frac{1}{x + \arcsin(2\sqrt{x})}\left(x + \arcsin(2\sqrt{x})\right)'$$

We differentiate the composite function

$$(\ln x)' = \frac{1}{x} \qquad \left(\ln f(x)\right)' = \frac{1}{f(x)} f'(x)$$

Differentiate $y = \ln\left(x + \arcsin(2\sqrt{x})\right)$

$$y' = \frac{1}{x + \arcsin(2\sqrt{x})}\left(x + \arcsin(2\sqrt{x})\right)'$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{1-(2\sqrt{x})^2}}(2\sqrt{x})'\right)$$

The sum rule and the chain rule.

$$\left(\arcsin f(x)\right)' = \frac{1}{\sqrt{1-f^2(x)}}f'(x)$$

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Differentiate $y = \ln\left(x + \arcsin(2\sqrt{x})\right)$

$$y' = \frac{1}{x + \arcsin(2\sqrt{x})}\left(x + \arcsin(2\sqrt{x})\right)'$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{1 - (2\sqrt{x})^2}}(2\sqrt{x})'\right)$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{1 - 4x}} \cdot 2 \cdot \frac{1}{2} \cdot x^{-1/2}\right)$$

We differentiate the composite function

$$\sqrt{x} = x^{\frac{1}{2}} \qquad (\sqrt{x})' = \frac{1}{2}x^{\frac{1}{2}-1}$$

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Differentiate $y = \ln\left(x + \arcsin(2\sqrt{x})\right)$

$$y' = \frac{1}{x + \arcsin(2\sqrt{x})}\left(x + \arcsin(2\sqrt{x})\right)'$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{1-(2\sqrt{x})^2}}(2\sqrt{x})'\right)$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{1-4x}} \cdot 2 \cdot \frac{1}{2} \cdot x^{-1/2}\right)$$

$$= \frac{1}{x + \arcsin(2\sqrt{x})}\left(1 + \frac{1}{\sqrt{x}\sqrt{1-4x}}\right)$$

We simplify.

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Differentiate $y = \arcsin\sqrt{\frac{x}{x+1}}$.

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Differentiate $y = \arcsin\sqrt{\frac{x}{x+1}}$.

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{\frac{x}{x+1}}\right)^2}} \cdot \left(\sqrt{\frac{x}{x+1}}\right)'$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arcsin f(x))' = \frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x)$$

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Differentiate $y = \arcsin\sqrt{\frac{x}{x+1}}$.

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{\frac{x}{x+1}}\right)^2}} \cdot \left(\sqrt{\frac{x}{x+1}}\right)'$$

$$= \frac{1}{\sqrt{\frac{x+1}{x+1} - \frac{x}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x}{x+1}\right)^{-\frac{1}{2}} \cdot \left(\frac{x}{x+1}\right)'$$

$$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\left(\sqrt{f(x)}\right)' = \frac{1}{2}\left(f(x)\right)^{-1/2} \cdot f'(x)$$

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Differentiate $y = \arcsin\sqrt{\frac{x}{x+1}}$.

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{\frac{x}{x+1}}\right)^2}} \cdot \left(\sqrt{\frac{x}{x+1}}\right)'$$

$$= \frac{1}{\sqrt{\frac{x+1}{x+1}-\frac{x}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x}{x+1}\right)^{-\frac{1}{2}} \cdot \left(\frac{x}{x+1}\right)'$$

$$= \frac{1}{\sqrt{\frac{1}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x+1}{x}\right)^{\frac{1}{2}} \cdot \frac{1\cdot(x+1)-x\cdot(1+0)}{(x+1)^2}$$

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Differentiate $y = \arcsin\sqrt{\frac{x}{x+1}}$.

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{\frac{x}{x+1}}\right)^2}} \cdot \left(\sqrt{\frac{x}{x+1}}\right)'$$

$$= \frac{1}{\sqrt{\frac{x+1}{x+1}-\frac{x}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x}{x+1}\right)^{-\frac{1}{2}} \cdot \left(\frac{x}{x+1}\right)'$$

$$= \frac{1}{\sqrt{\frac{1}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x+1}{x}\right)^{\frac{1}{2}} \cdot \frac{1\cdot(x+1)-x\cdot(1+0)}{(x+1)^2}$$

$$= \sqrt{x+1} \cdot \frac{1}{2} \cdot \frac{\sqrt{x+1}}{\sqrt{x}} \frac{1}{(x+1)^2}$$

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Differentiate $y = \arcsin\sqrt{\dfrac{x}{x+1}}$.

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{\frac{x}{x+1}}\right)^2}} \cdot \left(\sqrt{\frac{x}{x+1}}\right)'$$

$$= \frac{1}{\sqrt{\frac{x+1}{x+1}-\frac{x}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x}{x+1}\right)^{-\frac{1}{2}} \cdot \left(\frac{x}{x+1}\right)'$$

$$= \frac{1}{\sqrt{\frac{1}{x+1}}} \cdot \frac{1}{2} \cdot \left(\frac{x+1}{x}\right)^{\frac{1}{2}} \cdot \frac{1\cdot(x+1)-x\cdot(1+0)}{(x+1)^2}$$

$$= \sqrt{x+1} \cdot \frac{1}{2} \cdot \frac{\sqrt{x+1}}{\sqrt{x}} \frac{1}{(x+1)^2} = \frac{1}{2(x+1)\sqrt{x}}$$

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Differentiate $y = (x^3 + 2x)e^{-2x}$.

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Differentiate $y = (x^3 + 2x)e^{-2x}$.

$$y' = (x^3 + 2x)' e^{-2x} + (x^3 + 2x)\left(e^{-2x}\right)'$$

Product rule $(uv)' = u'v + uv'$

Differentiate $y = (x^3 + 2x)e^{-2x}$.

$$y' = (x^3 + 2x)' e^{-2x} + (x^3 + 2x) \left(e^{-2x}\right)'$$
$$= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2x)'$$

Chain rule

$$(e^x)' = e^x \qquad \left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$$

Differentiate $y = (x^3 + 2x)e^{-2x}$.

$$\begin{aligned} y' &= (x^3 + 2x)'e^{-2x} + (x^3 + 2x)\left(e^{-2x}\right)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2x)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2) \end{aligned}$$

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Differentiate $y = (x^3 + 2x)e^{-2x}$.

$$\begin{aligned} y' &= (x^3 + 2x)' e^{-2x} + (x^3 + 2x)\left(e^{-2x}\right)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2x)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2) \\ &= e^{-2x}\left(3x^2 + 2 - 2(x^3 + 2x)\right) \end{aligned}$$

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Differentiate $y = (x^3 + 2x)e^{-2x}$.

$$\begin{aligned} y' &= (x^3 + 2x)' e^{-2x} + (x^3 + 2x)\left(e^{-2x}\right)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2x)' \\ &= (3x^2 + 2)e^{-2x} + (x^3 + 2x)e^{-2x}(-2) \\ &= e^{-2x}\left(3x^2 + 2 - 2(x^3 + 2x)\right) \\ &= e^{-2x}\left(-2x^3 + 3x^2 - 4x + 2\right) \end{aligned}$$

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Differentiate $y = (x^2 - 1)\sin(2x) - (3x - 1)\cos(2x)$.

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Differentiate $y = (x^2 - 1)\sin(2x) - (3x - 1)\cos(2x)$.

$$y = (x^2-1)'\sin(2x) + (x^2-1)\Big(\sin(2x)\Big)' - \left[(3x-1)'\cos(2x) + (3x-1)\Big(\cos(2x)\Big)'\right]$$

Two times product rule.

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Differentiate $y = (x^2 - 1)\sin(2x) - (3x - 1)\cos(2x)$.

$$y = (x^2-1)'\sin(2x) + (x^2-1)\Big(\sin(2x)\Big)'$$
$$- \left[(3x-1)'\cos(2x) + (3x-1)\Big(\cos(2x)\Big)'\right]$$
$$= 2x\sin(2x) + (x^2-1)\cos(2x)2$$
$$- \left[3\cos(2x) + (3x-1)\Big(-\sin(2x)\Big)2\right]$$

The chain rule.

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Differentiate $y = (x^2 - 1)\sin(2x) - (3x - 1)\cos(2x)$.

$$y = (x^2-1)'\sin(2x) + (x^2-1)\Big(\sin(2x)\Big)'$$
$$- \Big[(3x-1)'\cos(2x) + (3x-1)\Big(\cos(2x)\Big)'\Big]$$
$$= 2x\sin(2x) + (x^2-1)\cos(2x)2$$
$$- \Big[3\cos(2x) + (3x-1)\Big(-\sin(2x)\Big)2\Big]$$
$$= \sin(2x)\Big[2x + 2(3x-1)\Big] + \cos(2x)\Big[2(x^2-1) - 3\Big]$$

We factor.

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Differentiate $y = (x^2 - 1)\sin(2x) - (3x - 1)\cos(2x)$.

$$
\begin{aligned}
y &= (x^2-1)'\sin(2x) + (x^2-1)\Big(\sin(2x)\Big)' \\
&\quad - \Big[(3x-1)'\cos(2x) + (3x-1)\Big(\cos(2x)\Big)'\Big] \\
&= 2x\sin(2x) + (x^2-1)\cos(2x)2 \\
&\quad - \Big[3\cos(2x) + (3x-1)\Big(-\sin(2x)\Big)2\Big] \\
&= \sin(2x)\Big[2x + 2(3x-1)\Big] + \cos(2x)\Big[2(x^2-1) - 3\Big] \\
&= \sin(2x)\Big[8x - 2\Big] + \cos(2x)\Big[2x^2 - 5\Big]
\end{aligned}
$$

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Differentiate $y = \sqrt{2 + \cos(2x)}$

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Differentiate $y = \sqrt{2 + \cos(2x)}$

$$y' = \left[(2 + \cos(2x))^{\frac{1}{2}}\right]'$$
$$= \frac{1}{2} \cdot [2 + \cos(2x)]^{-\frac{1}{2}} \cdot [2 + \cos(2x)]'$$

- The root is differentiated as a power function with an exponent $\frac{1}{2}$.
- The expression below the radical is the inside function. We multiply by the derivative of the inside function.

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Differentiate $y = \sqrt{2 + \cos(2x)}$

$$y' = \left[(2+\cos(2x))^{\frac{1}{2}}\right]'$$
$$= \frac{1}{2} \cdot [2+\cos(2x)]^{-\frac{1}{2}} \cdot [2+\cos(2x)]'$$
$$= \frac{1}{2\sqrt{2+\cos(2x)}} \cdot [0 - \sin(2x) \cdot 2]$$

- The sum rule.
- The function $\cos(2x)$ is differentiated as a composite function with inside function $(2x)$.

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Differentiate $y = \sqrt{2+\cos(2x)}$

$$y' = \left[(2+\cos(2x))^{\frac{1}{2}}\right]'$$
$$= \frac{1}{2} \cdot [2+\cos(2x)]^{-\frac{1}{2}} \cdot [2+\cos(2x)]'$$
$$= \frac{1}{2\sqrt{2+\cos(2x)}} \cdot [0 - \sin(2x) \cdot 2]$$
$$= -\frac{\sin(2x)}{\sqrt{2+\cos(2x)}}$$

We simplify.

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Differentiate $y = \ln \sqrt{\frac{1}{\sin x}}$

Differentiate $y = \ln \sqrt{\frac{1}{\sin x}}$

$$y' = \frac{1}{\sqrt{\frac{1}{\sin x}}} \cdot \frac{1}{2} \left(\frac{1}{\sin x} \right)^{-\frac{1}{2}} \cdot (-1)(\sin x)^{-2} \cdot \cos x$$

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Differentiate $y = \ln\sqrt{\frac{1}{\sin x}}$

$$y' = \frac{1}{\sqrt{\frac{1}{\sin x}}} \cdot \frac{1}{2}\left(\frac{1}{\sin x}\right)^{-\frac{1}{2}} \cdot (-1)(\sin x)^{-2} \cdot \cos x$$

$$= \sqrt{\sin x} \cdot \frac{1}{2} \cdot \sqrt{\sin x} \cdot (-1)\frac{1}{\sin^2 x}\cos x$$

Differentiate $y = \ln \sqrt{\dfrac{1}{\sin x}}$

$$y' = \frac{1}{\sqrt{\frac{1}{\sin x}}} \cdot \frac{1}{2} \left(\frac{1}{\sin x} \right)^{-\frac{1}{2}} \cdot (-1)(\sin x)^{-2} \cdot \cos x$$

$$= \sqrt{\sin x} \cdot \frac{1}{2} \cdot \sqrt{\sin x} \cdot (-1) \frac{1}{\sin^2 x} \cos x$$

$$= -\frac{1}{2} \operatorname{cotg} x$$

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Differentiate $y = \ln \sqrt{\dfrac{1}{\sin x}}$.

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Differentiate $y = \ln\sqrt{\frac{1}{\sin x}}$.

$$y' = -\frac{1}{2} \cdot (\ln \sin x)'$$

We simplify.

$$y = \ln\sqrt{\sin^{-1} x} = \ln \sin^{-\frac{1}{2}} x = -\frac{1}{2} \cdot \ln \sin x$$

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Differentiate $y = \ln \sqrt{\frac{1}{\sin x}}$.

$$y' = -\frac{1}{2} \cdot (\ln \sin x)'$$
$$= -\frac{1}{2} \cdot \frac{1}{\sin x} \cdot \cos x$$

We differentiate composite function. The outside function is $\ln(\cdot)$ and the inside function is $\sin(x)$.

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Differentiate $y = \ln\sqrt{\dfrac{1}{\sin x}}$.

$$y' = -\frac{1}{2}\cdot(\ln\sin x)'$$
$$= -\frac{1}{2}\cdot\frac{1}{\sin x}\cdot\cos x$$
$$= -\frac{1}{2}\cdot\operatorname{cotg} x$$

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Differentiate $y = \ln \sin e^{3x}$.

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Differentiate $y = \ln \sin e^{3x}$.

$$y' = \frac{1}{\sin e^{3x}} \cdot \left(\sin e^{3x}\right)'$$

We differentiate the logarithm, the inside function is $\sin e^{3x}$.

$$\boxed{(\ln x)' = \frac{1}{x}}$$

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Differentiate $y = \ln \sin e^{3x}$.

$$y' = \frac{1}{\sin e^{3x}} \cdot \left(\sin e^{3x}\right)'$$
$$= \frac{1}{\sin e^{3x}} \cdot \cos e^{3x} \cdot \left(e^{3x}\right)'$$

We differentiate the sine function, the inside function is e^{3x}.

$$(\sin x)' = \cos x$$

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Differentiate $y = \ln \sin e^{3x}$.

$$y' = \frac{1}{\sin e^{3x}} \cdot \left(\sin e^{3x}\right)'$$
$$= \frac{1}{\sin e^{3x}} \cdot \cos e^{3x} \cdot \left(e^{3x}\right)'$$
$$= \operatorname{cotg}\left(e^{3x}\right) \cdot e^{3x}(3x)'$$

We differentiate the exponential, the inside funciton is $3x$.

$$\boxed{(e^x)' = e^x}$$

Differentiate $y = \ln \sin e^{3x}$.

$$y' = \frac{1}{\sin e^{3x}} \cdot \left(\sin e^{3x}\right)'$$

$$= \frac{1}{\sin e^{3x}} \cdot \cos e^{3x} \cdot \left(e^{3x}\right)'$$

$$= \operatorname{cotg}\left(e^{3x}\right) \cdot e^{3x}(3x)'$$

$$= \operatorname{cotg}\left(e^{3x}\right) \cdot e^{3x} \cdot 3$$

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Differentiate $y = \sqrt{x + \ln(9 - x)}$

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Differentiate $y = \sqrt{x + \ln(9 - x)}$

$$y' = \frac{1}{2} \cdot \Big(x + \ln(9 - x)\Big)^{-\frac{1}{2}} \cdot \Big(x + \ln(9 - x)\Big)'$$

$$\left(\sqrt{f(x)}\right)' = \frac{1}{2}\Big(f(x)\Big)^{-\frac{1}{2}} f'(x)$$

Differentiate $y = \sqrt{x + \ln(9 - x)}$

$$y' = \frac{1}{2} \cdot \left(x + \ln(9 - x)\right)^{-\frac{1}{2}} \cdot \left(x + \ln(9 - x)\right)'$$
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x + \ln(9 - x)}} \left(1 + \frac{1}{9 - x} \cdot (0 - 1)\right)$$

Differentiate $y = \sqrt{x + \ln(9 - x)}$

$$\begin{aligned} y' &= \frac{1}{2} \cdot \Big(x + \ln(9 - x)\Big)^{-\frac{1}{2}} \cdot \Big(x + \ln(9 - x)\Big)' \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{x + \ln(9 - x)}} \left(1 + \frac{1}{9 - x} \cdot (0 - 1)\right) \\ &= \frac{8 - x}{2(9 - x)\sqrt{x + \ln(9 - x)}} \end{aligned}$$

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Differentiate $y = \dfrac{x^2}{(x+1)^3}$.

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Differentiate $y = \dfrac{x^2}{(x+1)^3}$.

$$y' = \frac{(x^2)'(x+1)^3 - x^2\left[(x+1)^3\right]'}{(x+1)^{3\cdot 2}}$$

The quotient rule.

$$\frac{u}{v} = \frac{u'v - uv'}{v^2}$$

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Differentiate $y = \dfrac{x^2}{(x+1)^3}$.

$$y' = \frac{(x^2)'(x+1)^3 - x^2\left[(x+1)^3\right]'}{(x+1)^{3\cdot 2}}$$
$$= \frac{2x(x+1)^3 - x^2 3(x+1)^2 \cdot 1}{(x+1)^6}$$

We evaluate derivatives. The function $(x+1)^3$ is a composite function.

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Differentiate $y = \dfrac{x^2}{(x+1)^3}$.

$$y' = \frac{(x^2)'(x+1)^3 - x^2\Big[(x+1)^3\Big]'}{(x+1)^{3\cdot 2}}$$

$$= \frac{2x(x+1)^3 - x^2 3(x+1)^2 \cdot 1}{(x+1)^6}$$

$$= \frac{x(x+1)^2\Big[2(x+1)+3x\Big]}{(x+1)^6}$$

We factor.

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Differentiate $y = \dfrac{x^2}{(x+1)^3}$.

$$y' = \frac{(x^2)'(x+1)^3 - x^2\Big[(x+1)^3\Big]'}{(x+1)^{3\cdot 2}}$$
$$= \frac{2x(x+1)^3 - x^2 3(x+1)^2 \cdot 1}{(x+1)^6}$$
$$= \frac{x(x+1)^2\Big[2(x+1)+3x\Big]}{(x+1)^6}$$
$$= \frac{x(5x+2)}{(x+1)^4}$$

We simplify.

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Differentiate $y = x \ln \frac{x^2}{x+1}$.

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Differentiate $y = x \ln \frac{x^2}{x+1}$.

$$y' = (x)' \cdot \ln \frac{x^2}{x+1} + x \cdot \left(\ln \frac{x^2}{x+1} \right)'$$

The product rule.

$$(uv)' = u'v + uv'$$

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Differentiate $y = x \ln \dfrac{x^2}{x+1}$.

$$y' = (x)' \cdot \ln \frac{x^2}{x+1} + x \cdot \left(\ln \frac{x^2}{x+1} \right)'$$

$$= 1 \cdot \ln \frac{x^2}{x+1} + x \cdot \frac{x+1}{x^2} \cdot \left(\frac{x^2}{x+1} \right)'$$

We differentiate the remaining terms. The logarithm is differentiated as a composite function.

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Differentiate $y = x \ln \frac{x^2}{x+1}$.

$$y' = (x)' \cdot \ln \frac{x^2}{x+1} + x \cdot \left(\ln \frac{x^2}{x+1} \right)'$$

$$= 1 \cdot \ln \frac{x^2}{x+1} + x \cdot \frac{x+1}{x^2} \cdot \left(\frac{x^2}{x+1} \right)'$$

$$= 1 \cdot \ln \frac{x^2}{x+1} + 1 \cdot \frac{x+1}{x} \cdot \frac{2x \cdot (x+1) - x^2 \cdot (1+0)}{(x+1)^2}$$

The inside function of the logarithm is a quotient and we use the quotient rule.

$$\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$$

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Differentiate $y = x \ln \dfrac{x^2}{x+1}$.

$$y' = (x)' \cdot \ln \frac{x^2}{x+1} + x \cdot \left(\ln \frac{x^2}{x+1} \right)'$$

$$= 1 \cdot \ln \frac{x^2}{x+1} + x \cdot \frac{x+1}{x^2} \cdot \left(\frac{x^2}{x+1} \right)'$$

$$= 1 \cdot \ln \frac{x^2}{x+1} + 1 \cdot \frac{x+1}{x} \cdot \frac{2x \cdot (x+1) - x^2 \cdot (1+0)}{(x+1)^2}$$

$$= \ln \frac{x^2}{x+1} + \frac{1}{x} \cdot \frac{x^2+2x}{x+1}$$

We cancel by $(x+1)$ a nd simplify the mnumerator of the last fraction.

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Differentiate $y = x \ln \dfrac{x^2}{x+1}$.

$$y' = (x)' \cdot \ln \frac{x^2}{x+1} + x \cdot \left(\ln \frac{x^2}{x+1}\right)'$$
$$= 1 \cdot \ln \frac{x^2}{x+1} + x \cdot \frac{x+1}{x^2} \cdot \left(\frac{x^2}{x+1}\right)'$$
$$= 1 \cdot \ln \frac{x^2}{x+1} + 1 \cdot \frac{x+1}{x} \cdot \frac{2x \cdot (x+1) - x^2 \cdot (1+0)}{(x+1)^2}$$
$$= \ln \frac{x^2}{x+1} + \frac{1}{x} \cdot \frac{x^2+2x}{x+1} = \ln \frac{x^2}{x+1} + \frac{x+2}{x+1}$$

We do some final simplifications.

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Differentiate $y = 2x \operatorname{arctg} x - \ln(1 + x^2)$.

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Differentiate $y = 2x \operatorname{arctg} x - \ln(1 + x^2)$.

$$y' = (2x)' \cdot \operatorname{arctg} x + 2x \cdot (\operatorname{arctg} x)' - \frac{1}{1 + x^2} \cdot (1 + x^2)'$$

We use the product rule and the chain rule.

$$(uv)' = u'v + uv'$$

$$\Big(u(v(x))\Big)' = u'(v(x)) \cdot v'(x)$$

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Differentiate $y = 2x \operatorname{arctg} x - \ln(1+x^2)$.

$$y' = (2x)' \cdot \operatorname{arctg} x + 2x \cdot (\operatorname{arctg} x)' - \frac{1}{1+x^2} \cdot (1+x^2)'$$

$$= 2 \cdot \operatorname{arctg} x + 2x \cdot \frac{1}{1+x^2} - \frac{1}{1+x^2} \cdot 2x$$

Differentiate $y = 2x \operatorname{arctg} x - \ln(1 + x^2)$.

$$y' = (2x)' \cdot \operatorname{arctg} x + 2x \cdot (\operatorname{arctg} x)' - \frac{1}{1+x^2} \cdot (1 + x^2)'$$

$$= 2 \cdot \operatorname{arctg} x + 2x \cdot \frac{1}{1+x^2} - \frac{1}{1+x^2} \cdot 2x$$

$$= 2 \operatorname{arctg} x$$

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Differentiate $y = x^3 \arcsin x + \sqrt{1 - x^2}$.

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Differentiate $y = x^3 \arcsin x + \sqrt{1 - x^2}$.

$$y' = (x^3)' \cdot \arcsin x + x^3 \cdot (\arcsin x)' + \frac{1}{2} \cdot (1 - x^2)^{-\frac{1}{2}} \cdot (1 - x^2)'$$

The product rule and the chain rule.

$$(uv)' = u'v + uv'$$

$$\Big(u(v(x))\Big)' = u'(v(x)) \cdot v'(x)$$

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Differentiate $y = x^3 \arcsin x + \sqrt{1-x^2}$.

$$y' = (x^3)' \cdot \arcsin x + x^3 \cdot (\arcsin x)' + \frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}} \cdot (1-x^2)'$$

$$= 3x^2 \cdot \arcsin x + \frac{x^3}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)$$

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Differentiate $y = x^3 \arcsin x + \sqrt{1-x^2}$.

$$y' = (x^3)' \cdot \arcsin x + x^3 \cdot (\arcsin x)' + \frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}} \cdot (1-x^2)'$$

$$= 3x^2 \cdot \arcsin x + \frac{x^3}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)$$

$$= 3x^2 \arcsin x + \frac{x^3 - x}{\sqrt{1-x^2}}$$

Differentiate $y = x^3 \arcsin x + \sqrt{1 - x^2}$.

$$\begin{aligned} y' &= (x^3)' \cdot \arcsin x + x^3 \cdot (\arcsin x)' + \frac{1}{2} \cdot (1 - x^2)^{-\frac{1}{2}} \cdot (1 - x^2)' \\ &= 3x^2 \cdot \arcsin x + \frac{x^3}{\sqrt{1 - x^2}} + \frac{1}{2\sqrt{1 - x^2}} \cdot (-2x) \\ &= 3x^2 \arcsin x + \frac{x^3 - x}{\sqrt{1 - x^2}} \\ &= 3x^2 \arcsin x - x \cdot \frac{1 - x^2}{\sqrt{1 - x^2}} \end{aligned}$$

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Differentiate $y = x^3 \arcsin x + \sqrt{1-x^2}$.

$$y' = (x^3)' \cdot \arcsin x + x^3 \cdot (\arcsin x)' + \frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}} \cdot (1-x^2)'$$

$$= 3x^2 \cdot \arcsin x + \frac{x^3}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)$$

$$= 3x^2 \arcsin x + \frac{x^3 - x}{\sqrt{1-x^2}}$$

$$= 3x^2 \arcsin x - x \cdot \frac{1-x^2}{\sqrt{1-x^2}}$$

$$= 3x^2 \arcsin x - x \cdot \sqrt{1-x^2}$$

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Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

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Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

$$y' = \frac{\left(\sqrt{1-x^2}\right)' \cdot \arcsin x - \sqrt{1-x^2} \cdot (\arcsin x)'}{\arcsin^2 x}$$

We use the quotient rule.

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

$$y' = \frac{\left(\sqrt{1-x^2}\right)' \cdot \arcsin x - \sqrt{1-x^2} \cdot (\arcsin x)'}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x) \cdot \arcsin x - \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}}{\arcsin^2 x}$$

- We evaluate the derivatives.
- The expression bellow the radical is an inside function.

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Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

$$y' = \frac{\left(\sqrt{1-x^2}\right)' \cdot \arcsin x - \sqrt{1-x^2} \cdot (\arcsin x)'}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x) \cdot \arcsin x - \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x)\arcsin x}{\arcsin^2 x} - \frac{1}{\arcsin^2 x}$$

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Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

$$y' = \frac{\left(\sqrt{1-x^2}\right)' \cdot \arcsin x - \sqrt{1-x^2} \cdot (\arcsin x)'}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x) \cdot \arcsin x - \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x)\arcsin x}{\arcsin^2 x} - \frac{1}{\arcsin^2 x}$$

$$= \frac{\frac{1}{\sqrt{1-x^2}} \cdot (-x)}{\arcsin x} - \frac{1}{\arcsin^2 x}$$

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Differentiate $y = \dfrac{\sqrt{1-x^2}}{\arcsin x}$.

$$y' = \frac{\left(\sqrt{1-x^2}\right)' \cdot \arcsin x - \sqrt{1-x^2} \cdot (\arcsin x)'}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x) \cdot \arcsin x - \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}}{\arcsin^2 x}$$

$$= \frac{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x)\arcsin x}{\arcsin^2 x} - \frac{1}{\arcsin^2 x}$$

$$= \frac{\frac{1}{\sqrt{1-x^2}} \cdot (-x)}{\arcsin x} - \frac{1}{\arcsin^2 x}$$

$$= \frac{-x}{\sqrt{1-x^2} \cdot \arcsin x} - \frac{1}{\arcsin^2 x}$$

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Differentiate $y = \sqrt{x+1}\,\mathrm{arctg}\,\sqrt{x+1}$.

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Differentiate $y = \sqrt{x+1}\,\mathrm{arctg}\,\sqrt{x+1}$.

$$y' = \frac{1}{2\sqrt{x+1}} \cdot (1+0) \cdot \mathrm{arctg}\,\sqrt{x+1} + \\ + \sqrt{x+1} \cdot \frac{1}{1+\left(\sqrt{x+1}\right)^2} \cdot \frac{1}{2\sqrt{x+1}} \cdot (1+0)$$

The product and the chain rule.

$$(uv)' = u'v + uv'$$

$$\Big(u(v(x))\Big)' = u'(v(x)) \cdot v'(x)$$

Differentiate $y=\sqrt{x+1}\operatorname{arctg}\sqrt{x+1}$.

$$y' = \frac{1}{2\sqrt{x+1}}\cdot(1+0)\cdot\operatorname{arctg}\sqrt{x+1} +$$

$$+\sqrt{x+1}\cdot\frac{1}{1+\left(\sqrt{x+1}\right)^2}\cdot\frac{1}{2\sqrt{x+1}}\cdot(1+0)$$

$$=\frac{\operatorname{arctg}\sqrt{x+1}}{2\sqrt{x+1}}+\frac{1}{2}\cdot\frac{1}{x+2}$$

We simplify.

That's all.

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