

$$y'' + 2y' + 2y = x^2$$

a) $y'' + 2y' + 2y = 0$

 $\alpha \pm i\beta$

$$\lambda^2 + 2\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2} =$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i \cdot 1$$

$$y_1 = e^{-x} \cos(x)$$

$$y_2 = e^{-x} \sin(x)$$

b) $\left. \begin{array}{l} y_p = ax^2 + bx + c \\ y_p' = 2ax + b \\ y_p'' = 2a \end{array} \right\} \begin{array}{l} y'' + 2y' + 2y = x^2 \\ 2a + 2(2ax + b) + 2(ax^2 + bx + c) = x^2 \end{array}$

$$x^2 \underbrace{2a}_1 + x \underbrace{(4a + 2b)}_0 + 2a + 2b + 2c \underbrace{+ 0}_0 = x^2 + 0x + 0$$

$$2a = 1 \Rightarrow a = \frac{1}{2}$$

$$\left. \begin{array}{l} 4a + 2b = 0 \\ 2a + 2b + 2c = 0 \end{array} \right\} \begin{array}{l} 2 + 2b = 0 \\ 1 + 2b + 2c = 0 \end{array} \Rightarrow \begin{array}{l} b = -1 \\ -1 + 2c = 0 \\ c = \frac{1}{2} \end{array}$$

$$y_p = \frac{1}{2}x^2 - x + \frac{1}{2}$$

Observeispiel:

$$y = C_1 e^{-x} \cos(x) + C_2 e^{-x} \sin(x) + \frac{1}{2}x^2 - x + \frac{1}{2}$$

$$y'' + 3y' + 6y = x - 1$$

a) $y'' + 3y' + 6y = 0$

$$\lambda^2 + 3\lambda + 6 = 0$$

$$\lambda_{1,2} = \frac{-3 \pm \sqrt{9-24}}{2} = \frac{-3 \pm \sqrt{15}}{2} = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}$$

$$y_1 = e^{-\frac{3}{2}x} \cos\left(\frac{\sqrt{15}}{2}x\right), \quad y_2 = e^{-\frac{3}{2}x} \sin\left(\frac{\sqrt{15}}{2}x\right)$$

b)

$$\left. \begin{array}{l} y_p = ax+b \\ y_p' = a \cdot 1 + 0 = a \\ y_p'' = 0 \end{array} \right\} \quad \begin{array}{l} y'' + 3y' + 6y = x - 1 \\ 0 + 3(a) + 6(ax+b) = x - 1 \\ 3a + 6ax + 6b = x - 1 \\ 6a \cdot x + (3a + 6b) = \underbrace{1 \cdot x}_{xxxxxx} + (-1) \end{array}$$

$$6a = 1$$

$$3a + 6b = -1$$

$$a = \frac{1}{6}, \quad \frac{1}{2} + 6b = -1 \\ 6b = -\frac{3}{2} \\ b = -\frac{3}{12} = -\frac{1}{4}$$

$$y = \frac{1}{6}x - \frac{1}{4} + C_1 e^{-\frac{3}{2}x} \cos\left(\frac{\sqrt{15}}{2}x\right) + C_2 e^{-\frac{3}{2}x} \sin\left(\frac{\sqrt{15}}{2}x\right)$$

$$y'' - 2y' - 4y = x^2 + 2x$$

$$\text{a) } y'' - 2y' - 4y = 0 \quad \lambda_{1,2} = \frac{2 \pm \sqrt{4 + 16}}{2} = \frac{2 \pm \sqrt{20}}{2} = \\ = 1 \pm \sqrt{5}$$

$$y = e^{(1+\sqrt{5})x}$$

$$y = e^{*(1-\sqrt{5})x}$$

$$\hookrightarrow \left. \begin{array}{l} y_p = ax^2 + bx + c \\ y'_p = 2ax + b \\ y''_p = 2a \end{array} \right\} y'' - 2y' - 4y = x^2 + 2x$$

$$2a - 2(2ax + b) - 4(ax^2 + bx + c) \\ = 1 \cdot x^2 + 2 \cdot x + 0$$

$$\underline{-4ax^2} + x \underbrace{(-4a - 4b)}_{+2a - 2b - 4c} = \\ = \underline{1 \cdot x^2} + \underline{2 \cdot x} + 0$$

$$-4a = 1 \Rightarrow a = -\frac{1}{4}$$

$$-4a - 4b = 2 \Rightarrow 1 - 4b = 2 \Rightarrow b = -\frac{1}{4}$$

$$2a - 2b - 4c = 0$$

$$-\frac{1}{2} + \frac{1}{2} - 4c = 0 \\ -4c = 0 \\ c = 0$$

Observejte:

$$y = C_1 e^{(1+\sqrt{5})x} + C_2 e^{(1-\sqrt{5})x} + (-\frac{1}{4})x^2 - \frac{1}{4}x$$

$$y'' - 3y' + 2y = 2 - x$$

a) $y'' - 3y' + 2y = 0$

$$x^2 - 3x + 2 = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$y_1 = e^{2x}$$

$$y_2 = e^x$$

b)

$$\left. \begin{array}{l} y_1 = ax + b \\ y_1' = a \\ y_1'' = 0 \end{array} \right\} \quad \begin{array}{l} y'' - 3y' + 2y = 2 - x \\ 0 - 3 \cdot a + 2(ax+b) = 2 - x \end{array}$$

$$\begin{array}{c} 2ax - 3a + 2b = (-1) \cdot x + 2 \\ \hline \end{array}$$

$$2a = -1 \Rightarrow a = -\frac{1}{2}$$

$$-3a + 2b = 2 \Rightarrow \frac{3}{2} + 2b = 2$$

$$2b = \frac{1}{2}$$

$$b = \frac{1}{4}$$

$$y = C_1 e^{2x} + C_2 e^x - \frac{1}{2}x + \frac{1}{4}$$

$$y'' + 4y = x + 2$$

$$y(0) = 0$$

$$y'(0) = 1$$

a) $y'' + 4y = 0$

$$\lambda^2 + 4 = 0$$

$$\lambda^2 = -4$$

$$\lambda = \pm 2i = 0 \pm 2i$$

$$y_1 = e^{0x} \cdot \cos 2x = \cos 2x$$

$$y_2 = x \sin 2x$$

$$\begin{aligned} \Rightarrow y_p &= ax + b \\ y_p' &= a \\ y_p'' &= 0 \end{aligned}$$

$$\left. \begin{aligned} y'' + 4y &= x + 2 \\ 0 + 4(ax + b) &= x + 2 \\ 4ax + 4b &= x + 2 \end{aligned} \right\}$$

$$4a = 1 \Rightarrow a = \frac{1}{4}$$

$$4b = 2 \Rightarrow b = \frac{1}{2}$$

Observe $y_p = ax + b$:

$$y = C_1 \cos(2x) + C_2 \sin(2x) + \frac{1}{4}x + \frac{1}{2}$$

$$\textcircled{1} \quad y(0) = 0 \quad 0 = C_1 \cdot 1 + C_2 \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{2}$$

$$y'(0) = 1 \quad y' = -2C_1 \sin(2x) + 2C_2 \cos(2x) + \frac{1}{4} + 0$$

$$1 = C_1 \cdot 0 + 2 \cdot C_2 + \frac{1}{4}$$

$$\text{t: } 0 = C_1 + \frac{1}{2} \Rightarrow C_1 = -\frac{1}{2}$$

$$1 = 2C_2 + \frac{1}{4} \Rightarrow 2C_2 = \frac{3}{4} \Rightarrow C_2 = \frac{3}{8}$$

$$\boxed{y = -\frac{1}{2} \cos(2x) + \frac{3}{8} \sin(2x) + \frac{1}{4}x + \frac{1}{2}}$$

$$y'' + \frac{1}{\sqrt{x}} y = 0 \quad \text{ma' r\"oser} \quad y_2 = \sqrt{x}.$$

Nojd\"e vi edme f\"oren'.

$$\begin{aligned} y_1 &= y_2 \int \frac{1}{y_2^2} dx = \sqrt{x} \int \frac{1}{(\sqrt{x})^2} dx = \\ &= \sqrt{x} \int \frac{1}{x} dx = \sqrt{x} \cdot \ln x \end{aligned}$$

$$y = C_1 y_1 + C_2 y_2 = \boxed{C_1 \sqrt{x} + C_2 \sqrt{x} \cdot \ln x}$$