

Upracuj krok po krok po dwoch różnych ścieżkach C_1 i C_2 .

$$C_1: \vec{r} = (\cos t, \sin t) \quad t \in (0, \frac{\pi}{2})$$

$$C_2: \vec{r} = (t, 1-t) \quad t \in (0, 1)$$

[C₁] $f(x, y) = y$ $f(\vec{r}(t)) = \frac{t}{y}$

$$\frac{d\vec{r}}{dt} = (-\sin t, \cos t) \quad \frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right| = \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1$$

$$\int_{C_1} y \, ds = \int_0^{\frac{\pi}{2}} \underbrace{\sin t}_{f(\vec{r}(t))} \cdot 1 \, dt = \int_0^{\frac{\pi}{2}} \sin t \, dt = \left[-\cos t \right]_0^{\frac{\pi}{2}} = 0 - (-1) = 1$$

[C₂] $f(x, y) = y$ $f(\vec{r}(t)) = \frac{1-t}{y}$

$$\frac{d\vec{r}}{dt} = (1, -1) \quad \frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right| = \sqrt{1+1} = \sqrt{2}$$

$$\int_{C_2} y \, ds = \int_0^1 \underbrace{(1-t)}_{f(\vec{r}(t))} \cdot \sqrt{2} \, dt = \sqrt{2} \int_0^1 1-t \, dt =$$

$$= \sqrt{2} \left[t - \frac{1}{2}t^2 \right]_0^1 = \sqrt{2} \left(1 - \frac{1}{2} - 0 \right) = \frac{\sqrt{2}}{2}$$

Uypotekte $\int_{C_1} y^2 ds$ po křivadloch ρ_1 a ρ_2 .

$$C_1: \vec{F}(t) = (\cos t, \sin t) \quad t \in (0, \frac{\pi}{2})$$

$$C_2: \vec{F}(t) = (t, 1-t) \quad t \in (0, 1)$$

C₁ $f(x, y) = y^2 \quad f(\vec{F}(t)) = \sin^2 t$

$$\frac{d\vec{r}}{dt} = (-\sin t, \cos t) \quad \frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1$$

$$\int_{C_1} y^2 ds = \int_0^{\pi/2} \underbrace{\sin^2 t}_{f(\vec{F}(t))} \underbrace{\frac{ds}{dt}}_{1} dt = \frac{\pi}{4} \quad (\text{bez užití})$$

C₂ $f(x, y) = y^2 \quad f(\vec{F}(t)) = (1-t)^2$

$$\frac{d\vec{r}}{dt} = (1, -1) \quad \frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\begin{aligned} \int_{C_2} y^2 ds &= \int_0^1 \underbrace{(1-t)^2 \cdot \sqrt{2}}_{f(\vec{F}(t))} \underbrace{dt}_{\frac{ds}{dt}} = \sqrt{2} \int_0^1 t^2 - 2t + 1 dt = \\ &= \sqrt{2} \left[\frac{1}{3}t^3 - t^2 + t \right]_0^1 = \sqrt{2} \left(\frac{1}{3} - 1 + 1 - 0 \right) = \underline{\underline{\frac{\sqrt{2}}{3}}} \end{aligned}$$

Pokl.: Uypotekte $\int_0^{\pi/2} \sin^2 t dt$ bez integraci.

$$1) y = \sin t: \begin{array}{c} \curvearrowleft \\ \uparrow \\ \frac{\pi}{2} \end{array} \quad y = \cos t: \begin{array}{c} \curvearrowright \\ \uparrow \\ \frac{\pi}{2} \end{array} \quad \leftarrow \text{stejný hrac, lze použít metodu občívku}$$

$$2) y = \sin^2 t \quad \text{a } y = \cos^2 t \text{ mají na } (0, \frac{\pi}{2}) \text{ stejný hrac, lze použít metodu občívku} \Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 t dt = \int_0^{\frac{\pi}{2}} \cos^2 t dt$$

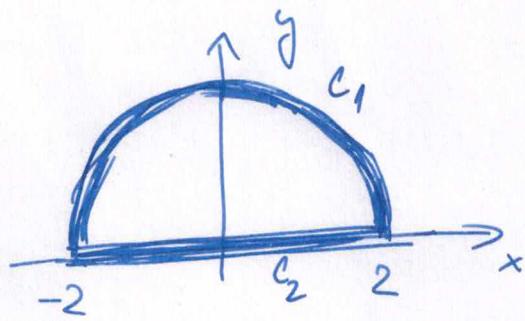
$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin^2 t dt &= \frac{1}{2} \cdot 2 \int_0^{\frac{\pi}{2}} \sin^2 t dt = \frac{1}{2} \left[\int_0^{\frac{\pi}{2}} \sin^2 t dt + \int_0^{\frac{\pi}{2}} \cos^2 t dt \right] = \\ &= \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin^2 t + \cos^2 t dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} 1 dt = \frac{1}{2} \cdot \frac{\pi}{2} = \underline{\underline{\frac{\pi}{4}}} \end{aligned}$$

Vypočítejte $\oint_C y \, ds$ kde C je

kružnice na obrázku.

(Půlkružnice a úsečka)

$$C_1 \quad C_2$$



$$C = C_1 \cup C_2$$

C_1 - půlkružnice

$$x = 2 \cdot \cos t$$

$$y = 2 \cdot \sin t \quad t \in [0, \pi]$$

$$\vec{r} = (2 \cos t, 2 \sin t), \frac{d\vec{r}}{dt} = (-2 \sin t, 2 \cos t)$$

$$ds = |\frac{d\vec{r}}{dt}| = \sqrt{4 \sin^2 t + 4 \cdot \cos^2 t} dt = \sqrt{4(\sin^2 t + \cos^2 t)} dt = 2 dt$$

$$\begin{aligned} \oint_{C_1} y \, ds &= \int_0^\pi 2 \cdot \sin t \cdot 2 dt = 4 \int_0^\pi \sin t dt = 4 \left[-\cos t \right]_0^\pi = \\ &= 4(-(-1) - (-1)) = 4 \cdot 2 = 8 \end{aligned}$$

$$\begin{aligned} \oint_{C_2} y \, ds &= \int_{-2}^2 0 \cdot ds = 0 \quad (\text{Integrovalu mohu, normativ} \\ &\quad \leftarrow \text{okolnou počítat sou s } d\vec{r} \text{ a } ds.) \end{aligned}$$

$$\oint_C y \, ds = \int_{C_1} y \, ds + \int_{C_2} y \, ds = 8 + 0 = 8$$

Vypočítat hmotnost závěru žroubovice

$$\vec{F} = (3 \cdot \cos t, 3 \cdot \sin t, 4t) \quad t \in [0, 2\pi]$$

$$jednotková hustota je \frac{\sqrt{2}}{x^2 + y^2}$$

$$\frac{d\vec{r}}{dt} = (-3 \sin t, 3 \cos t, 4)$$

$$|\frac{d\vec{r}}{dt}| = \sqrt{9 \cdot \sin^2 t + 9 \cdot \cos^2 t + 16} = \sqrt{25} = 5, \quad ds = 5 \cdot dt$$

$$f(\vec{r}) = \frac{\sqrt{4t}}{9 \cdot \cos^2 t + 9 \cdot \sin^2 t} = \frac{2}{9} \cdot \sqrt{t}$$

$$M_u = \int_C \frac{\sqrt{2}}{x^2 + y^2} ds = \int_0^{2\pi} \frac{2}{9} \cdot 5 \cdot \sqrt{t} dt = \frac{10}{9} \left[\frac{2}{3} \cdot t^{\frac{3}{2}} \right]_0^{2\pi} =$$

$$= \frac{10}{9} \cdot \left[\frac{2}{3} \cdot (2\pi)^{\frac{3}{2}} \right] =$$

$$= \frac{20}{27} (2\pi)^{\frac{3}{2}}$$

\cup j pocteké $\int_C \vec{F} d\vec{r}$ po třech různých způsobech.

$$\vec{F} = -y \vec{i} + x \vec{j}, \quad C_1: \vec{F} = (\cos t, \sin t) \quad t \in [0, \frac{\pi}{2}]$$

$$C_2: \vec{F} = (1-t, t) \quad t \in [0, 1]$$

$$C_3: \vec{F} = (1-t^2, t) \quad t \in [0, 1]$$

C_1 $\frac{d\vec{r}}{dt} = (-\sin t, \cos t) \quad \vec{F}(\vec{r}(t)) = \begin{pmatrix} \underbrace{x_1 t}_{-y} \\ \underbrace{x_2 t}_x \end{pmatrix}$

$$\int_{C_1} \vec{F} d\vec{r} = \int_0^{\frac{\pi}{2}} \sin^2 t + \cos^2 t dt = \int_0^{\frac{\pi}{2}} 1 dt = \frac{\pi}{2}$$

C_2 $\frac{d\vec{r}}{dt} = (-1, 1) \quad \vec{F}(\vec{r}(t)) = \begin{pmatrix} \underbrace{-t}_{-y} \\ \underbrace{1-t}_x \end{pmatrix}$

$$\int_{C_2} \vec{F} d\vec{r} = \int_0^1 -t + (1-t) dt = \int_0^1 1 dt = 1$$

C_3 $\frac{d\vec{r}}{dt} = (-2t, 1) \quad \vec{F}(\vec{r}(t)) = \begin{pmatrix} \underbrace{-t}_{-y} \\ \underbrace{1-t^2}_x \end{pmatrix}$

$$\int_{C_3} \vec{F} d\vec{r} = \int_0^1 2t^2 + 1 - t^2 dt = \int_0^1 t^2 + 1 dt =$$

$$= \left[\frac{1}{3} t^3 + t \right]_0^1 = \frac{1}{3} + 1 - 0 = \frac{4}{3}$$

Tři integrály kohoutkového pole po třech různých cestách z body $[0, 1]$ do body $[1, 0]$. Každý integral vychází jinak.

$$\underline{P} \leftarrow: \int_C x \, dx + y \, dy + (x+y-1) \, dz$$

$$C: \vec{F}(t) = (1+t, 1+2t, 1+3t) \quad t \in (0,1)$$

$$(N_{\text{rechts}} = (1,1,1) \text{ do } (2,3,4))$$

$$\vec{F}(x,y,z) = (x, y, x+y-1) \quad \frac{d\vec{F}}{dt} = (1, 2, 3)$$

$$\vec{F}(F(t)) = (1+t, 1+2t, 1+t + (1+2t) - \cancel{1}) = \\ = (1+t, 1+2t, 1+3t)$$

$$\vec{F}(F(t)) \cdot \frac{d\vec{F}}{dt} = 1 \cdot (1+t) + 2(1+2t) + 3(1+3t) = \\ = 1+2+3+t+4t+9t = 6+14t$$

$$\int_C \vec{F} \, d\vec{r} = \int_0^1 6+14t \, dt = [6t+7t^2]_0^1 =$$

$$= 6+7 - 0 = \underline{\underline{13}}$$

Pr.: Určete hodnotu parametru $a \in \mathbb{R}$ takovou, aby integrál \int_C

$$ax^3y \, dx + (x^3+1) \, dy$$

násobený $\approx \mathbb{R}^2$ může být integrální větší. Najděte kromě toho funkci a a počítejte integrál po čtvrtce z bodu $(0,0)$ do bodu $(1,1)$.

a) Podmínka pro násobení může být integrální větší:

$$\frac{\partial}{\partial y} (ax^3y) = \frac{\partial}{\partial x} (x^3+1)$$

$$ax^2 = 3x^2$$

$$\underline{a = 3}$$

\Rightarrow Kromě toho pro $\vec{F} = (3x^3y, x^3+1)$

$$\frac{\partial \varphi}{\partial x} = 3x^2y$$

$$\frac{\partial \varphi}{\partial y} = x^3+1 \quad \Rightarrow \quad \underline{\varphi(x,y) = x^3y + y + C}$$

Prokázejme $\int 3x^3y \, dx = x^3y + C(y)$ a

$$\frac{\partial}{\partial y} (x^3y + C(y)) = x^3 + C'(y) \quad a \quad \text{při } C'(y) = 1$$

Máme $\frac{\partial}{\partial y} (x^3y + C(y)) = x^3+1$. Odejdeme $C(y) = \int 1 \, dy = y$

c) C čtvrtce z $(0,0)$ do $(1,1)$

$$\int_C \vec{F} \, d\vec{r} = \varphi(1,1) - \varphi(0,0) = 1^3 + 1 - (0^3 \cdot 0 + 0) = \underline{\underline{2}}$$

d) Znaleźć takie gęsiony polinom φ , który po całkowaniu (integrowaniu) dałby \vec{F}

$$\int_C \left\{ 3x^2y \, dx + (x^3+1) \, dy \right\} \rightarrow \vec{F}$$

Miejsce z lewej $(0,0)$ do lewej (a,b)

$$\vec{F} = (a+t, b+t) \quad t \in (0,1)$$

$$\frac{d\vec{F}}{dt} = (a, b)$$

$$\vec{F} = (2x^2y, x^3+1)$$

$$\vec{F}(P(1)) = \left(3(a+)^2 b + 1, (a+)^3 + 1 \right) = \left(3a^2 b + 3, a^3 + 1 \right)$$

$$\vec{F}(P(t)) \frac{d\vec{r}}{dt} = 3a^2 b t^3 + a^3 b t^3 + b = 4a^3 b t^3 + b$$

$$\int_C \vec{F} \, d\vec{r} = \int_0^1 4a^3 b t^3 + b \, dt = \left[a^3 b t^4 + b t \right]_0^1 = a^3 b + b$$

Skonszt. p. pl.
 $\int_C \vec{F} \, d\vec{r} = \varphi(a,b) - \varphi(0,0)$

Proba pl.
 $\varphi(a,b) - \varphi(0,0) = a^3 b + b$.

Po przesunięciu promienia o konstantę ($a \rightarrow x, b \rightarrow y, \varphi(0,0) \rightarrow c$) do dolu

$$\varphi(x,y) = x^3 y + y + c$$