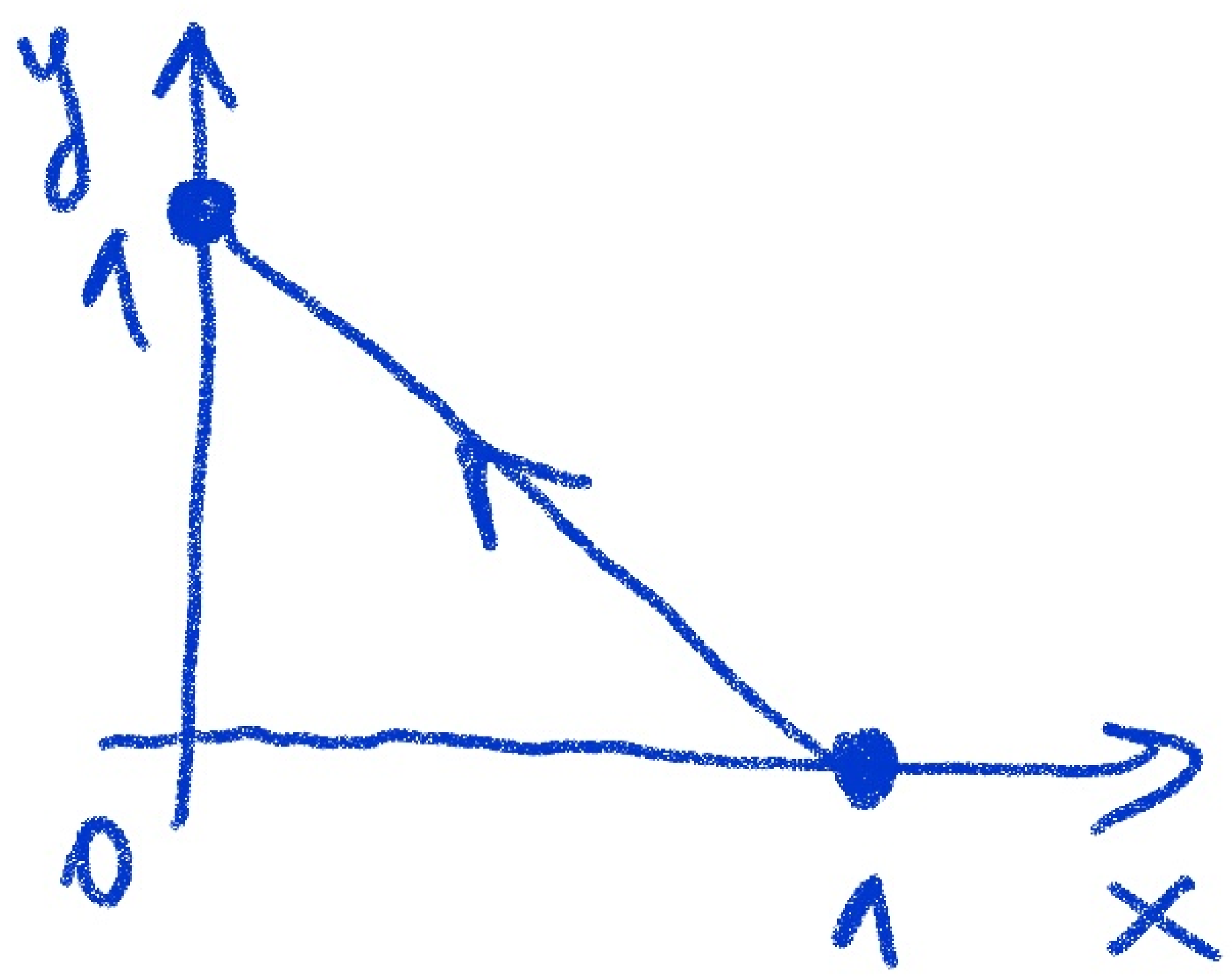
(1) $\int_C x^2 dx + xy\, dy$, C ... úsečka

[1,0] počát. bod, [0,1] koncový bod



x ... param.

$y = 1 - x$, $x \in [0,1]$

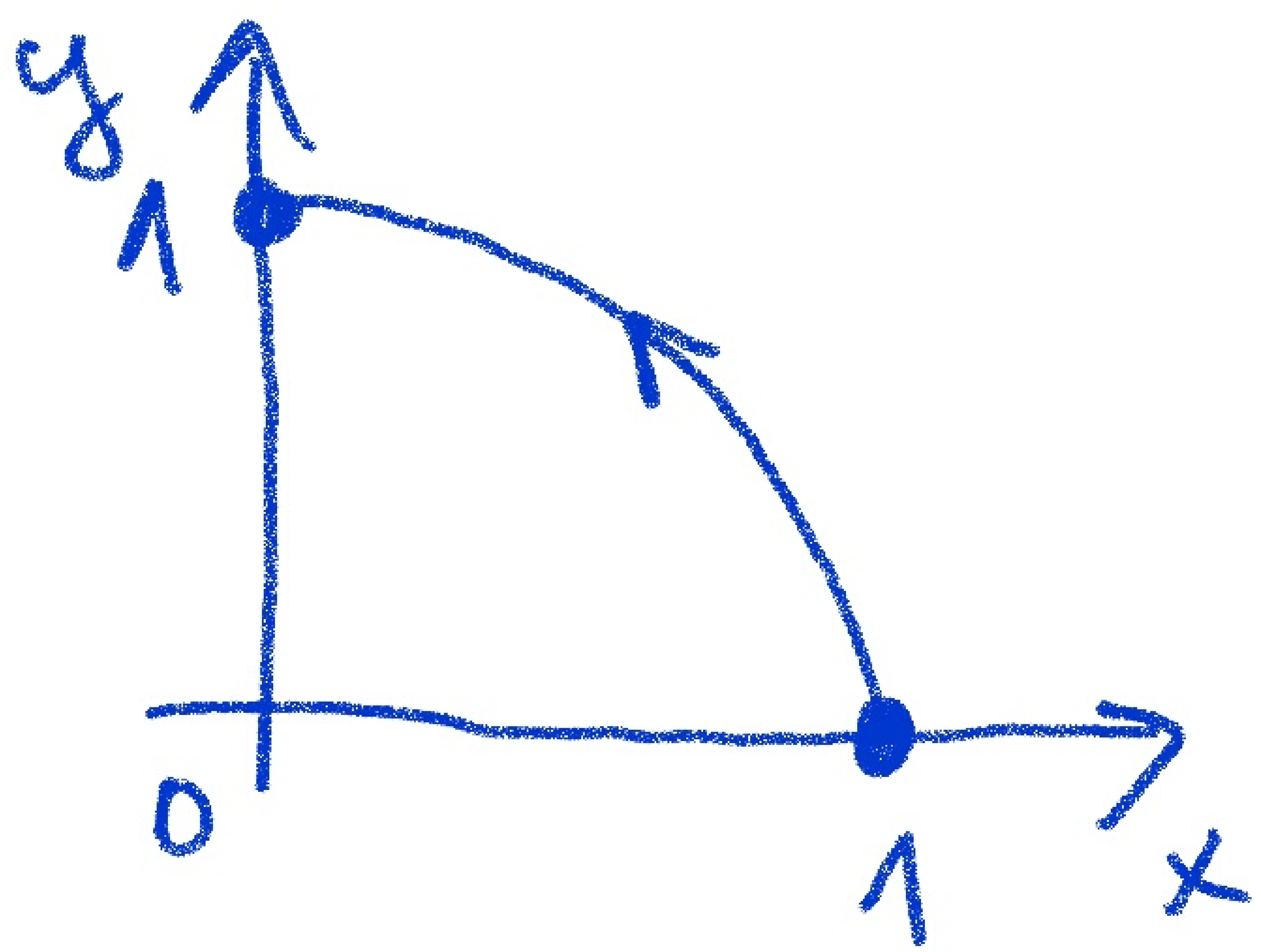
$dy = -dx$

$$\Rightarrow \int_1^0 x^2 \underline{dx} + x(1-x)(-\underline{dx}) = \int_1^0 (x^2 - x + x^2)\, dx = \int_1^0 (2x^2 - x)\, dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_1^0 = 0 - \frac{2}{3} + \frac{1}{2} = \underline{\underline{-\frac{1}{6}}}$$

② $\int_C x^2 dx + xy\,dy$, C … čtvrtkružnice ($x^2+y^2=1$) v 1. KVADRANTU

počát. bod $[1,0]$, koncový bod $[0,1]$



$x = \cos t \Rightarrow dx = -\sin t\,dt$

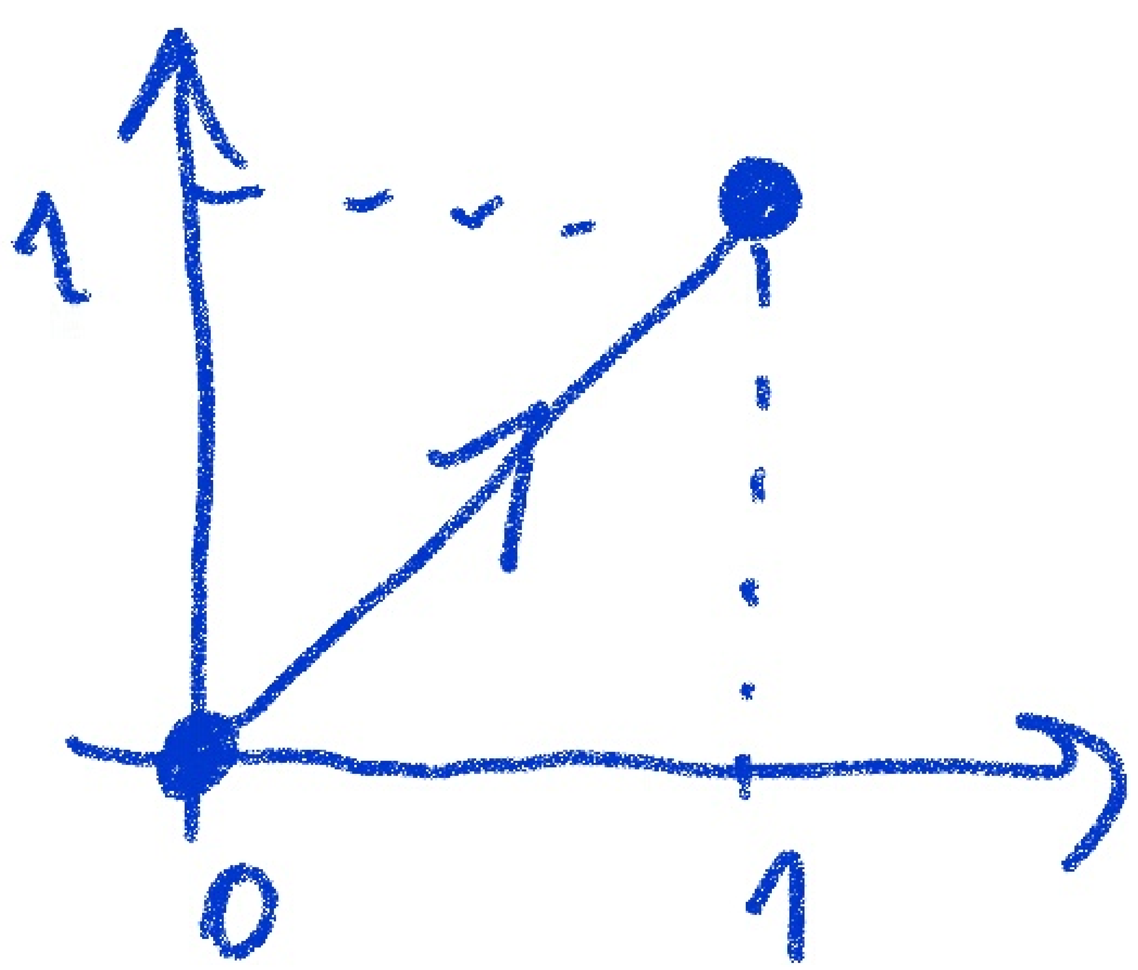
$y = \sin t \Rightarrow dy = \cos t\,dt$

$t \in [0, \frac{\pi}{2}]$

$\Rightarrow \int_0^{\frac{\pi}{2}} \cos^2 t(-\sin t)\,dt + \cos t \cdot \sin t \cdot \cos t\,dt = \underline{\underline{0}}$

③ $\int_C 3x^2y\,dx + (x^3+1)\,dy$, c … úsečka

[0,0] počát. bod , [1,1] koncový bod



$y = x \quad \Rightarrow dy = dx$

x … parametr

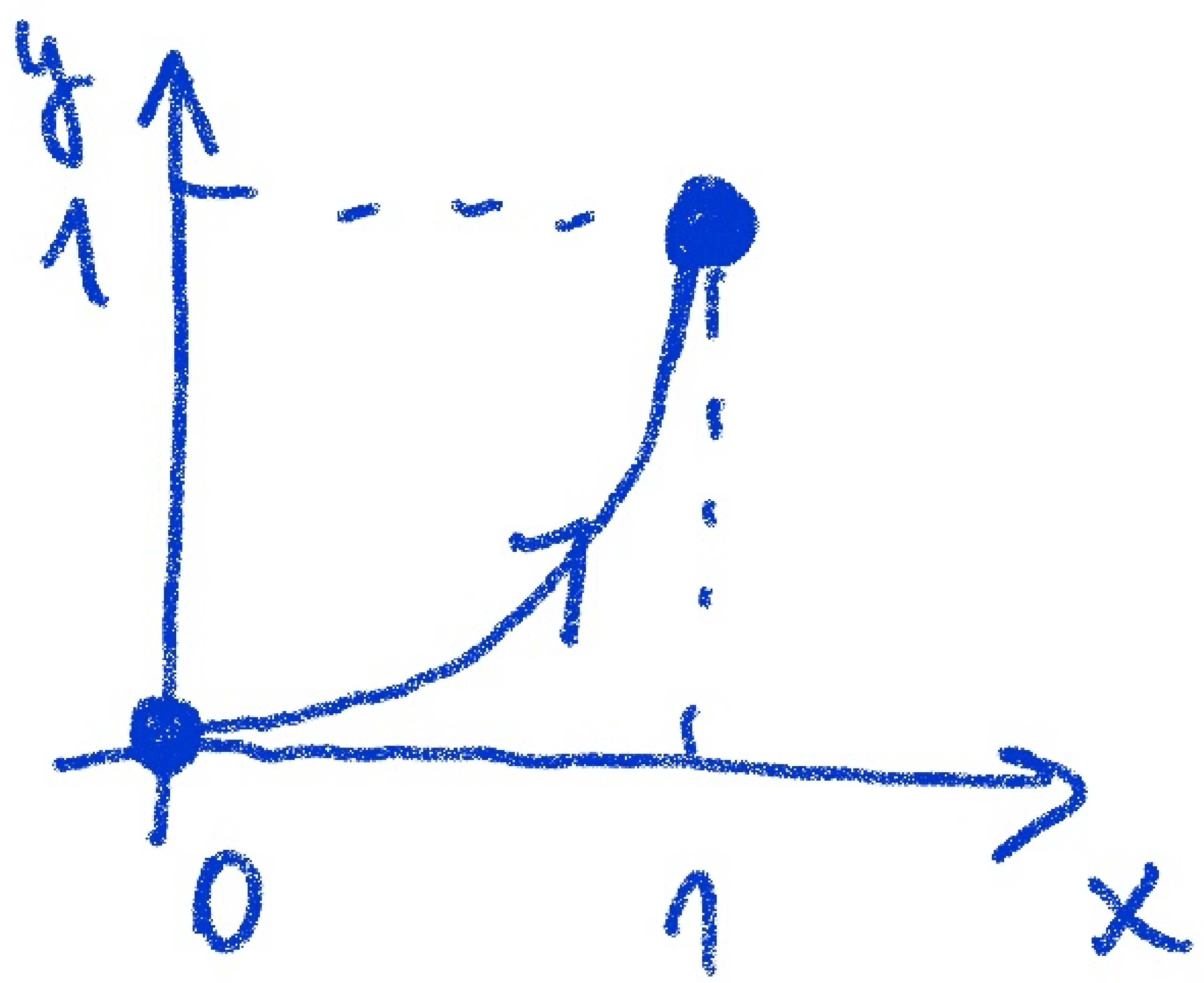
$x \in [0,1]$

$$\Rightarrow \int_0^1 3x^2 \cdot x\,\underline{dx} + (x^3+1)\,\underline{dx} = \int_0^1 (4x^3+1)\,dx$$

$$= \left[x^4 + x\right]_0^1 = 1 + 1 - 0 = \underline{\underline{2}}$$

④ $\int_C 3x^2y\,dx + (x^3+1)\,dy$, C … oblouk paraboly $y=x^2$

[0,0] počát. bod, [1,1] koncový bod



$y=x^2 \Rightarrow dy = 2x\,dx$

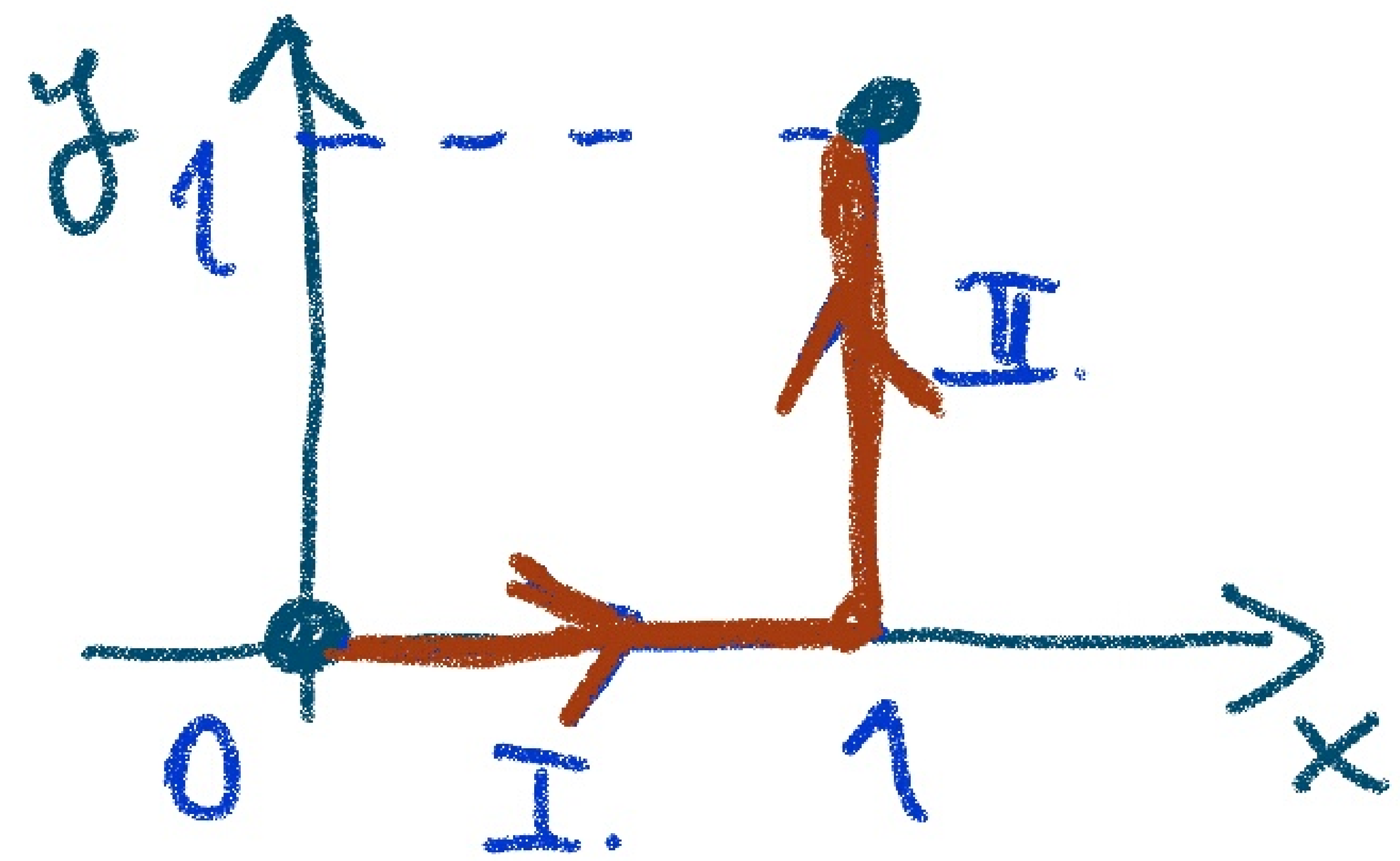
x … param.

$x \in [0,1]$

$$\Rightarrow \int_0^1 3x^2\cdot x^2\,\underline{dx} + (x^3+1)\cdot 2x\,\underline{dx}$$

$$= \int_0^1 (\underbrace{3x^4+2x^4}_{5x^4}+2x)\,dx = [x^5+x^2]_0^1 = 1+1-0 = \underline{\underline{2}}$$

5 $\int\limits_c 3x^2y\,dx + (x^3+1)\,dy$, c … lomená čára, viz. obr.



I. $y = 0 \Rightarrow dy = 0$

x param.

$x \in [0,1] \Rightarrow \int\limits_0^1 0\,dx = \underline{\underline{0}}$

II. $x = 1 \Rightarrow dx = 0 \Rightarrow \int\limits_0^1 3 \cdot 1^2 y \cdot 0 + (1^3+1)\,dy$

y … param.

$y \in [0,1]$

$= \int\limits_0^1 2\,dy = [2y]_0^1 = 2 - 0 = \underline{\underline{2}}$

$$\boxed{\int_C P\,dx + Q\,dy \quad : \quad P'_y = Q'_x \implies \text{nezávislost na integrační cestě}}$$

Př. 1, 2:

$$\int_C x^2 dx + xy\,dy$$

$$P = x^2 \quad , \quad Q = xy$$

$$\left.\begin{aligned} P'_y &= 0 \\ Q'_x &= y \end{aligned}\right\} \text{nejsou stejné!}$$

Př. 3, 4, 5:

$$\int_C 3x^2y\,dx + (x^3+1)\,dy$$

$$P = 3x^2y$$

$$Q = x^3+1$$

$$\left.\begin{aligned} P'_y &= 3x^2 \\ Q'_x &= 3x^2 \end{aligned}\right\} \checkmark$$