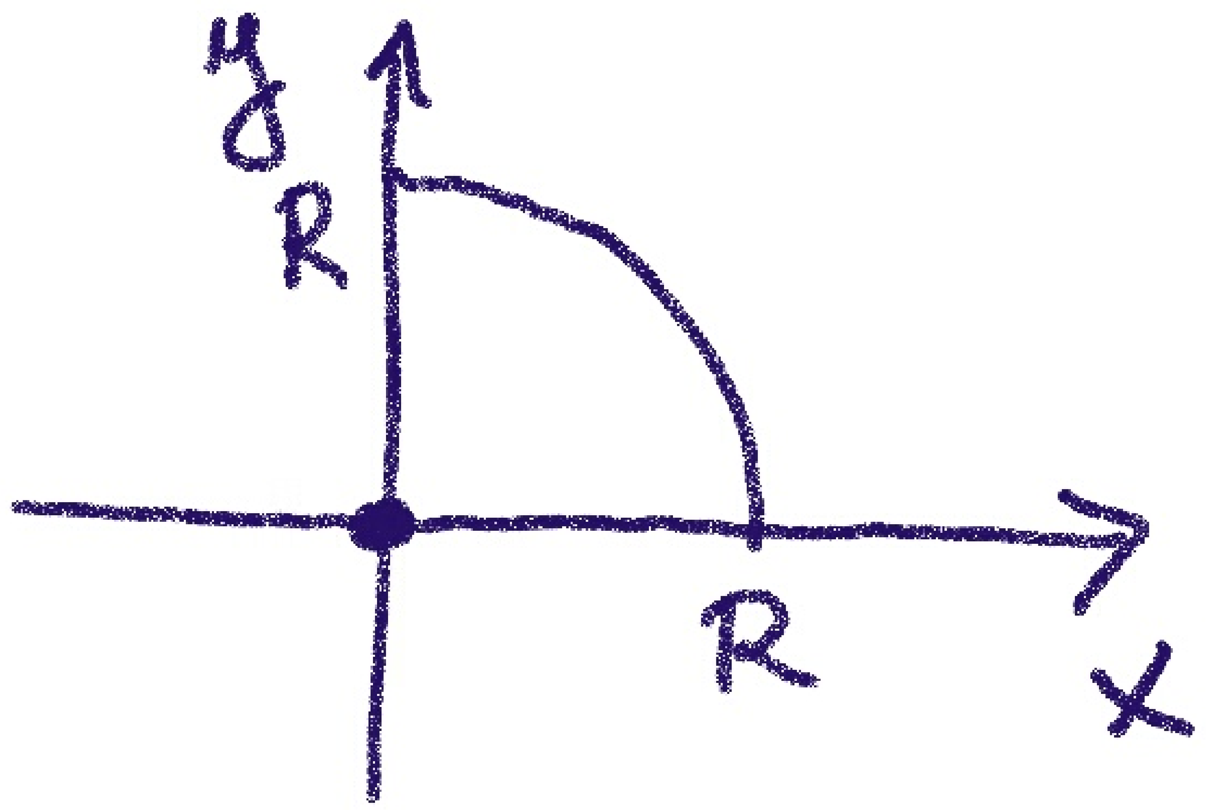
1. $\int_c xy\,ds$, c ... čtvrtina kružnice se středem v počátku o poloměru R (v 1. kvadrantu)



$$\left.\begin{aligned} x &= \varphi(t) \\ y &= \psi(t) \\ t &\in [\alpha, \beta] \end{aligned}\right\} \Rightarrow \int_c f(x,y)\,ds = \int_\alpha^\beta f(\varphi(t), \psi(t)) \cdot \sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\,dt$$

$$x = R\cdot\cos t \qquad \sqrt{(-R\cdot\sin t)^2 + (R\cdot\cos t)^2} = \sqrt{R^2(\underbrace{\sin^2 t + \cos^2 t}_{=1})} = R$$

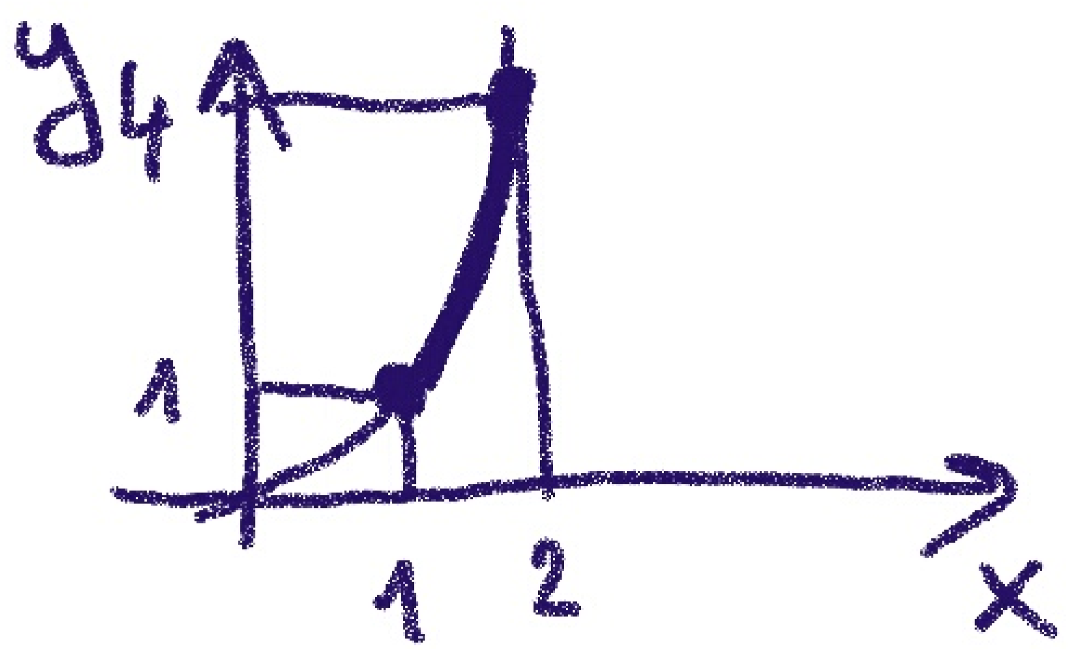
$$y = R\cdot\sin t$$

$$t \in [0, \tfrac{\pi}{2}] \Rightarrow \int_0^{\frac{\pi}{2}} R^2 \cdot \cos t \cdot \sin t \cdot R\,dt = R^3 \int_0^{\frac{\pi}{2}} \underbrace{\cos t \cdot \sin t}_{\frac{1}{2}\sin 2t}\,dt = \frac{1}{2}R^3 \int_0^{\frac{\pi}{2}} \sin 2t\,dt$$

$$= \frac{1}{2}R^3\left[-\frac{1}{2}\cos 2t\right]_0^{\frac{\pi}{2}} = \frac{1}{4}R^3\left[-\overbrace{\cos\pi}^{-1} + \overbrace{\cos 0}^{=1}\right] = \frac{1}{2}R^3$$

$$\boxed{\sin 2t = 2\sin t\cdot\cos t}$$

② $\int_C x \, ds$, C … oblouk paraboly $y = x^2$ mezi body $[1,1]$ a $[2,4]$



x … parametr

$y = x^2$

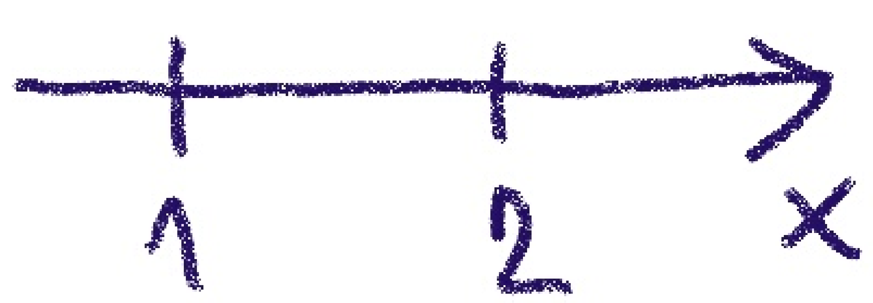
$x \in [1,2]$

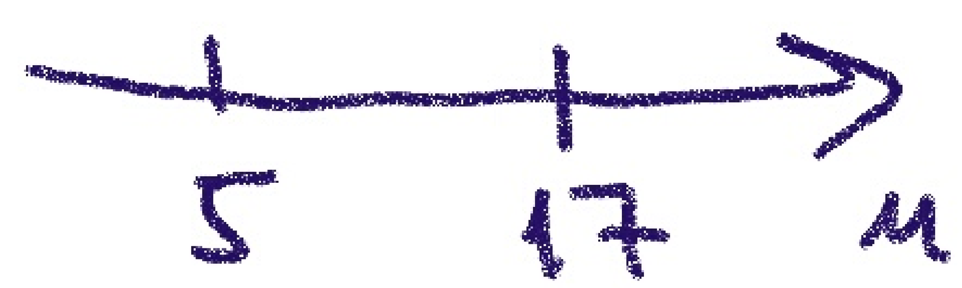
$$\int_C f(x,y)\, ds = \int_\alpha^\beta f(\varphi(t), \psi(t)) \cdot \underbrace{\sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\, dt}$$

$$\sqrt{1^2 + (2x)^2}\, dx$$

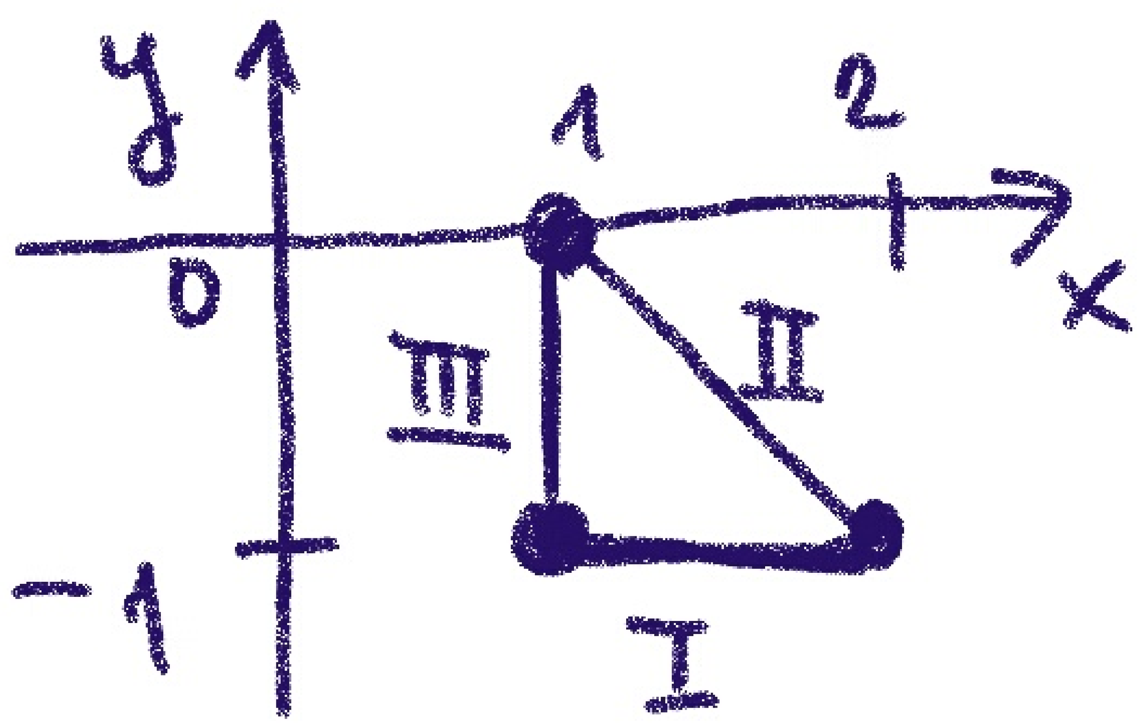
$$\Rightarrow \int_1^2 x \cdot \sqrt{1+4x^2}\, dx = \left| \begin{array}{l} u = 1+4x^2 \\ du = 8x\, dx \end{array} \right| = \int_5^{17} \sqrt{u} \cdot \frac{1}{8}\, du = \frac{1}{8} \int_5^{17} u^{\frac{1}{2}}\, du$$

$$= \frac{1}{8} \left[\frac{u^{3/2}}{\frac{3}{2}} \right]_5^{17} = \frac{1}{12} \left(17^{3/2} - 5^{3/2} \right)$$





③ $\int\limits_C (x+y)\,ds$, C… trojúhelník s vrcholy $[1,-1]$, $[2,-1]$, $[1,0]$



I. x…param.
$y=-1$
$x\in[1,2]$

$$\Rightarrow \int_1^2 (x-1)\sqrt{1+0}\,dx = \int_1^2 (x-1)\,dx = \left[\frac{x^2}{2}-x\right]_1^2 = 2-2-\left(\frac{1}{2}-1\right) = \frac{1}{2}$$

II. x…param.
$y=1-x$
$x\in[1,2]$

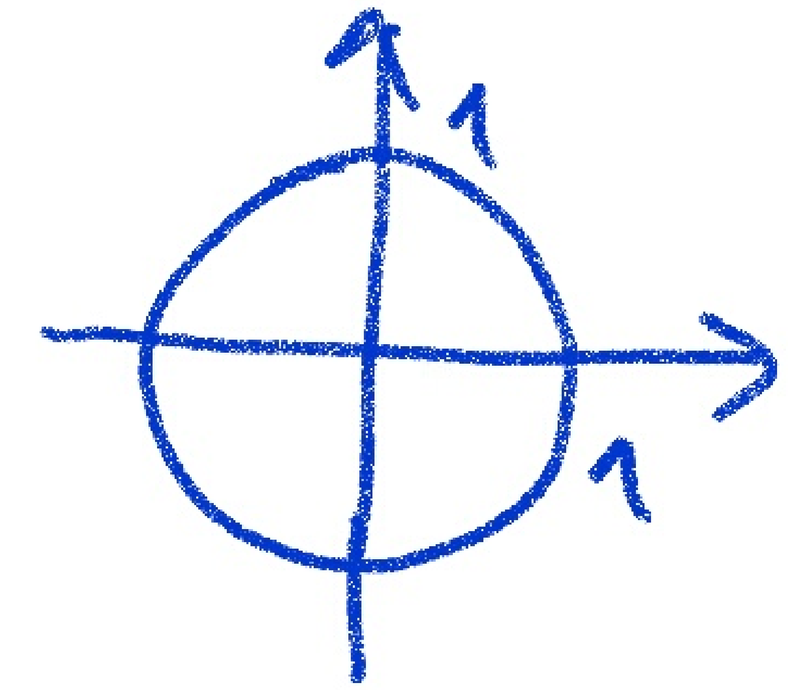
$$\Rightarrow \int_1^2 (x+1-x)\cdot\sqrt{1+1}\,dx = \int_1^2 \sqrt{2}\,dx = \left[\sqrt{2}\cdot x\right]_1^2 = \sqrt{2}\cdot 2-\sqrt{2}\cdot 1 = \sqrt{2}$$

III. y…param.
$x=1$
$y\in[-1,0]$

$$\Rightarrow \int_{-1}^0 (1+y)\cdot\sqrt{0+1}\,dy = \int_{-1}^0 (1+y)\,dy = \left[y+\frac{y^2}{2}\right]_{-1}^0 = 0-\left(-1+\frac{1}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2}+\sqrt{2}+\frac{1}{2} = 1+\sqrt{2}$$

④ $\int_C (x^2+y^2)\,ds$, C … kružnice o poloměru 1 se středem v počátku



a) $x=\cos t,\ y=\sin t,\ t\in[0,2\pi]$

$x^2+y^2=1$, $\quad ds=\sqrt{(-\sin t)^2+(\cos t)^2}\,dt = 1\,dt$

$$\Rightarrow \int_0^{2\pi} 1\,dt = [t]_0^{2\pi} = 2\pi - 0 = \underline{\underline{2\pi}}$$

b) $x=\cos 2t,\ y=\sin 2t,\ t\in[0,\pi]$

$x^2+y^2=1$, $\quad ds=\sqrt{(-\sin 2t\cdot 2)^2+(\cos 2t\cdot 2)^2}\,dt$

$$= \sqrt{4(\underbrace{\sin^2 2t+\cos^2 2t}_{=1})}\,dt = 2\,dt$$

$$\Rightarrow \int_0^{\pi} 2\,dt = [2t]_0^{\pi} = \underline{\underline{2\pi}}$$

c) $x = \cos t^2, \quad y = \sin t^2, \quad t \in [0, \sqrt{2\pi}]$

$$\boxed{\int_C (x^2+y^2)\, ds}$$

$x^2+y^2=1$

$$ds = \sqrt{(-\sin t^2 \cdot 2t)^2 + (\cos t^2 \cdot 2t)^2}\, dt$$

$$= \sqrt{4t^2 \underbrace{(\sin^2 t^2 + \cos^2 t^2)}_{=1}}\, dt = 2t\, dt$$

$$\Rightarrow \int_0^{\sqrt{2\pi}} 2t\, dt = \left[t^2\right]_0^{\sqrt{2\pi}} = (\sqrt{2\pi})^2 - 0^2 = \underline{\underline{2\pi}}$$