① $y' \cdot y(x+1) = 2 \,,\; y(0) = 2$

$$y' = f(x) \cdot g(y)$$

$$y' = \frac{2}{y(x+1)} = \frac{2}{x+1} \cdot \frac{1}{y}$$

- konst. řeš.: $\frac{1}{y} \neq 0 \Rightarrow$ nemá konst. řeš.

- $$\frac{dy}{dx} = \frac{2}{x+1} \cdot \frac{1}{y}$$

$$\int y \, dy = \int \frac{2}{x+1} \, dx$$

$$\int \frac{2}{x+1} dx = 2 \int \frac{1}{x+1} dx = 2 \ln|x+1|$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$$

$$\frac{1}{2} y^2 = 2 \ln(x+1) + c \quad / \cdot 2$$

$$y^2 = 4 \ln(x+1) + K \qquad (K = 2c)$$

$$y(0) = 2: \quad 2^2 = 4 \ln(0+1) + K$$

$$4 = K$$

$$\Rightarrow \quad y_p^2 = 4 \ln(x+1) + 4$$

$$y_p^2 = 4(\ln(x+1) + 1)$$

$$\boxed{y_p = 2 \cdot \sqrt{\ln(x+1) + 1}}$$

2. $\boxed{y' = xy^2}$

a) Najděte všechna řešení.
b) Najděte řešení splňující $y(1) = -2$.
c) Najděte řešení splňující $y(1) = 0$.

---

a) konst. řeš: $y^2 = 0 \Rightarrow \boxed{y = 0}$

$$\frac{dy}{dx} = xy^2$$

$$\int \frac{1}{y^2}\, dy = \int x\, dx$$

$$-\frac{1}{y} = \frac{x^2}{2} + c$$

$$y\left(\frac{x^2}{2} + c\right) = -1 \quad / \cdot 2$$

$$\boxed{y\left(x^2 + K\right) = -2}$$ obecné řešení

$(K = 2c)$

$$\int \frac{1}{y^2} dy = \int y^{-2} dy = \frac{y^{-1}}{-1} = -\frac{1}{y}$$

b) $y(1) = -2$:

$$-2\left(1^2 + K\right) = -2$$

$$\Rightarrow \underline{\underline{K = 0}}$$

$$y_p \cdot x^2 = -2 \Rightarrow \underline{\underline{y_p = -\frac{2}{x^2}}}$$

c) $y(1) = 0$

$\Downarrow$

$\underline{\underline{y_p = 0}}$

(3) $y' = \sqrt[3]{y}$

- konst. řešení: $\underline{y = 0}$

- $\frac{dy}{dx} = \sqrt[3]{y}$

$$\int \frac{1}{\sqrt[3]{y}}\, dy = \int dx$$

$$\frac{3}{2} \cdot \sqrt[3]{y^2} = x + c \quad \Big/ \cdot \frac{2}{3}$$

$$\sqrt[3]{y^2} = \frac{2}{3}(x+c) = \frac{2}{3}x + K \quad \left(K = \frac{2}{3}c\right)$$

$$\underline{y^2 = \left(\frac{2}{3}x + K\right)^3}$$

$$\int \frac{1}{\sqrt[3]{y}}\, dy = \int \frac{1}{y^{1/3}}\, dy = \int y^{-\frac{1}{3}}\, dy$$

$$= \frac{y^{\frac{2}{3}}}{\frac{2}{3}} = \frac{3}{2}\, y^{\frac{2}{3}} = \underline{\frac{3}{2}\sqrt[3]{y^2}}$$

$$④ \quad y' \cdot y\,(1+x^2) = x\,(1+y^2) \quad , \quad y(-2) = 1$$

$$\frac{dy}{dx} = y' = \frac{x}{1+x^2} \cdot \frac{1+y^2}{y}$$

- konst. rješ. nem

- $$\int \frac{2y}{1+y^2}\,dy = \int \frac{2x}{1+x^2}\,dx \quad / \cdot 2$$

$$\ln(1+y^2) = \ln(1+x^2) + C$$

$$1+y^2 = e^{\ln(1+x^2)+C}$$

$$1+y^2 = e^{\ln(1+x^2)} \cdot e^C \quad , \quad K = e^C$$

$$1+y^2 = (1+x^2)\cdot K$$

$$\underline{y^2 = (1+x^2)\cdot K - 1}$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|$$

$$\ln y = x \Rightarrow y = e^x$$

$$e^{\ln x} = x$$

$y(-2) = 1$:

$$1^2 = (1+(-2)^2)\cdot K - 1$$

$$2 = 5K \Rightarrow \underline{\underline{K = \frac{2}{5}}}$$

$$y_P^2 = (1+x^2)\cdot\frac{2}{5} - 1$$

$$\boxed{y_P = \sqrt{\frac{2}{5}(1+x^2) - 1}}$$