

$$\textcircled{1} \quad \begin{aligned} x' &= 6x - 5y \\ y' &= x + 3y \end{aligned}$$

Určete stacionární body a jejich typ

$$6x - 5y = 0$$

$$x + 3y = 0 \Rightarrow \underline{x = -3y}$$

$$-18y - 5y = 0$$

$$-23y = 0 \Rightarrow y = 0 \Rightarrow x = 0 \Rightarrow \underline{\underline{[0, 0]}}$$

$$J = \begin{pmatrix} 6 & -5 \\ 1 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} 6-\lambda & -5 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (6-\lambda)(3-\lambda) + 5$$

$$= 18 - 6\lambda - 3\lambda + \lambda^2 + 5$$

$$= \lambda^2 - 9\lambda + 23 = 0$$

$$\lambda_{1,2} = \frac{9 \pm \sqrt{81 - 92}}{2} = \frac{9 \pm i\sqrt{11}}{2}$$



NESTABILNÍ OHNISKO



$$\textcircled{2} \quad \begin{cases} x' = x^2 - y \\ y' = x - y^2 + 2x^4 \end{cases}$$

$$x^2 - y = 0 \Rightarrow \underline{y = x^2}$$

$$x - y^2 + 2x^4 = 0$$

$$x - x^4 + 2x^4 = 0$$

$$x + x^4 = 0$$

$$x(1 + x^3) = 0$$

$$x = 0 \Rightarrow y = 0 \Rightarrow [0, 0]$$

$$x = -1 \Rightarrow y = 1 \Rightarrow [-1, 1]$$

$$J(x, y) = \begin{pmatrix} 2x & -1 \\ 1 + 4x^3 & -2y \end{pmatrix}$$

$$1) \underline{J(0, 0)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1 \Rightarrow \lambda = \pm i$$

ohnisko nebo bod rotace

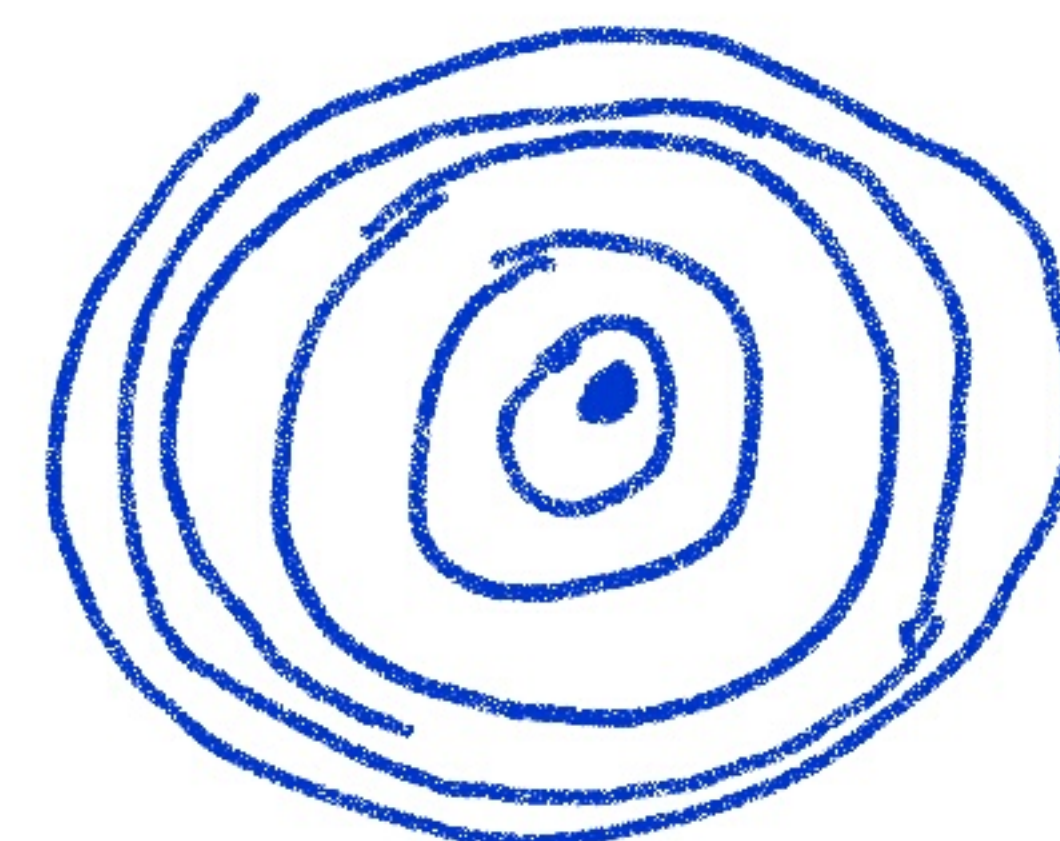
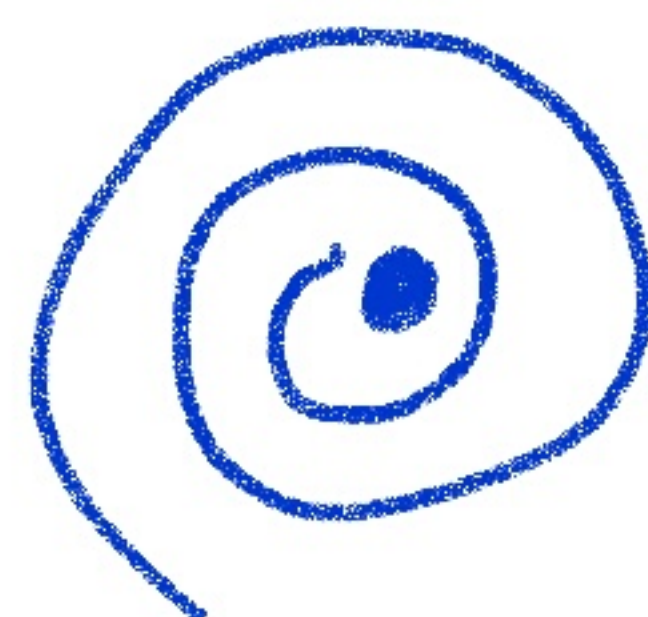
$$2) \underline{J(-1, 1)} = \begin{pmatrix} -2 & -1 \\ -7 & -2 \end{pmatrix} \Rightarrow \begin{vmatrix} -2-\lambda & -1 \\ -7 & -2-\lambda \end{vmatrix} = (-2-\lambda)^2 - 7 = 4 + 4\lambda + \lambda^2 - 7 = \lambda^2 + 4\lambda - 3 = 0$$

$$\left. \begin{array}{l} -2 + \sqrt{7} > 0 \\ -2 - \sqrt{7} < 0 \end{array} \right\} \Rightarrow \underline{\underline{SEDLO}}$$

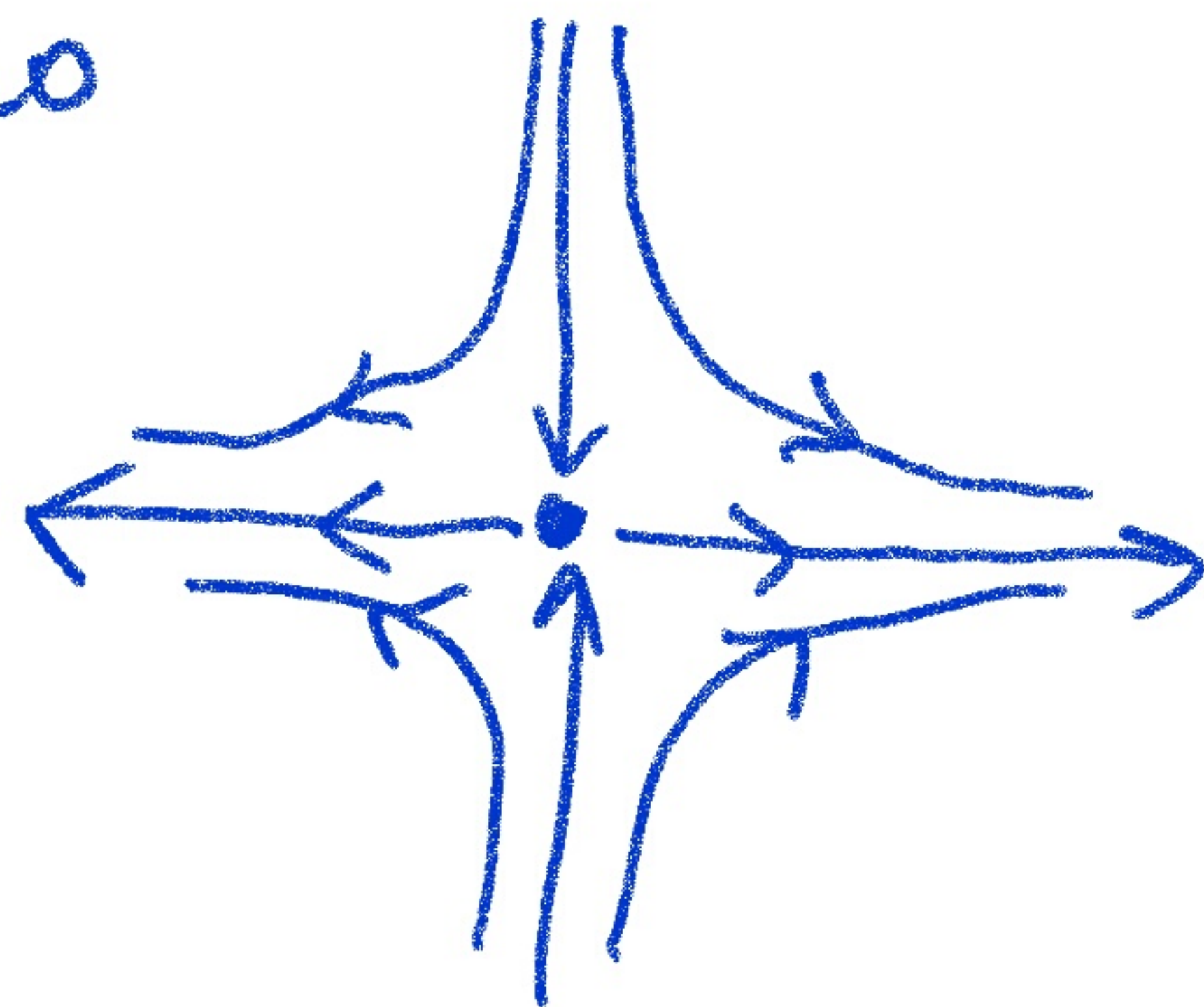
$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 + 12}}{2} = \frac{-4 \pm \sqrt{28}}{2} = \underline{\underline{-2 \pm \sqrt{7}}}$$



$[0,0]$  OHLUSZO NEBO BOD ROTACE



$[-1,1]$  SEDLO





$$\textcircled{3} \begin{cases} x' = x - 2y \\ y' = 5x - y \end{cases}$$

$$\begin{aligned} x - 2y &= 0 \Rightarrow x = 2y \\ 5x - y &= 0 \end{aligned}$$

$$10y - y = 0$$

$$9y = 0 \Rightarrow y = 0$$

$$\downarrow \\ x = 0$$

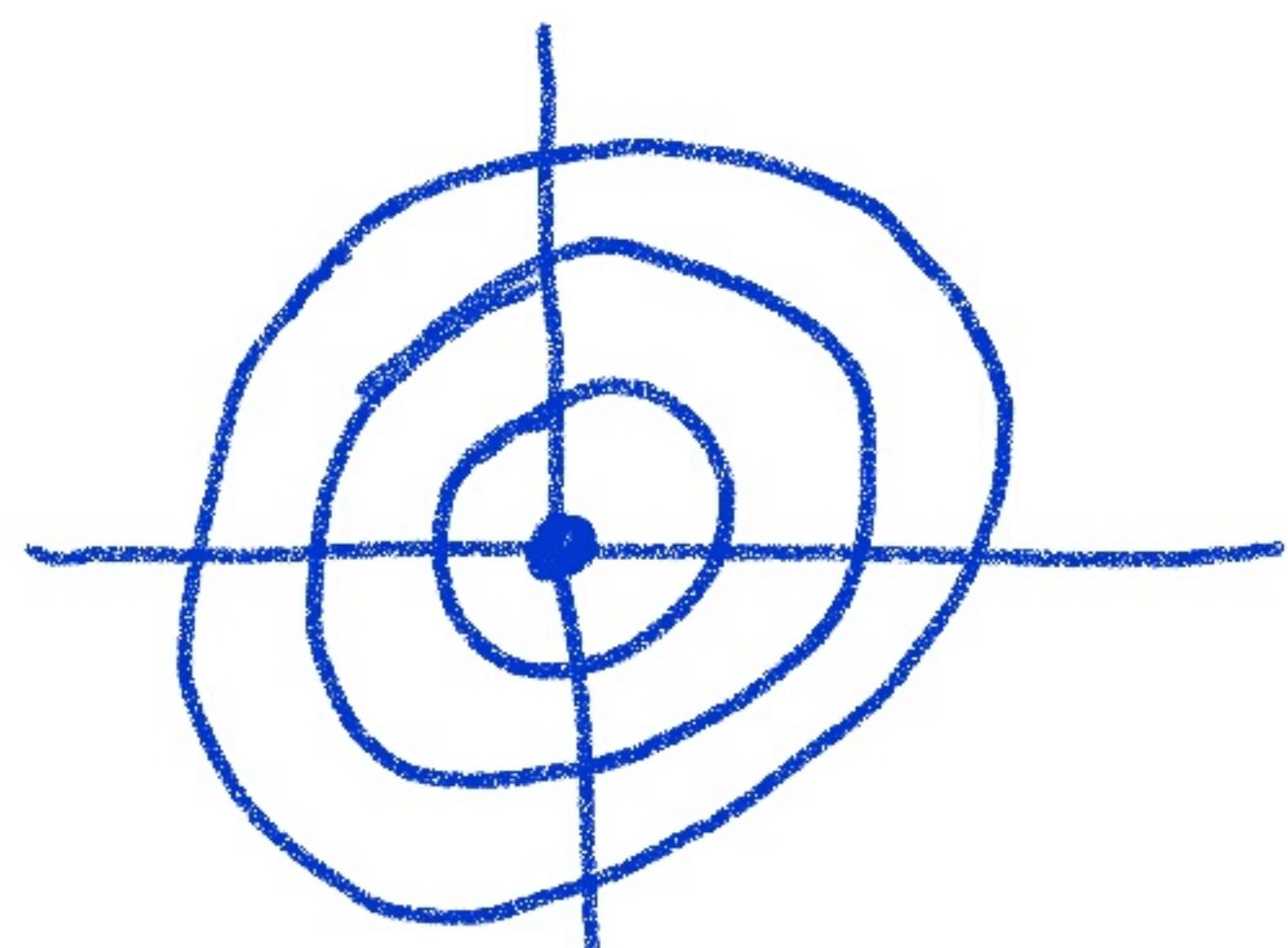
$$\underline{\underline{[0,0]}}$$

$$J = \begin{pmatrix} 1 & -2 \\ 5 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 5 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) - 5 \cdot (-2) = -1 - \lambda + \lambda + \lambda^2 + 10 \\ = \lambda^2 + 9 = 0 \Rightarrow \lambda^2 = -9$$

$$\underline{\underline{\lambda = \pm 3i}}$$

$\Rightarrow$  STRED





$$\textcircled{4} \begin{cases} x' = y^2 - x^2 \\ y' = 1 - y \end{cases}$$

$$y^2 - x^2 = 0$$

$$1 - y = 0 \Rightarrow \underline{y = 1}$$

$$1 - x^2 = 0$$

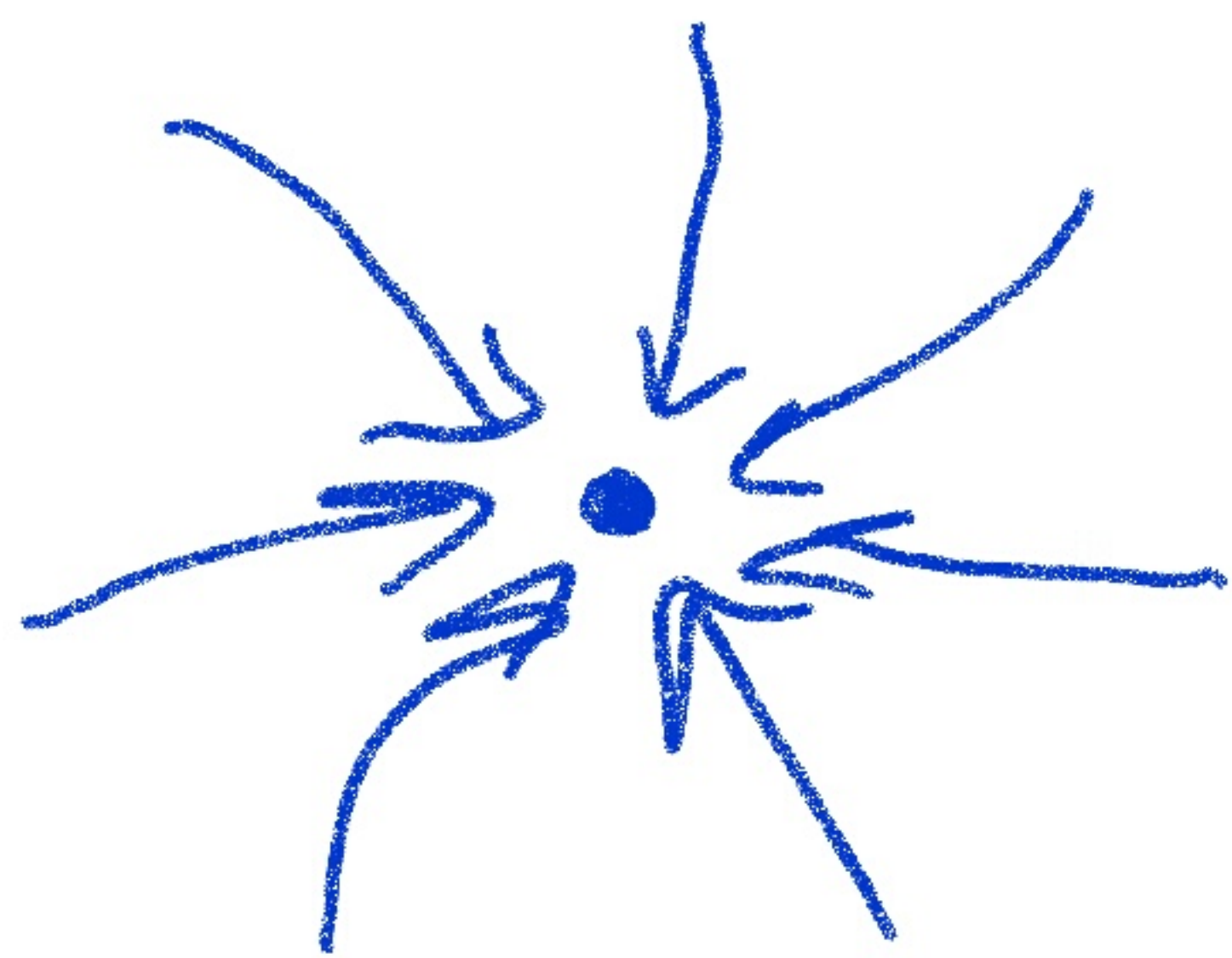
$$x^2 = 1 \Rightarrow \underline{x = \pm 1}$$

$$\Rightarrow \underline{[1, 1]}, \underline{[-1, 1]}$$

$$J(x, y) = \begin{pmatrix} -2x & 2y \\ 0 & -1 \end{pmatrix}$$

$$\textcircled{1} J(1, 1) = \begin{pmatrix} -2 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{vmatrix} -2 - \lambda & 2 \\ 0 & -1 - \lambda \end{vmatrix} = (-2 - \lambda) \cdot (-1 - \lambda) = 0$$
$$\underline{\lambda_1 = -2}, \underline{\lambda_2 = -1}$$



STABILNÍ UZEL

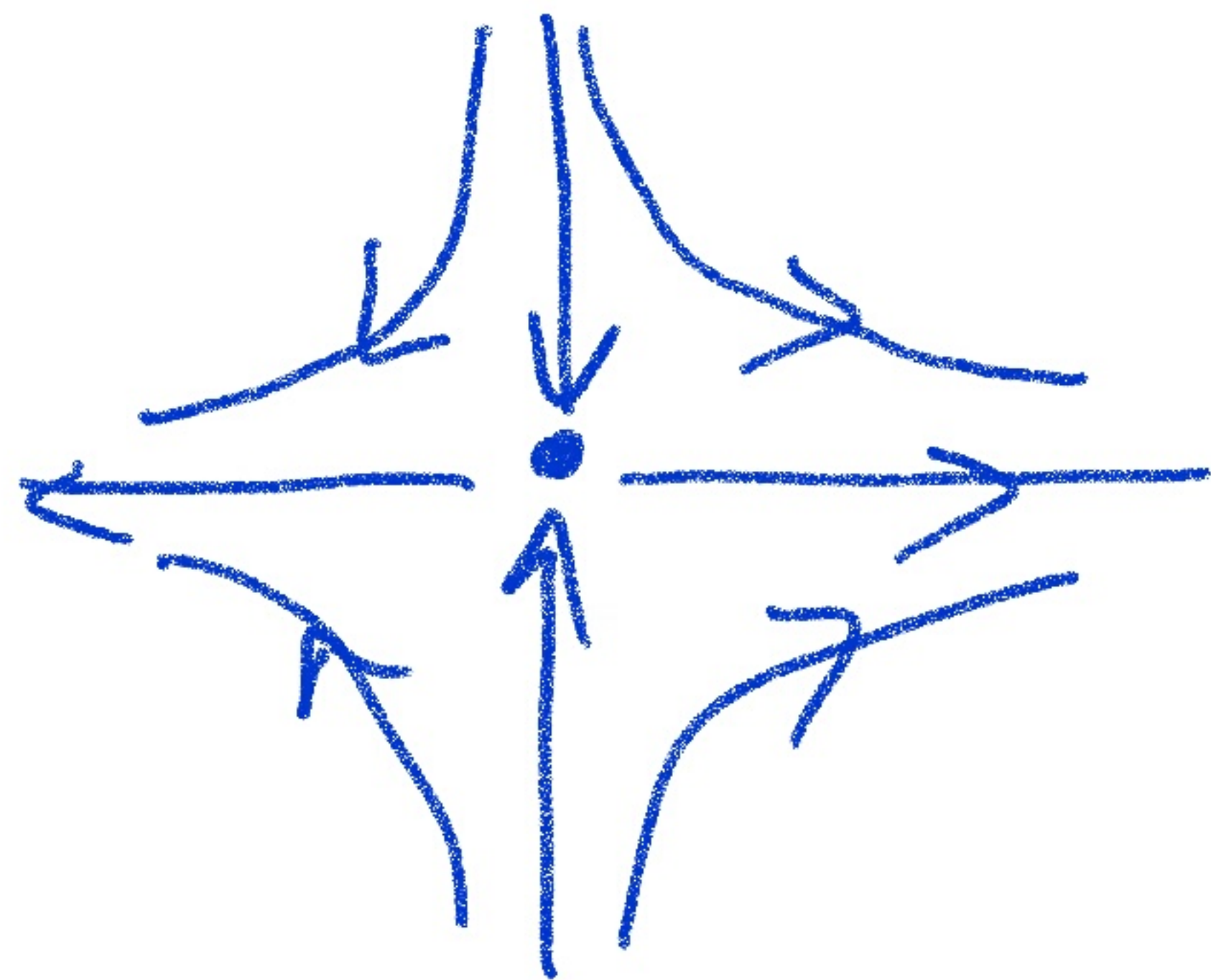


$$\textcircled{2} J(-1, 1) = \begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 2-\lambda & 2 \\ 0 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) = 0$$

$$\underline{\underline{\lambda_1 = 2}}, \quad \underline{\underline{\lambda_2 = -1}}$$

SADDLE





$$\textcircled{5} \quad \left. \begin{aligned} x' &= x + y - 2 \\ y' &= x + y^2 - 2 \end{aligned} \right\}$$

$$x + y - 2 = 0 \quad \Rightarrow \quad \underline{x = 2 - y}$$

$$x + y^2 - 2 = 0$$

$$2 - y + y^2 - 2 = 0 \quad \Rightarrow \quad y^2 - y = 0$$

$$y(y - 1) = 0$$

$$y = 0$$

$$x = 2$$

$$y = 1$$

$$x = 1$$



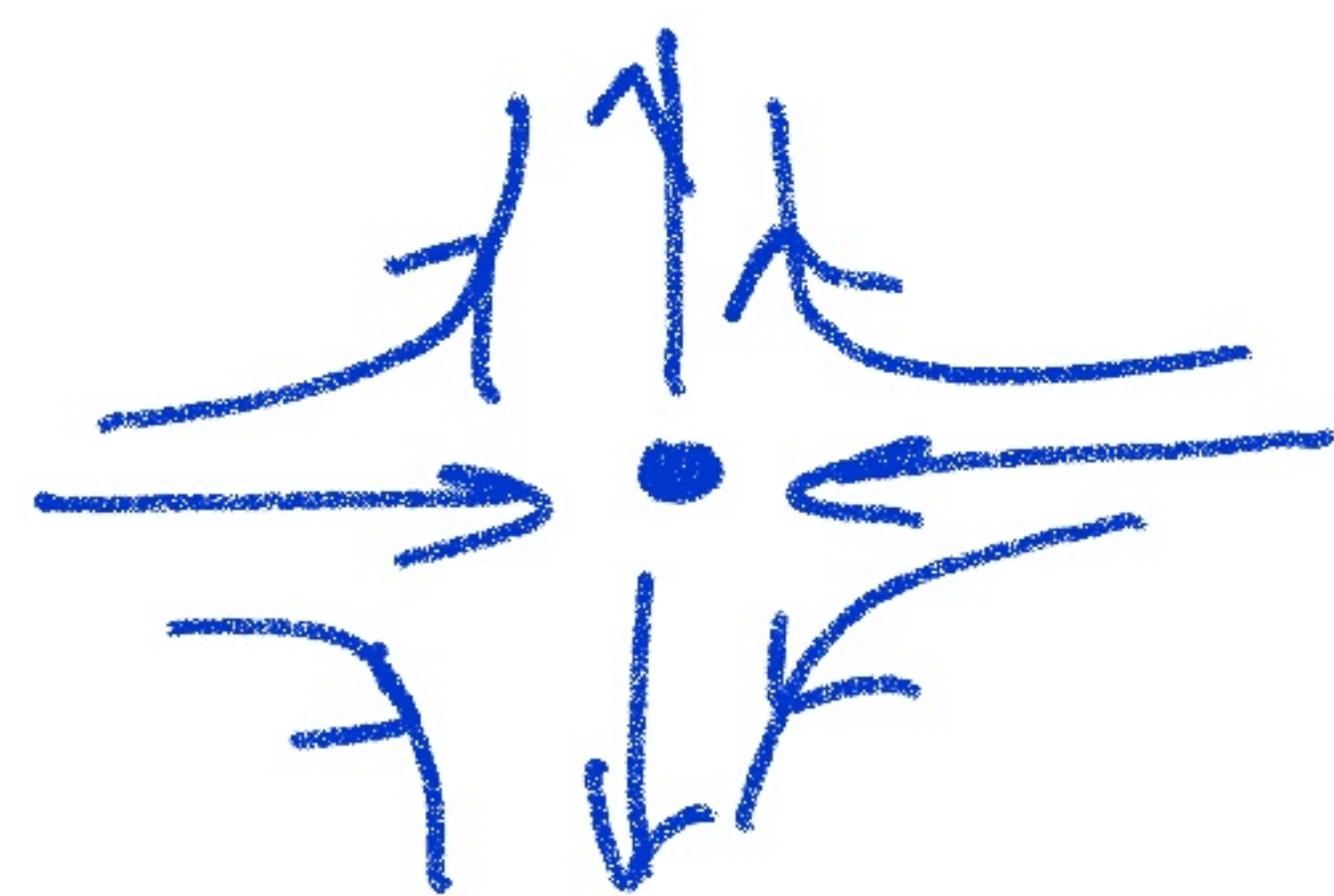
$$\underline{\underline{[2, 0]}}$$

$$\underline{\underline{[1, 1]}}$$

$$J(x, y) = \begin{pmatrix} 1 & 1 \\ 1 & 2y \end{pmatrix}$$



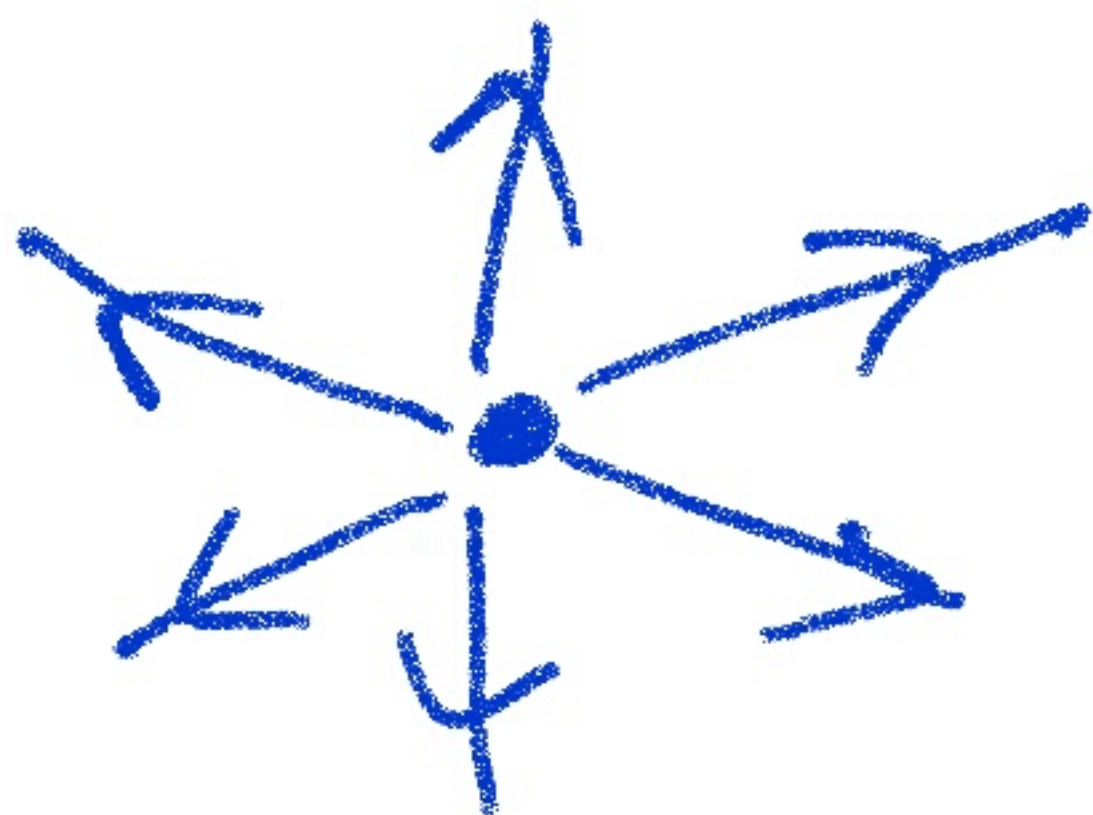
$$1) J(2,0) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$



$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = (1-\lambda) \cdot (-\lambda) - 1 = -\lambda + \lambda^2 - 1 \Rightarrow \underline{\lambda^2 - \lambda - 1 = 0}$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} \quad \left) \quad \begin{array}{l} \frac{1+\sqrt{5}}{2} > 0 \\ \frac{1-\sqrt{5}}{2} < 0 \end{array} \right. \quad \text{SADDLE}$$

$$2) J(1,1) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$



$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 1 = 2 - \lambda - 2\lambda + \lambda^2 - 1 = \lambda^2 - 3\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} > 0$$

UNSTABILNES KNOTEN