ŘEŠENÍ UKÁZKOVÉ PÍSEMKY II

(1)

$$y' + 2y = e^{3x} \quad / \cdot e^{2x} \qquad \text{integrační faktor: } e^{2x}$$

$$y' \cdot e^{2x} + 2y \cdot e^{2x} = e^{3x} \cdot e^{2x}$$

$$(y \cdot e^{2x})' = e^{5x}$$

$$y \cdot e^{2x} = \frac{1}{5} e^{5x} + c \quad \Rightarrow \quad y = e^{-2x}\left(\frac{1}{5} e^{5x} + c\right) = \frac{1}{5} e^{3x} + c \cdot e^{-2x}$$

(2)

$$\left.\begin{aligned} x^2 + x - y &= 0 \\ 2x - y &= 0 \quad \Rightarrow y = 2x \end{aligned}\right\} \Rightarrow \begin{aligned} x^2 + x - 2x &= 0 \\ x^2 - x &= 0 \\ x(x-1) &= 0 \quad \Rightarrow [0,0],\ [1,2] \end{aligned}$$

$$J = \begin{pmatrix} 2x+1 & -1 \\ 2 & -1 \end{pmatrix}$$

a) $$J(0,0) = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \Rightarrow \begin{vmatrix} 1-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) + 2 = \lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

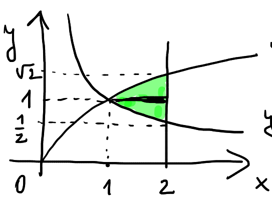
$\Rightarrow$ BOD ROTACE NEBO OHNISKO

b) $$J(1,2) = \begin{pmatrix} 3 & -1 \\ 2 & -1 \end{pmatrix} \Rightarrow \begin{vmatrix} 3-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-\lambda) + 2 = \lambda^2 - 2\lambda - 1 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$\Rightarrow$ SEDLO

(3)



$$y = \sqrt{x} \Rightarrow x = y^2$$

$$y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$$

a) $$\int_1^2 \left[\int_{\frac{1}{x}}^{\sqrt{x}} 2xy \, dy \right] dx$$

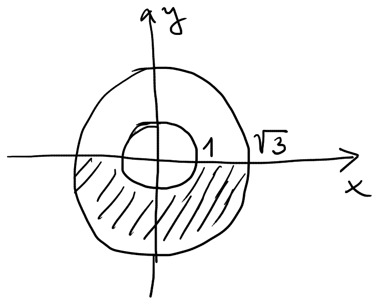
$$\int_0^{\frac{1}{2}} \left[\int_{\frac{1}{y}}^{2} 2xy \, dx \right] dy + \int_1^{\sqrt{2}} \left[\int_{y^2}^{2} 2xy \, dx \right] dy$$

b) $$\int_1^2 \left[\int_{\frac{1}{x}}^{\sqrt{x}} 2xy \, dy \right] dx = \int_1^2 \left[xy^2 \right]_{\frac{1}{x}}^{\sqrt{x}} dx = \int_1^2 \left(x^2 - \frac{1}{x} \right) dx = \left[\frac{x^3}{3} - \ln x \right]_1^2$$

$$= \frac{8}{3} - \ln 2 - \frac{1}{3} + \ln 1 = \frac{7}{3} - \ln 2$$

④

Ω:



$$\iint_{\Omega} \frac{x+y}{\sqrt{x^2+y^2}}\,dxdy = \int_{\pi}^{2\pi}\left[\int_{1}^{\sqrt{3}} \frac{r\cdot\cos\varphi + r\cdot\sin\varphi}{\sqrt{r^2}}\cdot r\,dr\right]d\varphi$$

$$= \int_{\pi}^{2\pi}\left[\int_{1}^{\sqrt{3}} r(\cos\varphi+\sin\varphi)\,dr\right]d\varphi$$

⑤

$P = 2xy - y \implies P'_y = 2x-1$

$Q = x^2 - x \qquad Q'_x = 2x-1$

$P'_y = Q'_x \implies$ nezávisí na int. cestě

kmenová funkce :

$$\Phi = \int P\,dx = \int(2xy-y)dx = x^2y - xy + c(y)$$

$$\Phi'_y = Q : \quad x^2 - x + c'(y) = x^2 - x$$

$$c'(y) = 0 \implies c(y) = 0 \implies \underline{\Phi = x^2y - xy}$$

$$Int = \Phi(1,1) - \Phi(0,0) = 1-1-0-0 = \underline{\underline{0}}$$

⑥ a) h ... výška stromu

h_{max} .... výška stromu v dospělosti

$$\frac{dh}{dt} = k\cdot h(h_{max} - h)$$

Rovnice se separ. prom., není lineární

b), c) – viz přednášky

⑦