ŘEŠENÍ UKÁZKOVÉ PÍSEMKY

(1) a) $\lambda^2 - 4\lambda + 4 = 0$

$(\lambda-2)^2 = 0 \Rightarrow \lambda_{1,2} = 2 \Rightarrow y_1 = e^{2x},\ y_2 = x \cdot e^{2x}$

$$\underline{y = C_1 \cdot e^{2x} + C_2 \cdot x\, e^{2x}}$$

b) $y_p = e^{2x} \cdot (ax+b) \cdot x^2$

$y_p = e^{2x}(ax^3 + bx^2)$

(2) $x' = x^2 + x - y$

$y' = 2x - y$

$x^2 + x - y = 0$

$2x - y = 0 \Rightarrow \underline{y = 2x}$

$x^2 + x - 2x = 0$

$x^2 - x = 0$

$x(x-1) = 0$ — $x_1 = 0 \Rightarrow y_1 = 0$; $x_2 = 1 \Rightarrow y_2 = 2$

$\Rightarrow [0,0],\ [1,2]$

$$J = \begin{pmatrix} 2x+1 & -1 \\ 2 & -1 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \Rightarrow \begin{vmatrix} 1-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} = (1-\lambda)\cdot(-1-\lambda) + 2$$

$$= -1 - \lambda + \lambda + \lambda^2 + 2 = \lambda^2 + 1 = 0$$

$\lambda^2 = -1 \Rightarrow \underline{\underline{\lambda = \pm i}}$

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$$J(1,2) = \begin{pmatrix} 3 & -1 \\ 2 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-\lambda) + 2$$

$$= -3 - 3\lambda + \lambda + \lambda^2 + 2$$

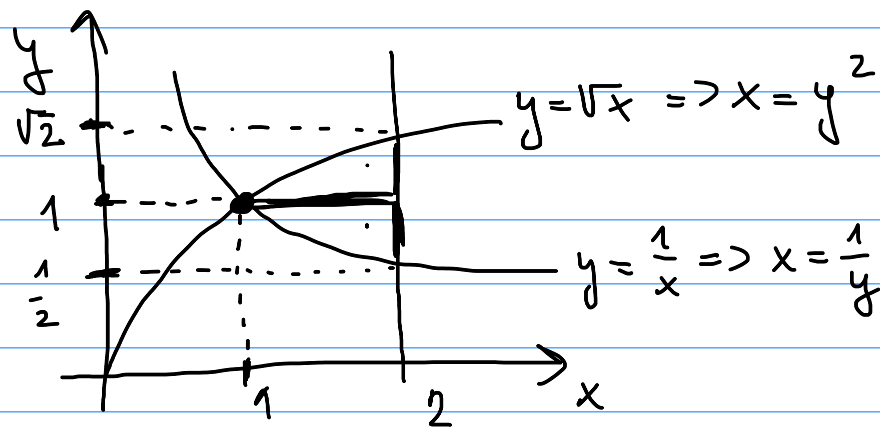
$$= \lambda^2 - 2\lambda - 1 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$1+\sqrt{2} > 0$

$1-\sqrt{2} < 0$ $\Rightarrow$ SEDLO

(3) $\iint\limits_{\Omega} 2xy \, dx dy$



a) $\int_1^2 \left[\int_{\frac{1}{x}}^{\sqrt{x}} 2xy \, dy \right] dx$

$\sqrt{x} = \frac{1}{x}$

$x = \frac{1}{x^2}$

$x^3 = 1 \Rightarrow x = 1$

druhé pořadí integrace:

$$\int_{\frac{1}{2}}^{1} \left[\int_{\frac{1}{y}}^{2} 2xy \, dx \right] dy + \int_{1}^{\sqrt{2}} \left[\int_{y^2}^{2} 2xy \, dx \right] dy$$

b) $\displaystyle\int_1^2 \left[\int_{\frac{1}{x}}^{\sqrt{x}} 2xy\,dy\right]dx = \int_1^2 \left[2x\cdot\frac{y^2}{2}\right]_{\frac{1}{x}}^{\sqrt{x}} dx$

$\displaystyle= \int_1^2 \left[xy^2\right]_{\frac{1}{x}}^{\sqrt{x}} dx = \int_1^2 \left[x(\sqrt{x})^2 - x\cdot\left(\frac{1}{x}\right)^2\right] dx$

$\displaystyle= \int_1^2 \left(x^2 - \frac{1}{x}\right) dx = \left[\frac{x^3}{3} - \ln x\right]_1^2$

$\displaystyle= \frac{8}{3} - \ln 2 - \frac{1}{3} + \underbrace{\ln 1}_{=0} = \underline{\underline{\frac{7}{3} - \ln 2}}$

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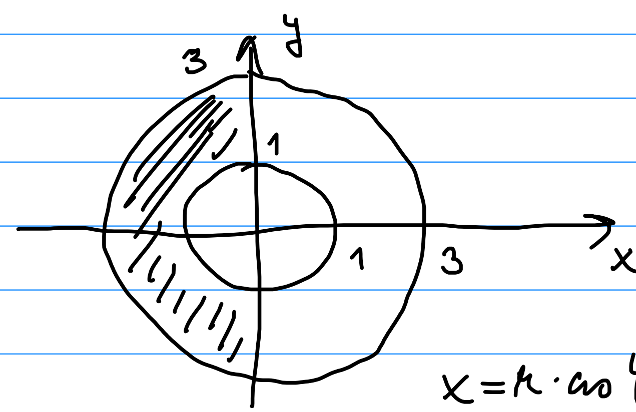
$\displaystyle\iint_\Omega \frac{xy}{\sqrt{x^2+y^2}}\,dx\,dy$



$\frac{\pi}{2} \le \varphi \le \frac{3\pi}{2}$

$1 \le r \le 3$

$x = r\cdot\cos\varphi$

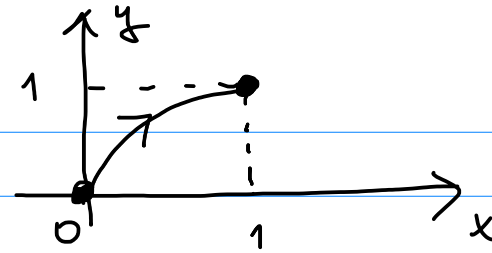
$y = r\cdot\sin\varphi$

$x^2 + y^2 = r^2$

$\displaystyle\Rightarrow \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\int_1^3 \frac{r\cdot\cos\varphi\cdot r\cdot\sin\varphi}{\sqrt{r^2}}\cdot r\,dr\right] d\varphi$

$\displaystyle= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\int_1^3 r^2\cdot\cos\varphi\cdot\sin\varphi\,dr\right] d\varphi$

(5) $$\int_C x\,dx + y\,dy$$



$y = \sqrt{x} = x^{1/2}$, $x \ldots$ parametr, $x \in [0,1]$

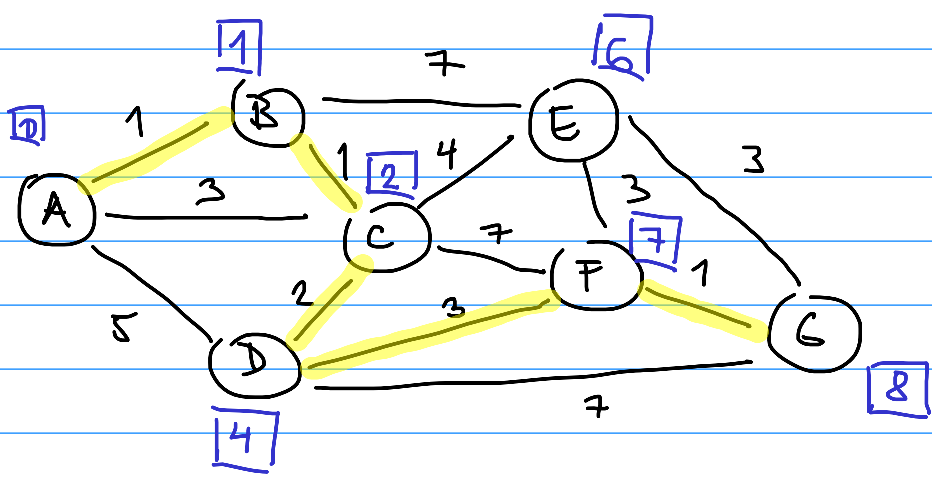
$dy = \frac{1}{2} \cdot x^{-1/2}\,dx$

$$\Rightarrow \int_0^1 x\,\underline{dx} + x^{1/2} \cdot \frac{1}{2} x^{-1/2}\,\underline{dx}$$

$$= \int_0^1 \left(x + \frac{1}{2}\right) dx = \left[\frac{x^2}{2} + \frac{1}{2} x\right]_0^1$$

$$= \frac{1}{2} + \frac{1}{2} - 0 - 0 = \underline{\underline{1}}$$

(7)



(6) Teorie – viz přednášky.