

Bonus 3 - Řešení'

①

Autonomní rovnice:

$$y' = y^2 - 1$$

$$\frac{dx}{dt} = \sqrt{x} - 2x$$

$$\frac{dy}{dt} = y^3 + y - 1$$

②

$$\left. \begin{array}{l} x' = 1 - y \\ y' = y^2 - x^2 \end{array} \right\} \Rightarrow \begin{array}{l} 1 - y = 0 \Rightarrow y = 1 \\ y^2 - x^2 = 0 \Rightarrow x = \pm 1 \end{array}$$

$\Rightarrow$ STAC. BODY: $[1, 1], [-1, 1]$

$$J(x, y) = \begin{pmatrix} 0 & -1 \\ -2x & 2y \end{pmatrix}$$

$$\bullet J(1, 1) = \begin{pmatrix} 0 & -1 \\ -2 & 2 \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & -1 \\ -2 & 2-\lambda \end{vmatrix} = -\lambda(2-\lambda) - 2 = \lambda^2 - 2\lambda - 2 = 0$$
$$\lambda_{1,2} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

$1 + \sqrt{3} > 0, 1 - \sqrt{3} < 0 \Rightarrow$ SEDLO

$$\bullet J(-1, 1) = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & -1 \\ 2 & 2-\lambda \end{vmatrix} = -\lambda(2-\lambda) + 2 = \lambda^2 - 2\lambda + 2 = 0$$
$$\lambda_{1,2} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$\Rightarrow$ NESTABILNÍ OHNISKO